The inadequacy of elastic properties from tensile tests for Lamb wave analysis

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Abstract

Lamb wave analyses have been conducted on metallic and composite structures. The dispersion characteristics for these plate structures are evaluated using the Rayleigh-Lamb equations and an assumption that the material properties are known. In most studies, the elastic moduli used are determined using standard tensile tests. These properties have been used to adequately model the dispersive behavior of Lamb wave in metallic plates. It has been shown that the use of elastic properties from tensile tests is able to model the dispersive Lamb wave behavior. However, this may not be the case for plates made from woven fibre reinforced plastics. The effects of material properties on the propagation of Lamb wave in fibre reinforced composite plates are significant. In most of these investigations, the elastic properties used are obtained from tensile tests. It will be shown that the use of such properties will lead to significant inaccuracy in the determination of the dispersion characteristics of a composite plate.

Keywords: Lamb wave, elastic properties of composite materials

Introduction

A modelling capability for propagating Lamb waves has always been desirable to gain understanding, reduce costly experimentation and perform analyses not possible experimentally. A key factor to accurate modelling of Lamb wave propagation is correctly defining the properties of the propagating medium. Much of the previous literature was focused on aluminium plates. In all these studies, the elastic properties used in structural analysis are also used in modelling Lamb wave propagation. This assumption was valid because such aluminium is a homogenous and isotropic material. Its elastic property does not exhibit significant strain rate effects.

Recently the use of advanced composite structures, in particular carbon fibre reinforced polymers (CFRP) have become commonplace and as a result accurate modelling in composite plates is now desirable. Unlike aluminum, the elastic properties of the fabricated CFRP plates are affected by the processing conditions and fibre compaction. The advances in the computational and experimental structural analyses have led to the use CFRP component in significant load bearing structures. Copious studies have reported good agreement between these analyses that suggest the validity of the elastic properties used.

Naik (1994) reported that stiffnesses of woven composite panels are governed by weave parameters such as weave architecture, yarn sizes, yarn spacing and yarn crimp [1]. The concept of the repeating unit cell (RUC) was used to help define the elastic properties of woven composites [2]. However, in the work by West [3], they showed that the axial stiffness of woven composite is not dependent on the crimp angle.

Lamb waves are low amplitude, high frequency elastic waves travelling on a medium. The length-scale of Lamb wave in the propagating direction is large compared with the microstructures of metallic plates. However, in this case of woven composite plates, the length-scale of the propagating Lamb wave may be comparable to that of its RUC. To this end, this paper will present a set of results that will highlight the apparent shortcoming in using elastic properties that are determined for structural analyses for analyzing Lamb wave propagation in a woven composite plate.

In this study, a set of results obtained with an aluminium plate will first be presented. The computational analyses of the Lamb wave propagation were conducted using published elastic
properties commonly used in structural analyses. The aim of this part of the work is to validate the computational capability and the experimental scanning laser vibrometry equipment. A quasi-isotropic test plate was fabricated from woven composite material. A series of tensile tests was performed to determine the elastic modulus of the test plate. These results were compared with theoretically derived elastic moduli using the manufacturer has published material properties. These properties were subsequently used to model the propagating Lamb wave on this woven composite plate. These computational results were compared with the experimental test data. This concerted work highlights the need for a new approach to establish a set of elastic properties of woven composite plates.

**Laser vibrometry**

The dispersion curves of the test plates were acquired using a laser vibrometry facility described in detail in [4, 5]. The PZT bonded to the plate is made to actuate Lamb waves by a 50V peak-peak drive signal from a Krohn-Hite model 7602 amplifier. The out-of-plane displacements from the Lamb waves are detected on the plate surface by a Polytec OFV 505 laser vibrometer. The positioning of the laser vibrometer relative to the plate is controlled by a stepper motor driven high resolution X-Y table set to 5000 steps per mm.

**Metallic plate**

An aluminium plate measuring 300x300x4mm had a Ferroperm Pz27 PZT Ø10mm x 1mm disc placed 75mm from the bottom edge as shown in Figure 1. Also shown is “line 1” which will be used to record the Lamb waves propagating across the plate by LV. Line 1 measured 210mm long and spans from the top edge of the plate to the edge of the PZT disc. During the laser vibrometry scan 256 points were collected along this line. A frequency sweep was conducted at each point using Hanning windowed sinusoids with centre frequencies ranging from 100 kHz to 1 MHz in 100 kHz intervals. At 200 kHz the window resulted in a 5 cycle sinusoid, the window was kept constant in time such that the energy at each frequency would be similar.

The data along line 1 was then transformed using a 2D FFT at each frequency and summated to reveal the full spectrum of Lamb modes shown in Figure 2. The theoretical Lamb wave modes are overlaid on the 2D FFT contour plot in the form of dispersion curves and demonstrate a good agreement. These curves were predicted using DISPERSE with the material properties summarized in Table 1. The properties are from the material library supplied by DISPERSE and the values common across engineering applications (such as FEA software) and literature. The excellent agreement between the theoretical and experimental dispersion curves show:

a. The use of elastic properties commonly used in structural analyses of aluminium structures can also be used in modelling Lamb wave propagation in aluminium plates.

b. The veracity and efficacy of the scanning laser vibrometry test facilities.

![Figure 1 Aluminium plate with PZT and scanned line.](image-url)
Figure 2 2D FFT on aluminium plate showing agreement with mathematical modelling.

Table 1 Aluminium material properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>2700</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3375</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>70.758</td>
</tr>
</tbody>
</table>

Composite Plate

A composite specimen was fabricated using Hexcel M18 prepreg with woven carbon G939 fabric. The quasi-isotropic test plate used is fabricated from 16 bi-axial plies in the layup [0/90, +/-45, +/-45, 0/90, 0/90, +/-45, +/-45, 0/90]. Upon curing, the plate was trimmed to a planar dimension of 400mm x 400mm. A Ferroperm Pz27 Ø10mm x 1mm PZT disc was placed at the centre of the plate as shown in Figure 3. The dimensions and the mass of the test plate were measured and the material density is calculated to be 1434 kg/m³.

A series of uniaxial test specimens were cut from the fabricated composite plate as shown in Figure 4a to measure the stiffness in the 0-degree direction. A unidirectional strain gauge was bonded at the centre of each of the coupon aligned with the tensile direction. An Instron tensile machine equipped with a 2kN load cell was used as shown in Figure 4b. Stress across the strain gauge was calculated by dividing the load by the cross sectional area of the test specimen. The stress-strain curves from the 4 test specimens are shown in Figure 5. The effective moduli of the test specimens were calculated from these results are shown in Table 2. The average modulus was found to be 43.9 GPa.

Using the in-plane modulus provided by Hexcel and estimates for the remaining values (Table 3), the effective modulus of the test plate was calculated using classical laminate theory. This method returned an effective modulus of 46.2 GPa which agreed well with the experimental value of 43.9 GPa, which corresponds to a 5% variation.

The material properties in Table 3 were used to calculate the dispersion curves in DISPERSE. Figure 6 shows the comparison between the experimentally measured dispersion curve compared with those calculated using DISPERSE. The measured wavenumbers of majority of all modes are higher than the theoretical values. Curiously, there is a region where there is apparent agreement between the theoretical and the experimental dispersion characteristics (see Figure 6). The deviation of the experimental and theoretical dispersion curves are greater than 5%.
Figure 3 Photograph of specimen showing scan lines.

400 mm x 400 mm plate

Test coupons

Figure 4 a) Test coupons cut from plate. b) Test coupon in tension loading.

Figure 5 Stress-strain curve using strain gauges.
Table 2 Effective tension moduli.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Laminate theory using initial properties</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T1-4 Average</th>
</tr>
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<tbody>
<tr>
<td>Effective modulus (GPa)</td>
<td>46.2</td>
<td>45.7</td>
<td>44.3</td>
<td>43.0</td>
<td>42.7</td>
<td>43.9</td>
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</table>

Table 3 Published and estimated material properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$ = $E_{33}$ (GPa)</td>
<td>67</td>
</tr>
<tr>
<td>$E_{22}$ (GPa)</td>
<td>8.6</td>
</tr>
<tr>
<td>$G_{12}$ = $G_{13}$ = $G_{23}$ (GPa)</td>
<td>5</td>
</tr>
<tr>
<td>$\nu_{12}$ = $\nu_{13}$ = $\nu_{32}$</td>
<td>0.35</td>
</tr>
<tr>
<td>Ply thickness (mm)</td>
<td>0.269</td>
</tr>
</tbody>
</table>

Figure 6 Preliminary reference 2D FFT showing modes in close proximity.

Discussions

The results presented above can be summarized as follows:

a. Excellent agreement between the theoretical and experimental dispersion curves can be obtained when a metallic plate. The efficacy of the laser vibrometry test facility is verified.

b. The use of elastic properties as defined by tensile tests is sufficient for modelling the dispersive characteristics of Lamb wave.

c. The elastic modulus of a composite specimen determined from a tensile test agreed well with the theoretical modulus calculated using manufacturer’s data.

This discussion establishes a possible explanation for the disagreement between the measured and theoretical dispersion curves obtained for the composite panel. According to Naik [1] elastic stiffnesses of woven composite panels are governed by weave parameters such as weave architecture, yarn sizes, yarn spacing and yarn crimp. The concept of the repeating unit cell (RUC) was used to help define the elastic properties of woven composites [2]. The definition of the RUC shows that the need to consider the length-scales when defining the elastic property of woven composite. Figure 7 shows the weave pattern of the test plate used in this paper. The width of the
tow is approximately 1.9 mm. Figure 7 outlines the concept of the RUC for the test plate used in this paper. The important length-scales associated with this RUC are shown in Figure 7.

The measured wavelength of the Lamb wave modes ranges from 3 mm to 25 mm. Whilst these are small compared with the thickness of the plates, it is evident that they are comparable to the length-scale of the RUC as defined in Figure 7. Therefore, it is expected that the perturbations associated with the propagating Lamb wave mode will have length scales that are comparable to the RUC. In this respect, one can expect the yarn crimp angle to have an effect on the elastic property of the woven composite. To this end, one can expect a reduction in the effective elastic modulus that will bring about a reduction in the group velocity of the Lamb wave modes. This is consistent with the results shown in Figure 6. However, the region where modes are in close proximity (see Figure 6) show that merely altering the elastic moduli by trial-and-error is not sufficient. A systematic approach based on the dispersive characteristics of the propagating Lamb wave mode including the higher order modes is required to establish a set of elastic properties that will allow for an accurate representation of the dispersion curves of woven composites.

Conclusions

The efficacy of the non-contact laser vibrometry for studying the propagation of Lamb wave propagation in plate-like structures was first demonstrated. In the experiments where an aluminium plate was used, there was an excellent agreement between the theoretical and experimental dispersion curves. The elastic constants for aluminium used in the theoretical analyses were obtained from standard tensile tests.

Similar investigations were repeated on a composite plate incorporating a satin weave carbon fibre fabric. Firstly, the results from a series of tensile test confirmed that the in-plane elastic constant of were consistent with manufacturer’s data. The elastic properties were then used to determine the dispersion curves of Lamb waves propagating in the test specimen. The agreement between the theoretical and experimentally measured Lamb modes was poor.

An explanation for this disagreement is attributed to the length-scale of the RUC of the woven fabric of the CFRP. It was found that the wavelength of the propagation Lamb wave is comparable with the length-scale of the RUC. In this respect, one expects the elastic constant to be less than that determined from standard tensile tests which is consistent with the higher wave number measured. These results show a need for an appropriate test method for determining the elastic constants that is governed by the dispersion relationship of the propagating Lamb waves.
Acknowledgements

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References:


