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Form-finding of Tensegrity Structures by using Non-linear Analysis

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Abstract

In the form-finding of tensegrity structures, it is usually difficult to derive the final self-equilibrium configurations as expected. To achieve such goal, we adopt the non-linear analysis to find the self-equilibrium configurations which are as close as possible to the originally given configurations. To deal with the singularity of the resulting tangent stiffness matrix due to being free-standing, we utilize Moore-Penrose pseudo-inverse of the stiffness matrix for the analysis. It is demonstrated by the numerical examples that it is easy to get the self-equilibrium configurations close to original design, thus, it is considered to be a promising method for free-form design of tensegrity structures. Moreover, the method is easy to understand because it is extended from usual structural analysis.

Keywords: Tensegrity, Form-finding, Non-linear Analysis, Moore-Penrose Pseudo-inverse

Introduction

Tensegrity structures are always statically indeterminate, and their members carry prestresses, tension or compression, even when no external load is applied. Therefore, every node or member of a tensegrity structure is always in the state of (self-)equilibrium, and furthermore, its (self-equilibrated) configuration cannot be arbitrarily designed as expected. The process of determination of the self-equilibrated configuration associated with the distribution of prestresses is called form-finding or shape-finding, which is one of the key problems in the design of tensegrity structures.

To tackle the form-finding problem, there have been a number of methods developed so far, see for example the review paper [Juan and Tur (2008)]. In this study, we make use of the concept of non-linear static structural analysis for the purpose of finding the self-equilibrated configuration of a tensegrity structure as close as possible to its initially expected configuration.

Form-finding algorithm

In conventional structural analysis, rigid-body motions of the structure have to be appropriately constrained so that the corresponding (tangent) stiffness matrix is full-rank, and thus, invertible. Instead of computation of inverse of the tangent stiffness matrix in conventional structural analysis, we apply singular value decomposition (SVD) in this study to derive its Moore-Penrose generalized inverse. Because the (rigid-body) motions corresponding to zero stiffness will not occur by applying SVD, the rigid-body motions are then naturally ruled out [Zhang and Ohsaki (2013)].

The tangent stiffness matrix **K** is decomposed as follows by using the technique of SVD

$$\mathbf{K} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^{\mathrm{T}}, \tag{1}$$

where $\mathbf{\Lambda}$ is a diagonal matrix, the diagonal entries Λ_{ii} of which are the singular values of **K**. The Moore-Penrose generalized inverse $\mathbf{K}^{-}$ of **K** is calculated as follows:

$$\mathbf{K}^{-} = \mathbf{\Psi}\mathbf{\Lambda}^{-1}\mathbf{\Psi}^{\mathrm{T}}, \tag{2}$$

where

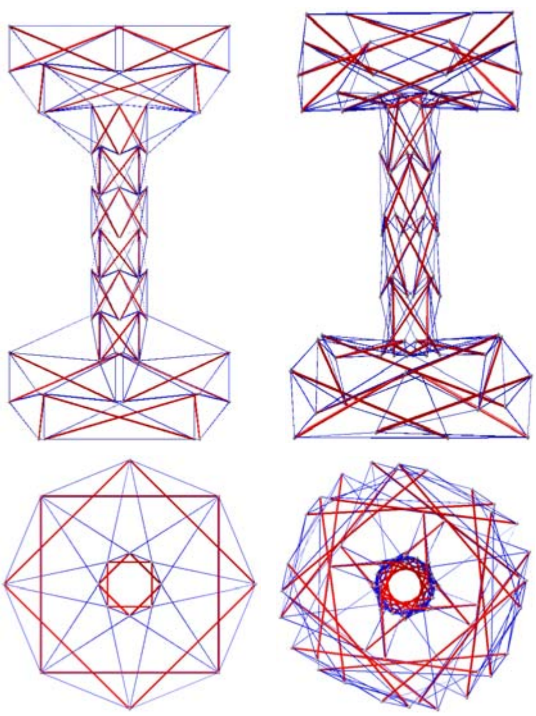
$$\Lambda_{ii}^{-1} = \begin{cases} 1/\Lambda_{ii} & \text{if } \Lambda_{ii} \neq 0, \\ 0 & \text{if } \Lambda_{ii} = 0. \end{cases} \tag{2}$$

Therefore, the nodal displacements **d** subjected to out-of-balance forces **f** can be calculated as

$$\mathbf{d} = \mathbf{K}^{-}\mathbf{f}. \tag{4}$$

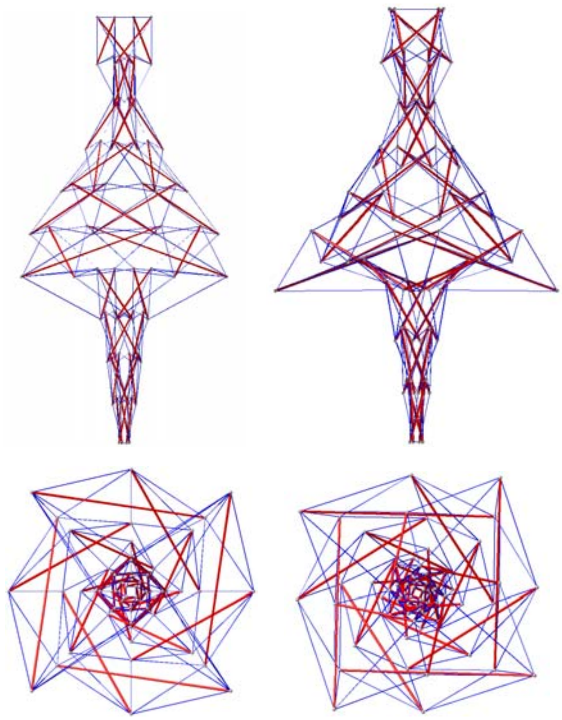
Numerical examples

The initial and final configurations of two numerical examples by using the proposed method are shown in Figures 1 and 2.



(a) Initial (b) Final

Figure 1. Example 1



(a) Initial (b) Final

Figure 2. Example 2

Conclusions

In this study, we have presented an approach for free-form design of tensegrity structures, based on the basic concept in non-linear static analysis. In the proposed method, it is no need to constrain the rigid-body motions of the structure. Variation of the predefined parameters, including the initial configuration, member stiffness and initial prestresses, would certainly lead to different self-equilibrated configurations. This might give users some freedom to get control over the final configurations. However, this process of controlling the final configurations is not direct, and might need trial and error in the design.

References

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