Reliability-Based Study of Well Casing Strength Formulation

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Abstract

The increasing development of materials technology and the consequent rise in complexity of structures, demand a proper knowledge of the safety levels involved in the design. The reliability techniques applied to structural analysis allow dealing with the uncertainties inherent in the design of structures, so that the failure probability can be predicted since the design stage. In this context, the design variables are described as random variables, within the choice of an adequate distribution model to represent data is required. In an oil and gas well scenario, the casing design is a crucial stage of the whole project, representing the major structural elements responsible to maintain the well integrity through its lifetime, allowing adequate production activities. The occurrence of failures in casing systems can lead to irreversible safety problems in well operation. For the evaluation of the strength of the tubes used in casing design, the code API 5C3:1994 is widely applied by several companies overall. This deterministic formulation cannot deal with the uncertainties associated with the tube manufacturing process, as variations in geometrical and mechanical properties. This paper addresses the analysis of casing strength in a reliability-based approach, regarding the failure modes usually verified in well casing design. The reliability analysis is performed by the Monte Carlo simulation and the First/Second Order transformation methods (FORM/SORM). The safety levels associated to the referred formulation are estimated and discussed.

Keywords: OCTG, Well casing design, Structural reliability, Burst, Collapse

Introduction

Casing systems in wells play an important role as the major structural system that keeps the well integrity since the drilling and along its lifetime. The main elements in the casing are the tubulars and connections between them, which have to be designed in order to support the external loadings that they are subjected. In offshore well construction, which includes drilling, casing and completion stages, the casing system may represent around 15% to 20% of the total cost. The search for oil and gas in increasingly higher depths exposes the casing to extreme conditions, including high levels of pressure and temperature, besides chemical attack as, for instance, corrosion due to hydrogen sulfide gas. In this scenario, the adequate balance between cost and safety levels has to be reached in the design, and the structural reliability theory can assist the designer in this decision-making process.

The main loadings experienced by casing in vertical wells are represented by internal pressure, external pressure and axial force. The failure modes associated to internal and external pressure, so-called burst and collapse, respectively, are the governing failure modes in the problem. Tensile or compressive forces occur along the casing, but rarely induce tube failure alone. In the case of directional wells, not assessed in this work, additional torsion and bending effects can stand out. The strength equations of tubulars are posed in the code API 5C3:1994, which is widely applied by oil and gas companies. This formulation refers to a serviceability limit state (SLS) related to the elastic regime. Therefore, the tubes are designed to bear loads up to reaching the steel yielding limit, disregarding its bearing capacity after this point. This is one of the reasons that makes this code seems conservative, underestimating the actual strength of the tube. The API 5C3:1994 normative code suggests the Barlow's equation for burst design strength, whose derivation is done by assuming thin wall hypothesis, which can be inadequate in some tubes commonly adopted in casing design. In terms of external pressure, the code provides four minimum collapse formulas for design strength, developed on the 1960's, being each formula suitable for a distinct D/t interval.

A new version of the code (API 5C3:2008, identical to ISO/TR 10400:2007) introduces the ultimate limit state (ULS) philosophy in casing design and suggests that reliability-based procedures can be performed in order to enhance the strength evaluation. An ultimate limit state equation is usually related with experimental rupture test results, since it tries to estimate the load at which the casing actually fails. Therefore, a proper equation is usually chosen by fitting experimental test data. Although this proposed improvements, the older version from 1994 is still the most adopted in design routines. The new paradigm of ULS associated to probabilistic design is slowly being introduced in the companies by consulting and research activities.

The increasing development of materials and structural modeling, and the consequent growing in complexity of structures, demand a proper knowledge of safety levels involved in the design. The structural reliability theory provides methods to evaluate these safety (or risk) levels, accounting for the uncertainties inherent to the design. In engineering applications, the uncertainties commonly verified relates to mechanical (material) parameters, as Young modulus or tensile strength, and dimensional parameters, as lengths and masses, for instance. It refers directly to non-uniformity on the manufacturing process of structural materials and elements. In the light of probability and statistics concepts, these uncertainties are modeled as random variables, and collected together in a framework of mathematical models that estimate the probability of failure associated to a specific failure mode defined by the user. Fundamentals and applications of the structural reliability theory can be found in Melchers (1999), Ang and Tang (2007), Ditlevsen and Madsen (2005), among others.

Specifically, in oil and gas industry, well casing design is related to steel tubular manufacturing, whose production quality and inspection procedures have been improved significantly in the last decades. The suggested casing strength formulas are dependent on the outside diameter (D) and wall thickness (t) – usually referred by the slenderness ratio D/t – and yield stress. In this case, the old version of API code recommends adopting conservative minimal or nominal values for these parameters and, additionally, in the design process, safety factors are applied to ensure implicitly a tolerable risk level (TRL). On the other hand, if the supplier guarantees accuracy in the manufacture and inspection processes, with lower dispersion levels in the tube performance values, and consequently the TRL can be kept by allowing the structural element to bear a higher load than the one predicted by the standard.

Some works in the literature address the recommended formulas from API 5C3:1994 and discuss its seeming conservative nature, besides proposing ULS formulations. Some works can be found in the literature with distinct ultimate limit state equation suggestions for burst [Klever and Stewart (1998); Klever (2010)] and collapse [Abbassian and Parfitt (1995); Klever and Tamano (2006); Tamano et al. (1985)]. As previously stated, new design codes also proposes to going beyond the elastic limits, enforcing this ultimate limit state philosophy.

The probabilistic analysis of casing design have been studied by some authors, since the 1990's. Adams et al. (1998) present the behavior of failure probabilities for API 5C3 strength in the ULS analysis provided by Tamano et al. (1985), and verify that it gives a wide range of variation for failure probabilities over a D/t range, concluding that this behavior is not desirable and suggesting a new reliability-based method for collapse casing design. Ju et al. (1998) proposes a different formulation. The code API 5C3:2008 itself does a very similar development, but it adopts Klever and Tamano (2006) as the ultimate state limit equation suggesting a more robust probabilistic method for collapse casing design. Burres et al. (1998) propose an interesting discussion, working on the calibration of safety factors in design equations in order to reach a specific TRL.

This paper addresses two kinds of reliability analyses. The first one is done by verifying the probabilities of API 5C3:1994 design strength be exceeded when the maximum deterministic load that it was designed is reached, instead of the failure probability of the casing actually fails if the same load is achieved. It means that this paper is not going to analyze the probability of failure for an ultimate limit state, but it is going to do it for the design equation, which is really used in design procedures. This kind of analysis can be useful to verify the influence of the dispersion of design variables in each design equation, across the D/t range, and to check the probability of this design strength be exceeded, leading the casing tube to transcend the elastic limit. A second analysis is developed by addressing a hypothetical design scenario, including defined loading profiles, for which the probability failures are evaluated along the depth of the well. This application becomes useful when the results are compared to the safety factors adopted by each company for each failure mode in the deterministic design.

The First and Second Order Reliability Methods (FORM/SORM) and the Monte Carlo simulation are used in this paper to perform the probabilistic analysis. The evolution of safety levels implicitly associated to the referred equations across D/t and for distinct grades is investigated, and a performance comparison between these methods is carried out.

This work is divided in three main sections. The first one has a brief review of what is and how works a structural reliability analysis. The second one brings an overview of the recommended practices for casing burst and collapse design made by API 5C3:1994. Finally, in the last section, the concepts are combined and the simulations are presented, discussing the results.

Structural Reliability Analysis

Essentially, a structural reliability analysis needs a limit state function, some random variables and a reliability method. The limit state function must represent the problem which is going to be studied, in general, it gives positive values for safe events and negative values for failure events. The failure modes considered in this paper represent the safety margin of probabilistic API 5C3 casing design strength be exceeded by the deterministic corresponding strength. Equation (**Erro! Fonte de referência não encontrada.**) presents the failure mode adopted $G(\boldsymbol{X})$, where $R(\boldsymbol{X})$ is the resistance term and L is the load term.

$$G(\boldsymbol{X}) = R(\boldsymbol{X}) - L \qquad (1)$$

where $\boldsymbol{X}$ is a vector containing the random variables. As the focus of this paper consists on the analysis of casing strengths, only the resistance term is going to be assumed as probabilistic.

Random variables must estimate the behavior of geometric/mechanical properties and other design variables related to the structural element, that influence the resistance term in limit state function. On the other hand, the load term in limit state function also could have random variables, which could be the self-weight, some external mechanical load or the one caused by temperature variation, for instance. The correlation between random variables also can be attached on reliability-based problems, although the literature states that adopting the variables as independent is a conservative procedure. In the following analysis, random variables are going to be independent, but the correlation between them can be adopted in future works.
The reliability-based method is going to link the limit state function with the random variables to compute a failure probability. For structural reliability the Monte Carlo Method, FORM and SORM are the most known ones [Melchers (1999)]. In the next sections, these methods are briefly detailed.

Monte Carlo Simulation

The method consists in generate n random scenarios to be tested in the limit state function, computing the number of failure events n_f (when $G(\boldsymbol{X}) \leq 0$), and estimate the failure probability by $P_f = n_f/n$.
The random scenarios are defined by generating n aleatory values for each random variable assumed in the analysis. Therefore, the statistical characterization of each variable and a random number generator are required. An illustrative example of a Monte Carlo simulation is presented in Fig. **Erro! Fonte de referência não encontrada.**, in which a thousand events are generated. Each event is tested with the limit state function, where if the resistance (R) is higher than the load (L), there is a safe event, otherwise there is a failure event. In this hypothetical example R and L are Gaussian distributed random variables with means 115.0 and 90.0, and standard deviation equals to 4.0 and 10.0, respectively. It is usual to adopt the notation $R = N(115.0; 4.0)$ and $L = N(90.0; 10.0)$.

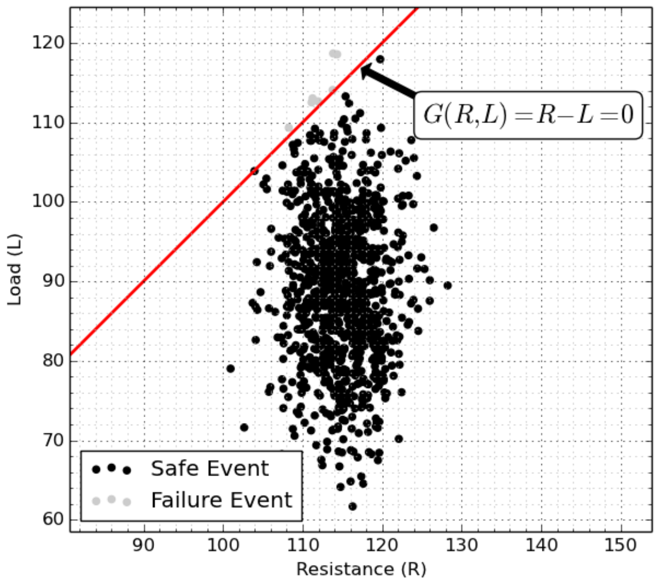

Figure 1. Monte Carlo illustrative example assuming R=N(115.0;4.0) and L=N(90.0;10.0)

By its nature, Monte Carlo provides very accurate results, since an adequate number of simulations is performed. However, this method may have issues with very low failure probabilities once it will need, at least, the inverse of the failure probability number of scenarios to possibly be capable to detect one failure event, i.e., if the problem has a probability of failure equals to 10^{-6}, a minimum of 10^{-6} scenarios has to be generated and simulated. It has to be regarded that the estimated value

P_f is also a random variable, and some expressions are proposed to define a minimum number of scenarios which leads to a desirable covariance of P_f. The computational cost in Monte Carlo simulations may increases substantially in some applications, being noted as a possible disadvantage. Some strategies of random data generation can be applied in order to improve performance in this method as, for instance, stratified sampling and other importance sampling procedures.

First/Second Order Reliability Methods

A reliability analysis problem can be mathematically expressed considering the limit state function ($G(\boldsymbol{X}) = 0$) and the adopted random variables $\boldsymbol{X}$, being P_f exactly evaluated by the integral:

$$\boldsymbol{P_f = P(G(X) \leq 0) = \int_{G(X)\leq 0} f_X(x)dx} \tag{2}$$

in which $\boldsymbol{f_X(x)}$ is the joint probability density function of the random variables $\boldsymbol{X}$. However, depending on the number of random variables, this integral is not easy to solve and numerical approximations should be applied, where Monte Carlo simulation is an option. Transformation methods as the First Order Reliability Method (FORM), which one is analytically derived and iteratively solved, stand out as an interesting choice. The method consists in transforming all random variables ($\boldsymbol{X}$) in its corresponding standardized normally distributed ones ($\boldsymbol{U}$), this is done by first applying a normal tail approximation and then reducing them to standard normal probability distribution function. It is also necessary to rewrite the limit state function for this standard normal space ($G(\boldsymbol{U}) = 0$). In this new space, the probability of failure concept can be associated with the shortest distance between the new adopted limit state function and the transformed random variables space origin. This distance is known as the reliability index β and its relation with P_f is provided by:

$$\boldsymbol{P_f} = \Phi(-\beta) \tag{3}$$

The reliability analysis is posed as a nonlinear optimization problem, in which one wants to minimize the distance β subject to the constraint function $G(\boldsymbol{U}) = 0$. The point $\boldsymbol{U}^*$ in which this condition is the most probable failure point, the so-called design point. Thus, the reliability index corresponds to the norm of the position vector of this point, i.e., $\beta = \|\boldsymbol{U}^*\|$. The random variables transformation is made as suggested by Hasofer and Lind (1974). The limit state function is approximated by a first order Taylor series at the current search point from the iterative optimization problem. The optimization problem can be expressed as follows:

$$\beta = \min(\|\boldsymbol{U}\|)\,, constrained\ to\ G(\boldsymbol{U}) = 0 \tag{4}$$

The algorithm HLRF (Hassofer, Lind, Rackwitz and Fiessler) is classically employed to the optimization problem solution. In general, in few iterations (less than 10) the convergence is reached. An advantage of this method is that it can be solved faster than Monte Carlo simulation, regarding a good level of accuracy, in many applications. Moreover, if the limit state function is linear on the random variables, these ones presenting Gaussian distribution, FORM results are exact. Figure 2 illustrates the procedure.

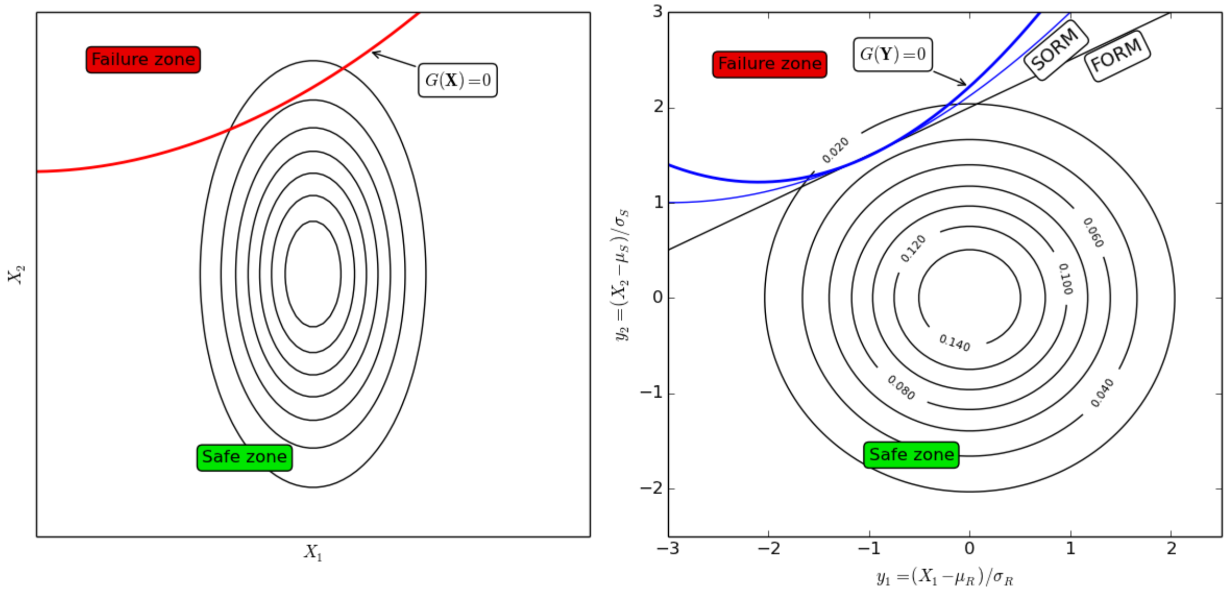

Figure 2. FORM illustrative Scheme: original and standard normal spaces

Another information extracted from FORM is the importance factor of each random variable, for the achieved failure probability. This information is associated with the position vector $\boldsymbol{U}^*$ and the partial derivatives $G(\boldsymbol{U})$ at this point. The importance factor give the influence of the random variable in the aleatory process.

It should be noted that, in the case of correlation between random variables, additional steps are necessary. In order to calculate the equivalent correlation coefficient for each pair of variables in the transformed normal space, the procedure proposed in Nataf (1962) can be applied. Moreover, this correlation has to be eliminated, so that the final transformation into standardized normal independent variables $\boldsymbol{U}$ can be performed. These procedures are detailed in Melchers (1999).

In some cases, in which the variables are tightly correlated, or present non-Gaussian distributions, or when the limit state function is strongly nonlinear, the use of a second order approximation of the limit state function can improve the accuracy of results obtained in the transformation method. This give rises to SORM (Second Order Reliability Method). This approximation demands more information over the limit state function, as its curvatures. The final approximation consists in a parabolic equation centered on the design point. In this work, the Breitung approximation is adopted [Breitung (1984)]. More details can be seen in Melchers (1999).

Casing Strength Formulation

The recommended practices for casing well design described by API 5C3:1994 are summarized in this section, focusing the axial, burst and collapse strengths.

Axial Strength

The axial strength that corresponds to a stress equal to the minimum yield strength, given as follows:

$$R_t = 0.7854(D^2 - d^2)Y_p \tag{5}$$

where:

R_t pipe body yield

Y_p material minimum yield strength
D specified outside diameter
d specified inside diameter

The axial force is the result of the balance between self-weight of the pipe and the pressure caused by the drilling fluid and other fluids from formation. As stated before in this text, it does not configure a governing failure mode by itself.

Burst Strength

The internal pressure that leads to a stress, on the inside wall, equals to the minimum yield strength. The failure mode associated is a brittle rupture of the tube. The equation is based on the Barlow's equation, suitable to thin wall tubes:

$$P_i = 0.875 \left(\frac{2\,Y_p\,t}{D}\right) \tag{6}$$

Where D is the outside diameter, t is wall thickness and Y_p the minimum yield strength of the steel. The reduction factor 0.875 refers to a tolerance of -12.5% in the wall thickness. This value is the allowable limit due non-uniformity in manufacture process, and is preconized by the code API 5CT:2010. This is one the reasons why the equation seems to be conservative.

Collapse Strength

When a pipe collapses due to external load, it changes the geometry to elliptical or other non-circular shape. It brings structural problems associated to loss of rigidity and local instability in the tubes, besides operational issues as blocking of passage of equipment into the tube. The external load is usually caused by pore-pressure, pressure from the drilling fluid, or fluid expansion due to temperature gradient. According to API 5C3:1994, four distinct casing slenderness D/t domains compose the collapse design strength. Yield strength collapse pressure formula (Eq. (7)) provides the load that generates minimum yield stress Y_p on the inside wall of the tube. This formula is achieved by means of Lamé's classical equation.

$$R_{cy} = 2Y_p \left[\frac{(D/t)-1}{(D/t)^2}\right] \tag{7}$$

Average plastic collapse pressure formula (Eq. (8)) was derived empirically from several collapse tests for casing tube grades K55, N80 and P110. This is the usual nomenclature for the steel which the casing tube has been made, where the first letter refers to its tensile strength and the following digits refers to its minimum yield stress. The data used by API 5C3:1994 authors was taken from a report made by a Workgroup composed by members from manufacturers and members from API itself. Collapse tests data were fitted separately for each grade, and then, constants A and B were empirically determined to generalize an average plastic collapse pressure formula. To obtain the minimum plastic collapse pressure formula a constant pressure for a particular grade, a constant C is subtracted from the average expression. This constant C is a tolerance limit and represents the conception that there is a 95% probability or confidence level that the collapse pressure will exceed the minimum stated with no more than 0.5% failures.

$$R_{cp} = Y_p \left[\frac{A}{(D/t)} - B\right] - C \tag{8}$$

Transition collapse pressure formula (Eq. (9)) overcomes an anomaly that happens between minimum plastic collapse formula and minimum elastic collapse formula: they do not intersect across the D/t range. Thus, this formula has been developed intersecting the D/t value where the average plastic collapse pressure formula gives a collapse pressure of zero and is tangent to the minimum elastic collapse pressure.

$$R_{ct} = Y_p \left[\frac{F}{(D/t)} - G\right] \tag{9}$$

Finally, the minimum elastic collapse pressure formula (Eq. (10)) was derived from theoretical elastic collapse pressure formula, resulting in the equation:

$$R_{ce} = \frac{46{,}95 \times 10^6}{(D/t) \times ((D/t) - 1)^2} \tag{10}$$

The D/t limits are the ones which define the collapse domains. They are determined by the intersection of the collapse pressure formulas described above and are shown below

$$(D/t)_{Yp} = \frac{\sqrt{(A-2)^2 + 8\left[B + \left(\frac{C}{Y_p}\right)\right]} + (A-2)}{2\left[B + \left(\frac{C}{Y_p}\right)\right]} \tag{11}$$

$$(D/t)_{PT} = \frac{Y_p(A-F)}{C + Y_p(B-G)} \tag{12}$$

$$(D/t)_{TE} = \frac{2 + B/A}{3B/A} \tag{13}$$

These D/t limits are dependent only on the yield stress (in psi) and must be calculated for each steel grade. Once they are determined, it is necessary to verify the casing design collapse domain by its thickness D/t. If casing D/t is lower than$(D/t)_{Yp}$, yield collapse pressure formula must be applied. If casing D/t is higher than $(D/t)_{Yp}$, but lower than $(D/t)_{PT}$, minimum plastic collapse pressure formula must applied. If casing D/t is higher than $(D/t)_{PT}$, but lower than $(D/t)_{TE}$, transition collapse pressure formula must applied. If casing D/t is higher than $(D/t)_{TE}$, minimum elastic collapse pressure formula must be applied. The coefficients A, B, C, F and G are shown below:

$$A = 2.8762 + 0.10679 \text{ x } 10^{-5}\, Y_p + 0.21301 \text{ x } 10^{-10}\, {Y_p}^2 - 0.53132 \text{ x } 10^{-16}\, {Y_p}^3$$
$$B = 0.026233 + 0.50609 \text{ x } 10^{-6}\, Y_p$$
$$C = -465.93 + 0.030867\, Y_p - 0.10483 \text{ x } 10^{-7}\, {Y_p}^2 + 0.36989 \text{ x } 10^{-13}\, {Y_p}^3$$
$$F = \frac{46.95 \text{ x } 10^6 \left[\frac{3(B/A)}{2+(B/A)}\right]^3}{Y_p \left[\frac{3(B/A)}{2+(B/A)} - (B/A)\right]\left[1 - \frac{3(B/A)}{2+(B/A)}\right]^2}$$

$$G = \frac{FB}{A}$$

The collapse resistance of casing in the presence of an axial stress is calculated by modifying the yield stress to an axial stress equivalent grade according to:

$$Y_{pa} = \left[\sqrt{1 - 0.75\left(S_a/Y_p\right)^2} - 0.5\left(S_a/Y_p\right)\right] Y_p \tag{14}$$

where:
S_a axial stress (pounds per square inch)
Y_p minimum yield strength of the pipe

Y_{pa} yield strength of axial stress equivalent grade, pounds per square inch.

Analysis and Results

In order to associate the concepts presented in previous sections, some reliability analyses are performed as follows. The transformation methods FORM and SORM, besides crude Monte Carlo simulation are applied. The set of random variables contains the yield strength Y_p, the outer diameter D and the wall thickness t, and its statistical parameters are taken from the code ISO 10400:2007. The referred statistical database compiles several manufacturing production data, between 1977-2004, being representative of different manufacturing technologies and quality levels. The statistical parameters are evaluated by using the coefficients shown in Table 1, for the three steel grades adopted, K55, N80 and P110. The grades represents that the steel used has a minimum yield strength of 55000 psi, 80000 psi and 110000 psi, respectively. In this table, *mean* is equal to the actual mean value divided by the nominal value, and *COV* is the standard deviation divided by the actual mean value. According to the reference, the variables are normally distributed.

The tolerable failure probability values are not an unanimity over the scientifical/technical community in structural engineering in general. It depends on the class of the structure, the failure cost, among others. The implication of human lives and environmental risks are also determinant aspects on the definition of a required safety level. Recommendations on some normative codes just begin to appear, e.g., the ones based on JCSS (Joint Committee on Structural Safety) suggestions. In well design industry, it consists in a subject of relatively incipient discussion. In the present text, probabilities of failure higher than 10^{-3} are considered unallowable, based on technical literature for applications in engineering.

Table 1. Statistical coefficients used to characterize the random variables

<table>
<tr><th colspan="2">Y_p</th><th colspan="2">D</th><th colspan="2">t</th></tr>
<tr><td>mean</td><td>COV</td><td>mean</td><td>COV</td><td>mean</td><td>COV</td></tr>
<tr><td colspan="2">K55</td><td rowspan="6">1.0059</td><td rowspan="6">0.00181</td><td rowspan="6">1.0069</td><td rowspan="6">0.0259</td></tr>
<tr><td>1.23</td><td>0.0719</td></tr>
<tr><td colspan="2">N80</td></tr>
<tr><td>1.21</td><td>0.0511</td></tr>
<tr><td colspan="2">P110</td></tr>
<tr><td>1.09</td><td>0.0377</td></tr>
</table>

Collapse and Burst Design Formulation Analyses

In that follows, both for collapse and burst, the reliability evaluation uses a limit state function that assumes the strength as probabilistic and the load as deterministic. The probabilistic strength formula is obtained by adopting the variables as random in the API 5C3 design equation. The deterministic load is set equals to the value of API 5C3 design strength, calculated on the nominal values. It means that the failure probability achieved represents the probability of the design strength be exceeded if there is a load equal to the minimum strength currently used in design.
Thus, the limit state function can be written as:

$$G(Y_p, D, t) = R_{conf}(Y_p, D, t) - L_{det} \tag{15}$$

It has to be noted that the correction factor 0.875 is not used in the term $R_{conf}(Y_p, D, t)$ in burst analysis. The variability of wall thickness is consistently treated here, by assuming it as a random variable. Figure 3 presents the results for collapse reliability analysis.

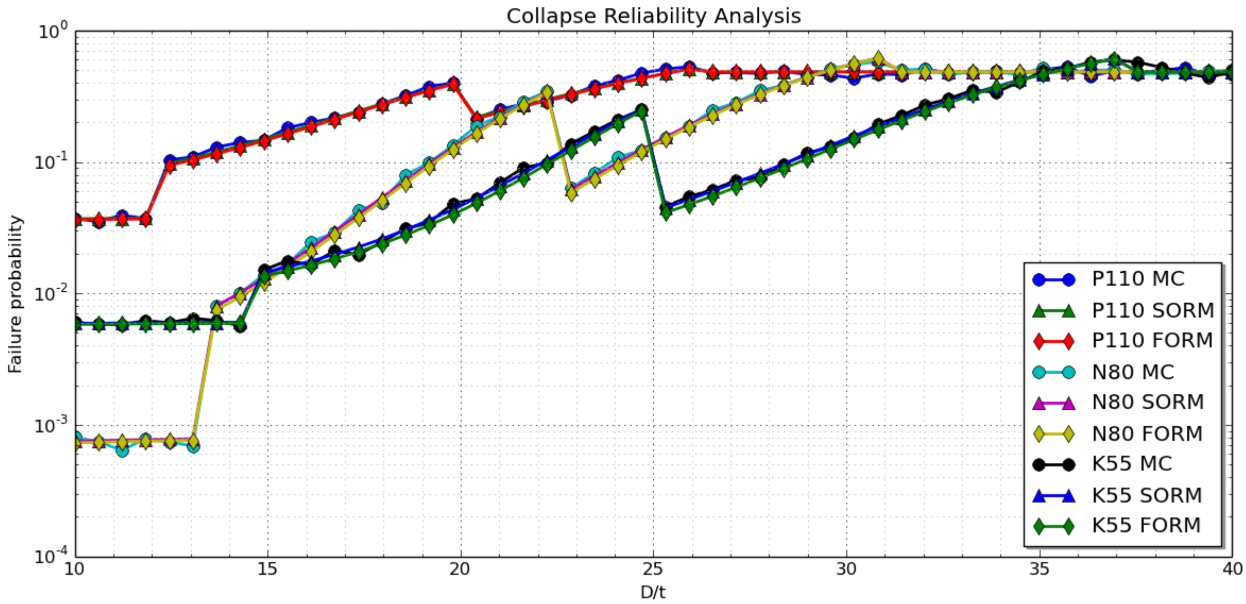

Figure 3. Collapse failure probabilities over D/t range

The four collapse domains described in the previous section present different failure probabilities over D/t range, besides the theoretical formulas give nearly constant failure probabilities (yield and elastic collapse pressure formulas) and the empirical formulas give variable failure probabilities (plastic and transition collapse pressure formulas). For the three grades it is noticed that the probability of failure grows as the casing tube D/t increases. This kind of behavior is not desirable in a design formulation, since it is expected constant safety level for all casing tubes designed by the same normative code. This is one of the main arguments posed by the committee which worked on the new version of the code (API 5C3:2008), proposing only one ULS formulation, and encouraging to use probabilistic analysis in design.

The analysis of burst formula is presented in Fig. 4.

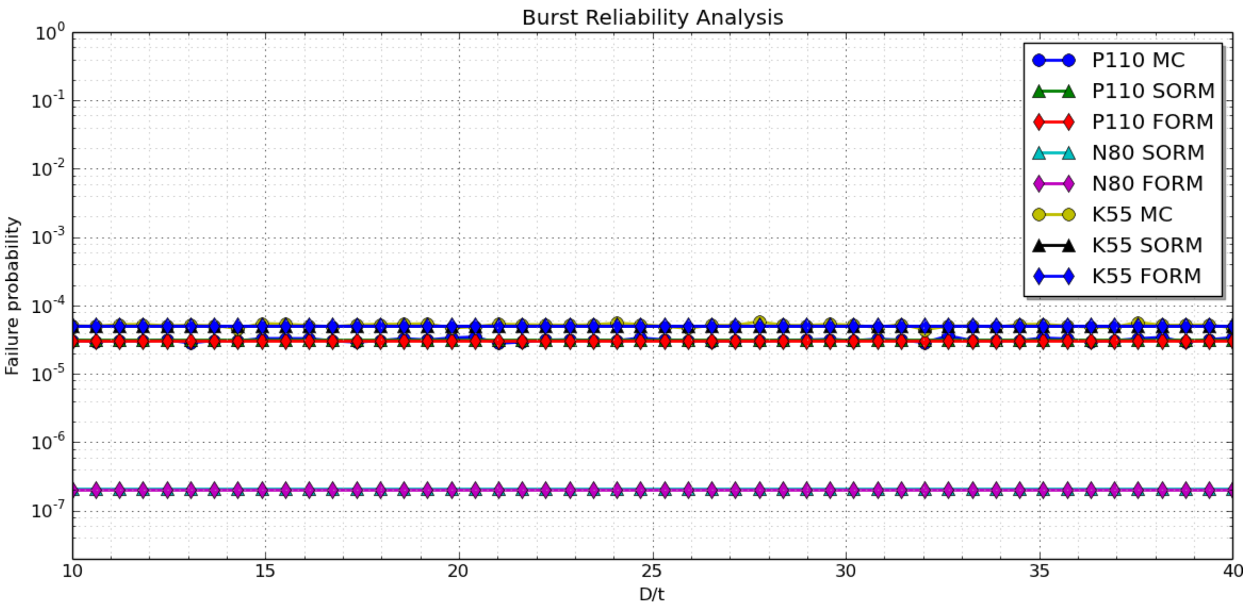

Figure 4. Burst failure probabilities over D/t range

In this case, the failure probabilities have a constant behavior over the D/t range. However, the failure probabilities are not constant when the casing grade varies. It is noticed that grade N80 has a much lower failure probability then the other two grades. This behavior may occur because *mean* and *COV* of Y_p vary with grade. However, the design formulation should have been calibrated to achieve a target reliability level considering the statistical data from production.

It should be noted that FORM results agree with SORM and Monte Carlo in both collapse and burst reliability analysis. The nature of these equations and its smooth nonlinearity contributes to this fact. It is possible that, in strongly nonlinear limit state functions, SORM provides quite different results. The maximum relative error observed between P_f values obtained by FORM and SORM is around XX% for collapse and XX% for burst. Monte Carlo is not compared with a numerical measurement error due its intrinsic random results, although the graphical visualization demonstrates a good agreement between Monte Carlo and the other reliability methods.

The importance factors obtained by FORM for collapse analysis are shown in the following Fig. 5.

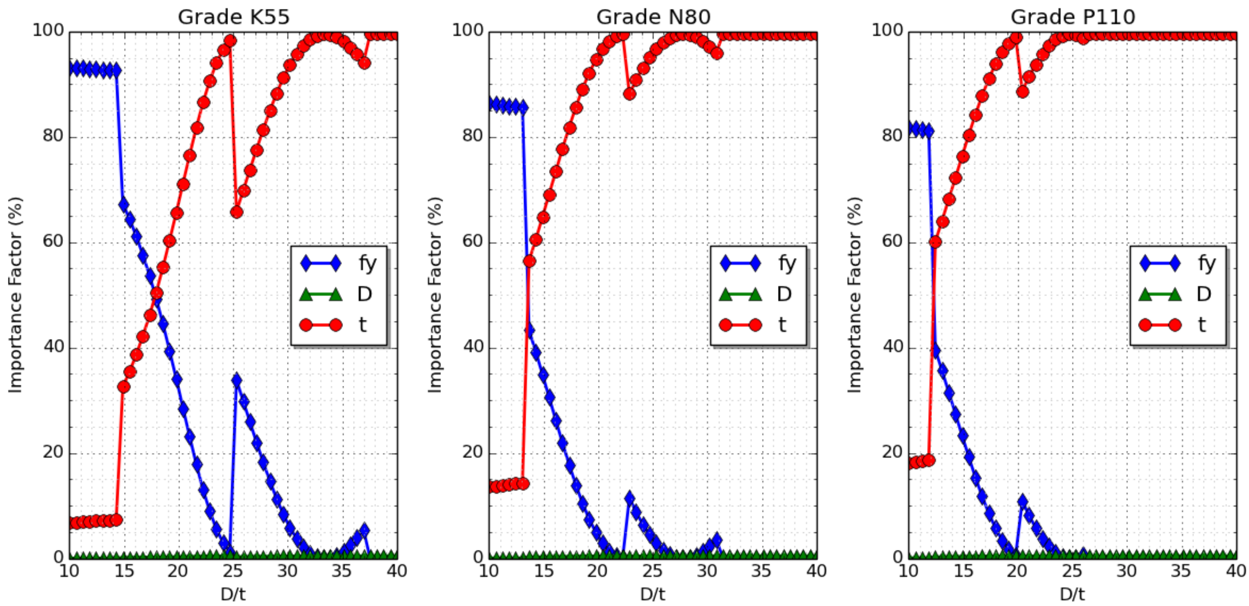

Figure 5. Importance factors over D/t range for collapse achieved probabilities of failure

For all grades, the most influent random variable in the process is the wall thickness, except for thick casing tubes in which the yield stress governs the probabilistic behavior. On the other hand, the diameter has a negligible influence in all these results. It means that considering it as deterministic will not affect significantly the failure probability values. These importance factors results are mainly impacted by the formulation used and by the adopted dispersion for each random variable, as it can be noticed in Table 1 that the diameter has the lowest *COV*.

FORM burst analysis provides the importance factors shown in Fig. 6. The yield stress is the most influent random variable in the achieved failure probability, followed by the wall thickness in all grades. Observing Table 1 and Fig. 6 it can be seen that the higher *COV* gives the higher importance factor for K55 grade. The others grades respect the following order. Once again, external diameter is the less important random variable, meaning that its dispersion is very small.

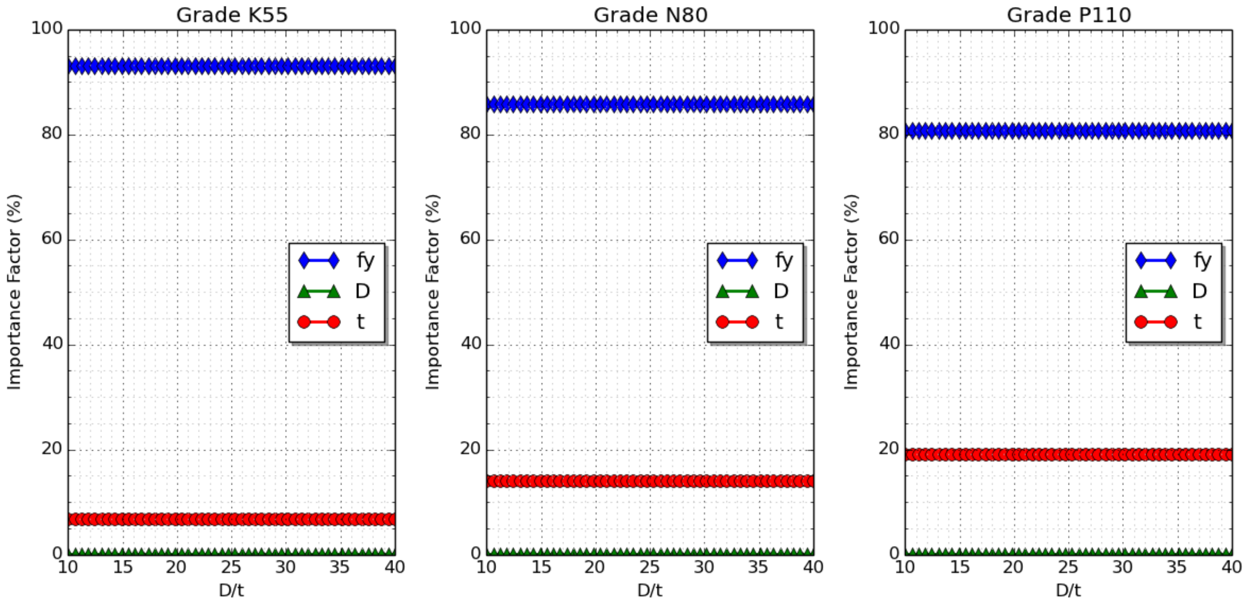

Figure 6. Importance factors over D/t range for burst achieved probabilities of failure

Analysis of an Extreme Design Scenario: Kick

In a casing design routine, the tubes are designed for different loading conditions throughout the well depth. Depending on the depth and the geomechanical conditions, extreme scenarios may occur along the drilling, casing, completion and production stages. These kind of scenarios has to be simulated in the well design. A *kick* situation is defined when a gas invades the drilling column, increasing drastically the expected internal pressure levels on the casing system, leading to possible accidents as a blow-out. For design purposes, it is considered that the last 2/3 of well depth are occupied by gas.

It is assumed a drilling of a 5700 m depth well, under 2000 m of water. The calculations of each pressure term are neglected, for sake of conciseness. The loading profile is shown in Fig. 7, in which is defined the differential pressure over the depth, resulting on a burst (internal pressure) failure mode overall. For this analysis is assumed a 10 3/4 in 85.3 lb/ft tube, which is widely applied in surface and intermediate casing structures. It has outer diameter of 10.75 in and wall thickness equals to 0.797 in.

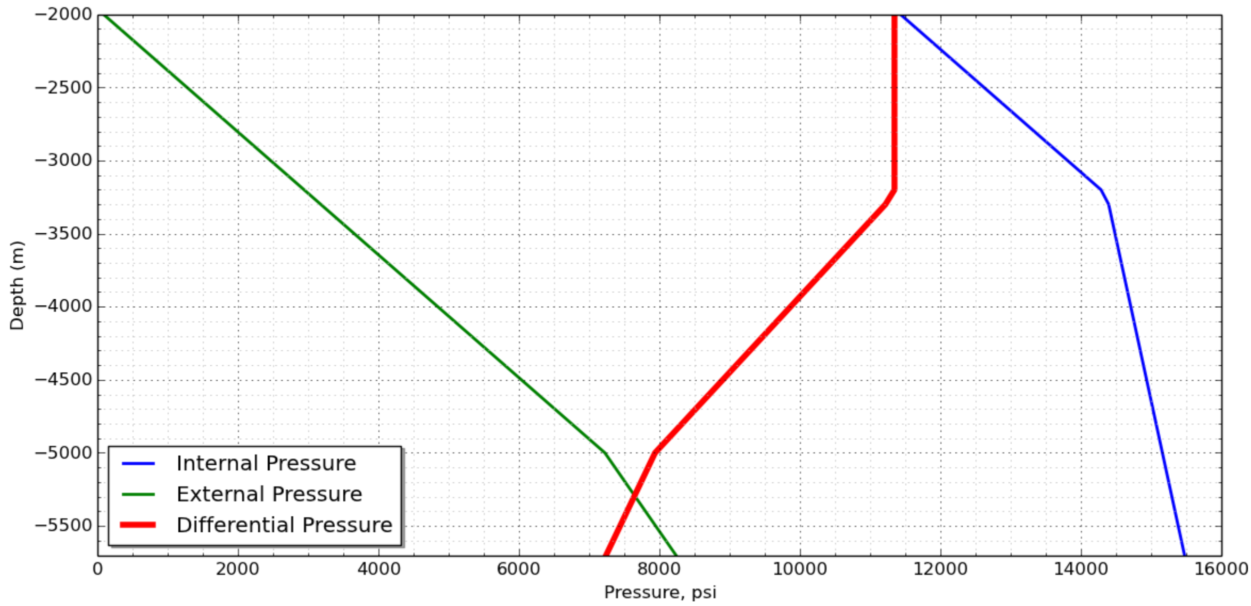

Figure 7. Loading Profile on the kick scenario

Regarding the reliability analysis, the limit state function adopted has the format:

$$G(Y_p, D, t) = R_{conf}(Y_p, D, t) - L_{det} \quad (16)$$

in which the load term is evaluated along the well depth according to the presented loading profile. The reliability evaluations are done every 100 m. The results are shown in Fig. 8, in which the failure probability values are log scaled.

As expected, the tube made of grade K55 reaches higher failure probability values. The severe values of pressure from 2000 m up to 4000 m leads to unallowable levels of P_f for this tube, showing its inadequacy for this scenario.

Considering that the differential pressure is constant up to around 3223 m, from which is considered the fluid inflow, the failure probabilities remain unchanged. From this point on, until 5700 m, it is observed a quasi-linear decrease of P_f values, referring to the reduction of applied pressure, due to the low specific weight of the invading fluid.

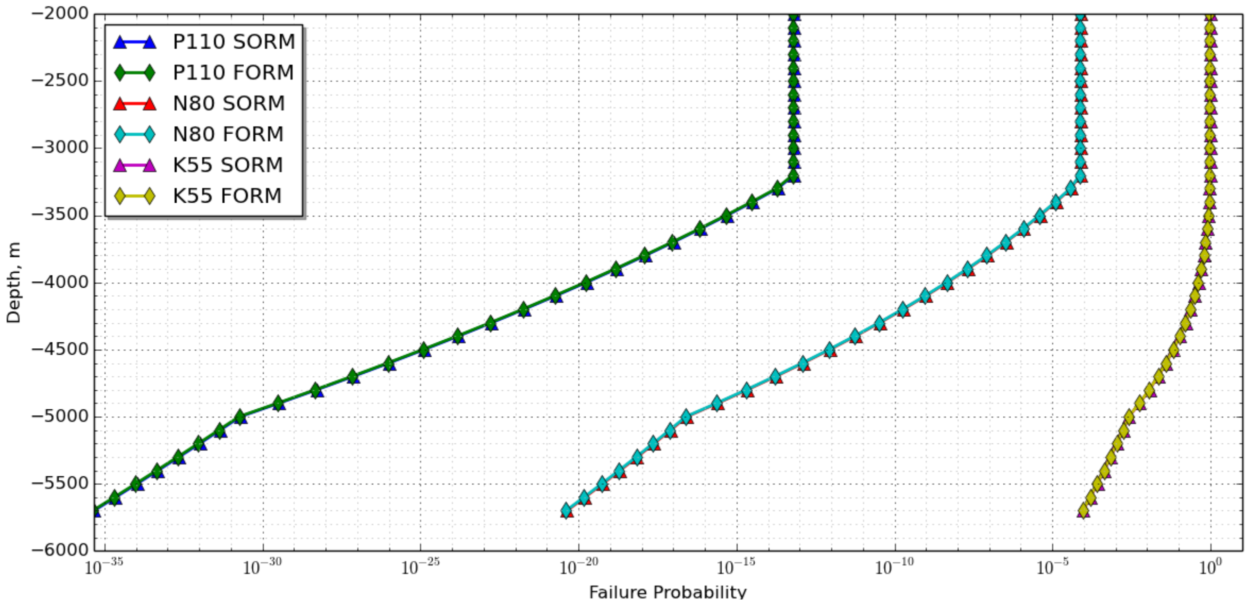

Figure 8. Evolution of failure probability along the depth

This kind of analysis allows to compare the P_f values in any point along the well depth with the safety factors usually employed in casing design. This discussion is not developed here, considering that these factors are defined by each oil company.

Conclusions

The classical casing collapse and burst strength equations are revisited in the light of a probabilistic approach. Moreover, the reliability analysis is applied to the verification of an extreme event kick scenario.

Regarding the analysis on the collapse design formulation, it is noticed that there are some high failure probability values associated to the collapse design strength, when a deterministic load equals to the minimum casing strength is considered. Moreover, the non-uniformity of the safety levels across the slenderness D/t is not a proper behavior, for structural design purposes. The importance factors values indicates that wall thickness is the most influent random variable in the achieved failure probability.

For burst analysis, moderate failure probability values are verified. The importance factors values indicates that the material yield limit is the most influent random variable in the achieved failure probability, followed by the wall thickness. The supposed conservative nature of the burst equation, posed by several authors in the literature, is apparently verified here.

The application of a probabilistic evaluation in the casing design practice can be done by procedures such as the scenario analysis presented. It brings robustness to the analysis, and assess the designer in decision-taking processes aiming both investment savings in simple wells and feasibility in complex wells.

In this context, the need of detailed analysis both on casing design formulation and about the non-deterministic nature of strength parameters stands out. The standardization codes and oil/gas companies are interested in these issues since the last two decades, and some scientific and technical publications has been developed. Some effort has also to be done in order to consider combined load cases in a probabilistic approach, focusing on the stochastic behavior of environmental load scenario. Reliability-based analysis also proves to be useful for industry and designers as a device to identifying aspects in which the manufacturing process has to be improved in its accuracy and quality inspection.

It should be remarked that the results presented in this paper are only indicatives of the probabilistic behavior associated to the design formulations studied. The P_f values themselves have to be interpreted with caution, as they reflect the behavior of a specific statistical database, provided by the code ISO 10400:2007.

This research group is engaged in probabilistic analysis of combined failure modes for well casing by both SLS and ULS approaches. A graphical user interface have been developed in order to disseminate the reliability analysis practice among casing designers.

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References

Abbassian, F. and Parfitt, S. (1995) Collapse and post collapse behaviour of tubulars: a simple approach. In *Proceedings of SPE Production Operations Symposium*.

Adams, A., Warren, A. and Masson, P. (1998) On the development of reliability-based design rules for casing collapse. In *Proceedings of SPE Applied Technology Workshop on Risk Based Design of Well Casing and Tubing*.

American Petroleum Institute – API (1994) API Bull 5C3: Bulletin on formulas and calculations for casing, tubing, drill pipe and line pipe properties. Bulletin.

American Petroleum Institute – API (2008) API TR 5C3: Technical report on equations and calculations for casing, tubing, and line pipe used as casing or tubing; and performance properties tables for casing and tubing. Technical report.

American Petroleum Institute – API (2010) API 5CT: Specification for casing and tubing.

Ang, A. H-S. and Tang, W. H. (2007) *Probability concepts in engineering: emphasis on applications to civil and environmental engineering*, John Wiley & Sons.

Breitung, K. (1984) Asymptotic approximations for multinormal integrals, *J. of Eng. Mech. Div.* **110**, 357–366.

Burres, C. V., Tallin, A. G. and Cernocky, E. P. (1998) Determination of casing and tubing burst and collapse design factors to achieve target levels of risk, including influence of mill source. In *Proceedings of SPE Applied Technology Workshop on Risk Based Design of Well Casing and Tubing*.

Ditlevsen, O. and Madsen, H. O. (2005) *Structural Reliability Methods*, Technical University of Denmark.

Hasofer, A. and Lind, N. (1974) An exact and invariant first order reliability format. *J. Eng. Mech. Div*. **100**, 111–121.

International Organization for Standardization – ISO (2007) ISO TR 10400: Petroleum, petrochemical and natural gas industries - Equations and calculations for the properties of casing, tubing, drill pipe and line pipe used as casing or tubing. Technical report.

Gouveia, L. P. (2014) *Avaliação da Confiabilidade de Tubos de Revestimento de Poços*. Federal University of Alagoas, Master's Thesis. *(in portuguese)*

Klever, F. (2010) Formulas for rupture, necking, and wrinkling of oil country tubular goods under combined loads, *SPE Journal* **15**, 834–855.

Klever, F. and Stewart, G. (1998) Analytical burst strength prediction of OCTG with and without defects. In *Proceedings of SPE Applied Technology Workshop on Risk Based Design of Well Casing and Tubing*.

Klever, F. and Tamano, T. (2006) A new OCTG strength equation for collapse under combined loads. *SPE Drilling and Completion* **21**, 164–179.

Melchers, R. E. (1999) *Structural reliability analysis and prediction*, Civil Engineering Series, John Wiley and Sons.

Nataf, M.A. (1962) Détermination des distributions de probabilités dont les marges sont données. *Comptes Rendus de l'Académie des Sciences* **225**, 42–43. *(in French)*

Scientific Computing and Visualization Laboratory – LCCV (2013) Development of failure criteria for well casing design based on risk analysis. Research Report.

Tamano, T., Mimaki, T. and Yanaimoto, S. (1985) A new empirical formula for collapse resistance of commercial casing. *Nippon Steel Tech. Report* **26**, 19–26.