Stochastic analysis of a radial-inflow turbine in the presence of parametric uncertainties

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Abstract

This paper presents an uncertainty quantification study of the performance analysis of the high pressure ratio single stage radial-inflow turbine used in the Sundstrand Power Systems T-100 Multi-purpose Small Power Unit. A deterministic 3D volume-averaged Computational Fluid Dynamics (CFD) solver is coupled with a non-statistical generalized Polynomial Chaos (gPC) representation based on a pseudo-spectral projection method. One of the advantages of this approach is that it does not require any modification of the CFD code for the propagation of random disturbances in the aerodynamic and geometric fields. The stochastic results highlight the importance of the blade thickness and trailing edge tip radius on the total-to-static efficiency of the turbine compared to the angular velocity and trailing edge tip length. From a theoretical point of view, the use of the gPC representation on an arbitrary grid also allows the investigation of the sensitivity of the blade thickness profiles on the turbine efficiency. The gPC approach is also applied to coupled random parameters. The results show that the most influential coupled random variables are trailing edge tip radius coupled with the angular velocity.

Keywords: Uncertainty Quantification, gPC, CFD, Radial Turbine

Introduction

In order to develop robust turbines' design, it is critical to consider uncertainties in the optimization process. Turbine design is nowadays mainly achieved using Computational Fluid Dynamics (CFD) technique. However, the design is made at the optimal operating conditions and any likely variations in operations can dramatically alter the efficiency of the turbine [Odabaee et al. (2014); Sauret and Gu (2014)]. The concern is even greater while working with high-density fluids in low-temperature Organic Rankine Cycles (ORC) which have the potential to extract more energy from the high-density fluids. Due to the complexity of the geometry and computational cost associated with these geometries and fluids, no advanced uncertainty quantification (UQ) has been proposed so far for these ORC turbines and very little work has been done on the uncertainty quantification in turbomachinery in general. Relatively recently, Gopinathrao et al. [Gopinathrao et al. (2009)] and Loeven and Bijl [Loeven and Bijl (2010)] successfully performed non-intrusive Polynomial Chaos and Probabilistic Collocation respectively on a transonic axial compressor but no study has been found on radial turbines.

UQ is a mathematical approach employed to determine the likely certain outcomes in an uncertain system. Any engineering system is subject to uncertainties, which can come from the random variation of geometric parameters and operating conditions for example. These uncertainties cannot be removed from the system and are called "aleatory" uncertainties [Faragher (2004)]. In addition, the numerical representation of this system also introduces uncertainties through the mathematical models and boundary conditions used. These "epistemic" uncertainties [Faragher (2004)], however, can be reduced as they are due to modelling errors. In the numerical simulations, all these parameters are constants, which make it impossible to isolate the influence of these parameters without using uncertainty quantification methods.

So far, different approaches have been developed in order to quantify these uncertainties. Monte Carlo (MC) technique is a typical approach employed to perform probabilistic analysis. However, it is suffering from expensive computational cost and poor convergence rates, especially for complex geometries [Sankaran and Marsden (2011)]. To overcome such issue, other approaches have been developed, such as so-called Polynomial Chaos (PC) method based on the spectral stochastic finite elements approach [Spanos and Ghanem (1989); Ghanem and Spanos (1991)], generalized Polynomial Chaos (gPC) as extended by Xiu [Xiu et al. (2002)], stochastic collocation method [Mathelin and Hussaini (2003)], and Multi-Element generalized Polynomial Chaos (ME-gPC) method [Wan and Karniadakis (2005); Kewlani and Iagnemma (2009)]. Non-intrusive methods are also becoming more popular as they don't require the modification of the deterministic solver. A comparison between intrusive and non-intrusive methods is presented by Onorato et al. [Onorato et al. (2010)] while non-intrusive approaches are detailed in [Loeven et al. (2007)].

In this work, a generalized Polynomial Chaos (gPC) method is coupled to the deterministic CFD solver and applied to the performance analysis of a radial-inflow turbine. A uniform distribution of the random parameters associated with Legendre polynomials is chosen. Due to the curse of dimensionality the uncertain parameters are investigated separately with high-order spectral projections while the combined effect of the parameters is initially investigated using low-order polynomials. The impact of the variable input parameters are evaluated on the total-to-static efficiency of the radial turbine. The stochastic space of each random variable is correlated to the range of uncertainty of the physical input parameters. The sensitivity to the uncertain parameters and their potential coupled effects on the stochastic turbine efficiency are discussed in details.

Generalized Polynomial Chaos Method

In this study, the generalized Polynomial Chaos (gPC) framework proposed by Spanos and Ghanem [Spanos and Ghanem (1989); Ghanem and Spanos (1991)] is used.
The gPC representation of a random process u is defined as:

$$u(x,\Theta) = \sum_{\alpha \in \mathrm{N}^{\mathrm{N}}} \hat{u}_\alpha(x)\phi_\alpha(\Theta) \tag{1}$$

where $\Theta = \{\Theta_i(\omega)\}_{i=1}^{N}$, $N \in \mathbb{N}^N$, is a $\mathbb{R}^N$ valued random array on a probability space (Ω, $\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{P}}$) with probability distribution $P_\Theta(d\theta)$ and $d\theta$ is the Lebesgue measure. $\phi_\alpha(\Theta)$ is the multivariate orthogonal polynomials, with total degree not greater than P. They are built as tensor products of orthogonal polynomials along each random dimension with respect to the probability measure $P_\Theta(d\theta)$. The modal coefficients in Eq. (1) are determined by:

$$\hat{u}_\alpha(x) = \mathrm{E}\{u(x,\Theta)\phi_\alpha(\Theta)\} / \mathrm{E}\{\phi_\alpha^2(\Theta)\} \quad \text{for } \alpha \in \mathbb{N}^N \tag{2}$$

where E represents the expectation. The order P of the polynomial basis is chosen based on accuracy requirements.
The modal coefficients can be re-written as:

$$\hat{u}_j(x) = \frac{1}{\mathrm{E}\{\phi_\alpha^2(\Theta)\}} \sum_{i=0}^{Nq} w_i u(x,\Theta)\phi_j(\Theta) \tag{3}$$

where the weights w_i and nodes Θ of the Gauss-Legendre quadrature are determined by solving an eigenvalue problem based on the Golub-Welsch algorithm.
$N_q=(N_d+1)^{Nd}-1$ is the number of cubature points, with Nd, the number of random parameters.

Interpolated gPC

As demonstrated by Sauret et al. [Sauret et al. (2014)] interpolated gPC can provide useful approximations of the gPC approach. The method uses the existing deterministic solutions as an arbitrary grid on which preferably high-order interpolations are performed to carry out the stochastic projection. This is of particular interest for the blade thickness profiles evaluation for which re-creating the profiles for each quadrature point is extremely time consuming. Thus, this approach is used here despite the reduced accuracy as a preliminary estimation of the sensitivity of the turbine performance to the blade thickness profiles.

Statistical Post-Processing of the gPC Method

Once the modal coefficients are determined using Eq. (1), the statistical properties of the random parameters are obtained thanks to the orthogonality of the polynomial basis. The mean μ and the variance σ^2 are thus obtained by:

$$\mu = \hat{u}_0 \tag{4}$$

$$\sigma^2 = \sum_{j=1}^{M} \hat{u}_j^2(x) E\left\{\phi_\alpha^2\right\} \tag{5}$$

Then the standard deviation, $\sigma=\sqrt{(\sigma^2)}$ and the coefficient of variation, CoV= σ/μ are obtained from Eqs. (3) and (4).

gPC-CFD coupling

The gPC method used in this study is non-intrusive and thus doesn't require any modification of the deterministic solver. The gPC method is implemented in Matlab and automatically coupled with the CFD solver *ANSYS-CFX* using Python scripting. The CFD results are then sent back into Matlab for the statistical post-processing.

Radial-Inflow Turbine

The radial-inflow turbine used in this work has been developed by Sundstrand and experimentally tested by Jones [Jones (1996)]. This geometry has become an open benchmark after the work of Sauret [Sauret (2012)] who reconstructed the geometry and provided initial CFD results.
The test case at nominal conditions is a 120 *kW*, 5.7 pressure ratio turbine used in the Sundstrand Power Systems T-100 Multi-purpose Small Power Unit. However, only the rig conditions have been experimentally tested which have a lower rotational speed and lower inlet pressure but the same pressure ratio. The rig conditions are used in this study for validation and application of the gPC method. The geometry is presented below in Figure 1 and the full details are presented in Sauret [Sauret (2012)]. The turbine has 19 stator blades and 16 rotor blades.

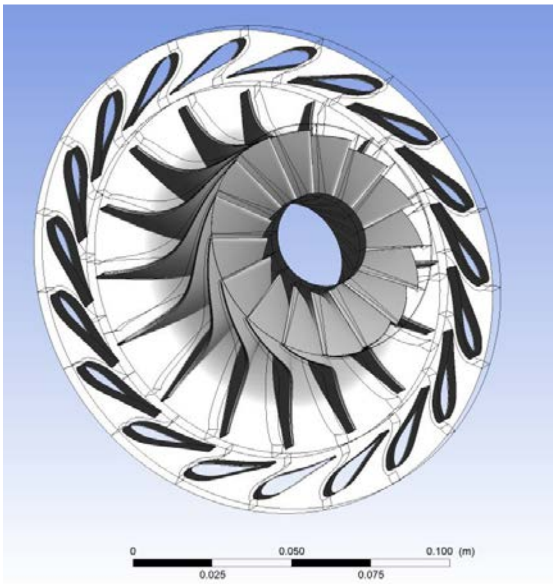

Figure 1. Rotor and Stator geometry of the radial-inflow turbine

Deterministic volume-averaged CFD solver

Three-dimensional geometry and mesh of one blade passage including stator, rotor and part of diffuser are reproduced in *ANSYS* turbomachinery package. Reynolds-Averaged Navier-Stokes equations are solved in this simulation using *ANSYS-CFX version 15*. The k-ω SST turbulence model is used for the simulations and high resolution schemes are used for both the advection and turbulence as recommended by [Louda et al. (2013)].

Boundary Conditions

For the rig condition, the temperature of inlet of stator is 477.6 *K* and the outlet static pressure is 72.4 kPa. The mass flow rate at the inlet of the stator is Qm = 0.0173 kg/s. The working fluid is air, considered as ideal gas. The rotational speed is 71700 RPM. [Sauret (2012)].

Mesh

The mesh is generated using *ANSYS-TurboGrid* for the flow passage for both rotor and stator. The non-dimensional grid spacing at the wall y_w^+ ranges from 20 to 140, which is the recommended range as the log-law wall function is valid for y_w^+ values above 15 and under 100 for machine Reynolds number of 1×10^5 where the transition affects the boundary layer formation and skin friction and up to 500 for Reynolds number of 2×10^6 when the boundary layer is mainly turbulent throughout [Manual (2000)]. The boundary layer refinement control is 4×10^6 with Near Wall Element Size Specification to reach the y_w^+ (non-dimensional wall element size) requirement for the k-ω SST turbulence model.
After a grid refinement study, the total mesh number is 712,082 including stator, rotor and part of diffuser. The grid quality was checked using indicators such as orthogonality of the cells and aspect ratios. The converged mesh is presented in Figure 2.
All of the computations were performed until full convergences of the flow variables were achieved. The residuals were dropped down below 10^{-6}.

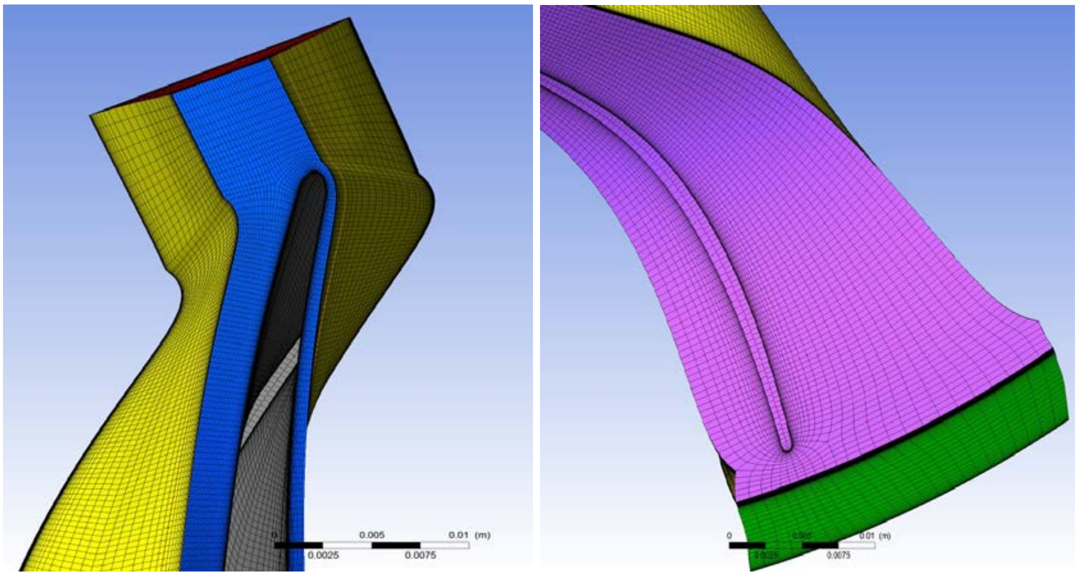

Figure 2. Three-dimensional view of the O–H grid at the rotor blade at the hub and shroud.

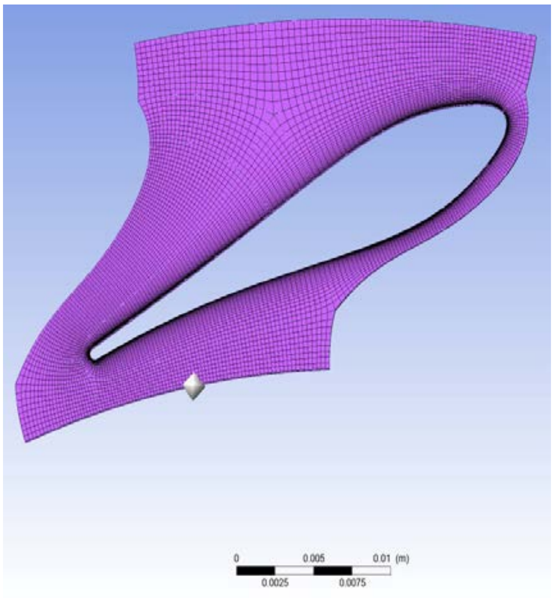

Figure 3. Two-dimensional view of the O–H grid at the stator blade.

Results

Validation

From Figure 4, three-dimensional CFD total-to-static efficiency is compared against the experimental data for the rig conditions. The results are in really good agreement with the experiments with a maximum difference less than 1%.

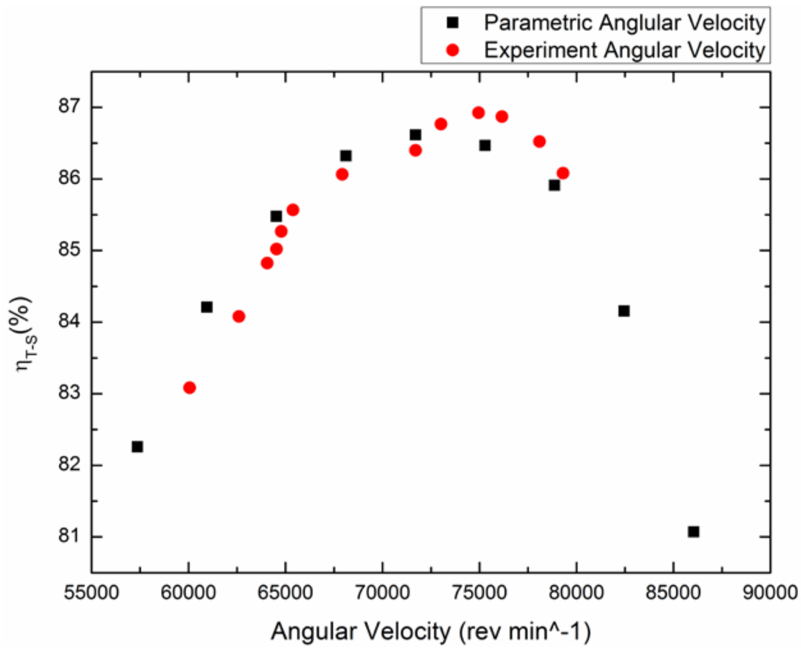

Figure 4. Variation of total-to-static efficiency with rotational speed.

Parametric Study

Four main parameters, angular velocity, TE (Trailing Edge) Tip Length, TE Tip Radius and blade thickness profiles (Table 1) have been initially investigated in order to identify a non-linear response surface on which the gPC method will be valuable to apply.

Table 1. Characteristics of the studied uncertain parameters

Uncertain Parameter	µ	Support
Angular Velocity ω (rev.min^{-1})	71700	[57360, 86040]
TE Tip Length L (mm)	35.0012	[33.1, 42]
TE Tip Radius R (mm)	36.83	[31.1, 37]
Blade Thickness peak position along the meridional length (%)	41	[21, 71]

The TE Tip Length and TE Tip Radius are defined in Figures 5 and 6. The red point "A" in Figures 5 and 6 is the geometry changing point, corresponding to the TE position at the shroud. The arrows' direction is the geometry changing direction. When "A" point is moving in horizontal direction, the TE tip length will vary. It is important to note that when "A" point is moving in the vertical direction (TE Tip Radius), the blade height will be modified and the tip clearance will be kept at the initial value.

In Figure 7, six different rotor blade thickness profiles have been manually established for the parametric study. The maximum value of the blade thickness is kept constant while its location is moved along the tip length, thus modifying the profile curve shape.

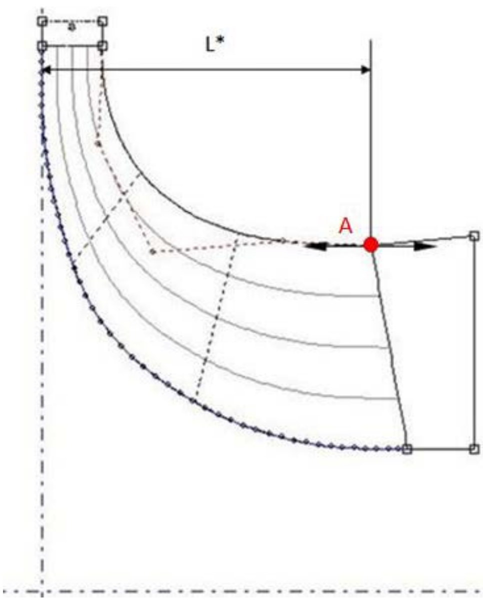

Figure 5. TE Length geometric study.

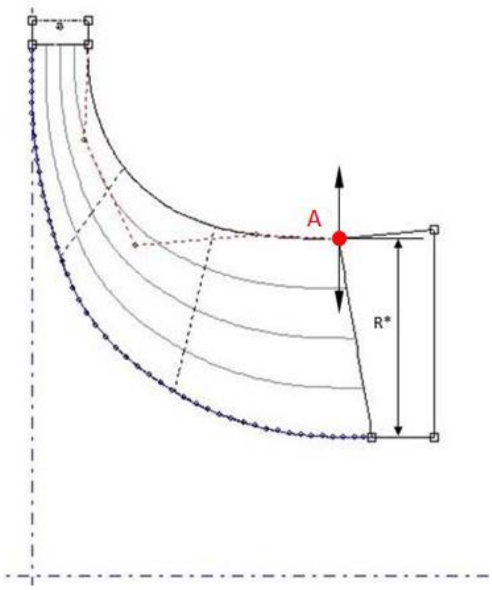

Figure 6. TE Tip Radius geometric study.

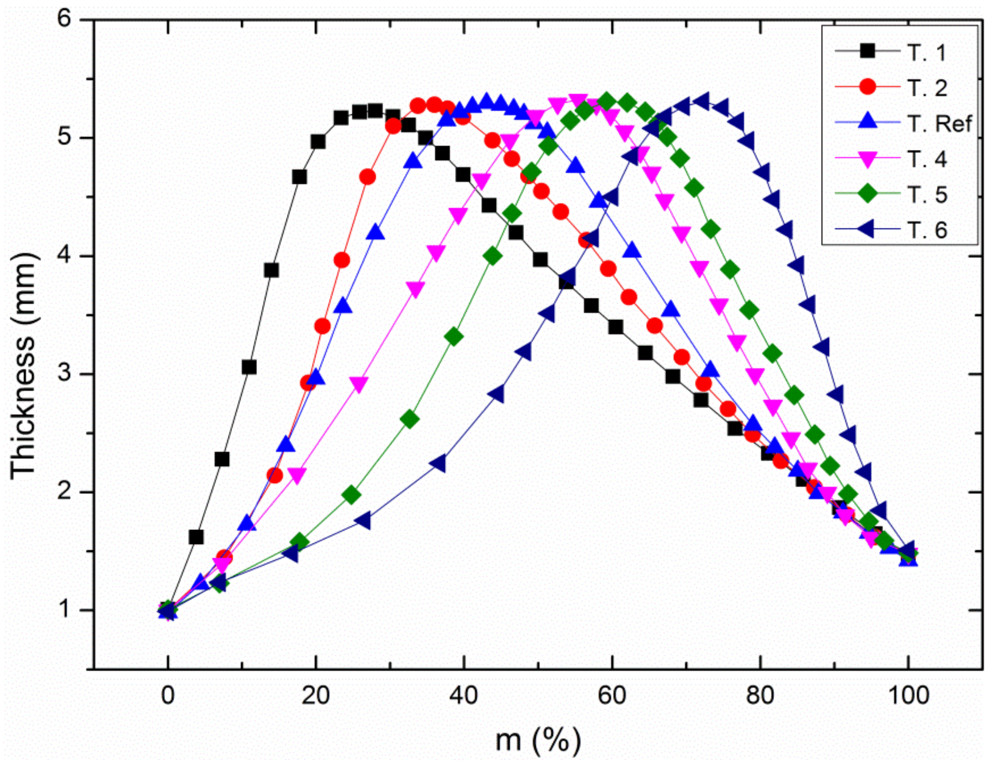

Figure 7. Blade thickness profile geometric study.

As shown in Figures 8-11, angular velocity, TE tip radius, TE tip length and blade thickness have non-linear response surfaces. One can also note that the maximum efficiency is obtained at values of the TE tip radius, TE tip length and blade thickness different from the initial Jones' geometry, indicating that optimization of this turbine can be achieved. Angular velocity, TE tip radius, TE tip length and blade thickness are thus used as random inputs for the application of the gPC method.

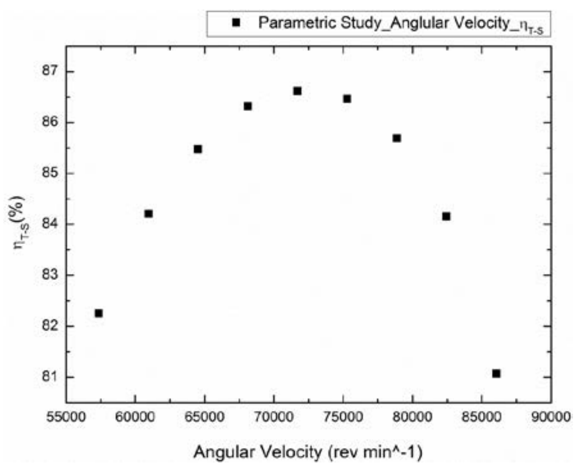

Figure 8. Evolution of the total-to-static efficiency with the angular velocity.

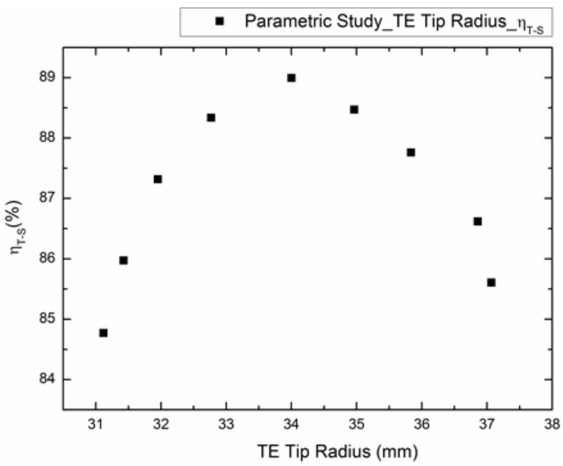

Figure 9. Evolution of the total-to-static efficiency with TE tip radius.

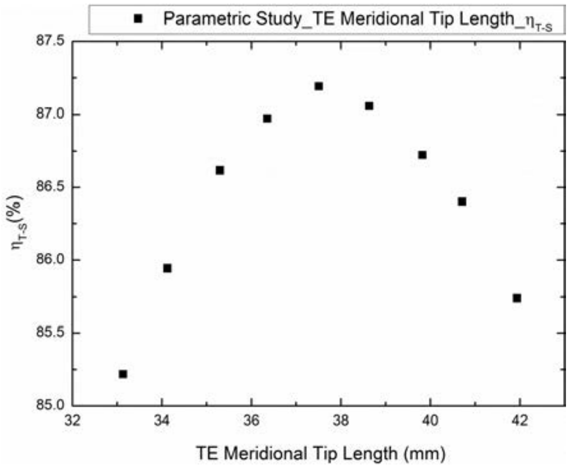

Figure 10. Evolution of the total-to-static efficiency with the TE tip length.

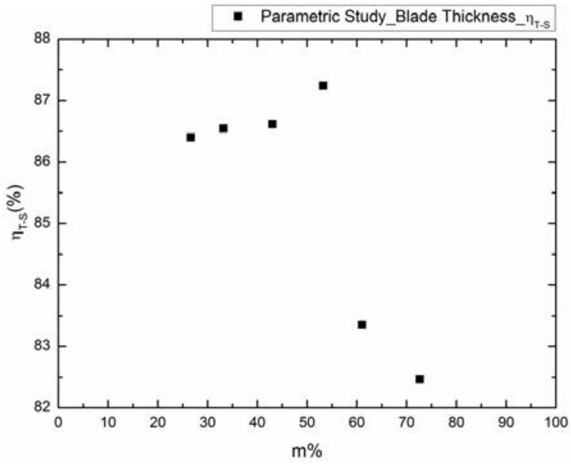

Figure 11. Evolution of the total-to-static efficiency with the blade thickness profiles.

Uncertainty Quantification

The mean and support for the 4 random parameters (angular velocity, TE tip radius, TE tip length and blade thickness) are summarized in Table 1.

Convergence Study

Figure 12 shows the CFD points and the gPC legendre quadrature points for *P* ranging is 1, 3, 5, 7, 9, 11 respectively, when angular velocity is the random variable.

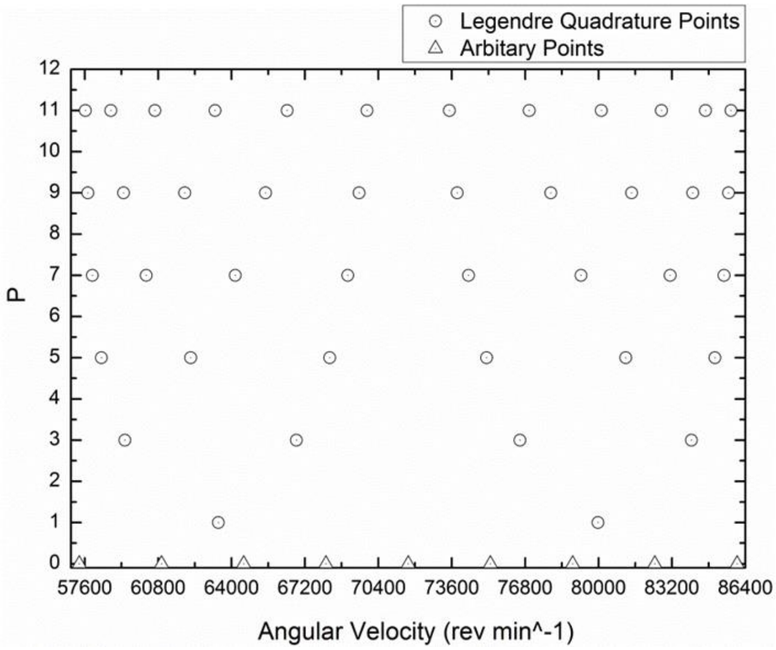

Figure 12. Legendre quadrature points and arbitrary support points for Angular Velocity for P=1, 3, 5, 7, 9, 11

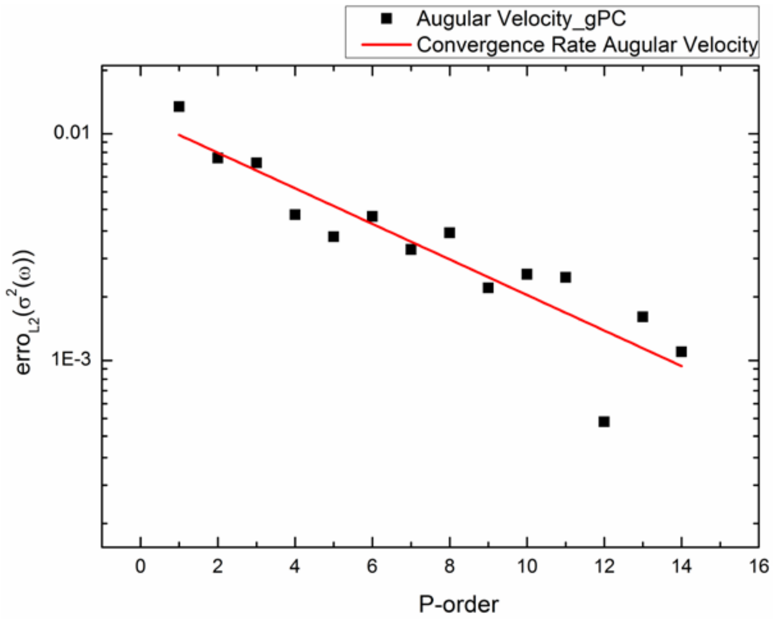

Figure 13. Convergence rates of the variance of the angular velocity in respect to the total-to-static efficiency.

In Figures 13, symbols represent simulations while lines are the corresponding linearly fitted decay rates. It can be seen that the error line trend decreases when the P-order is increasing, showing good convergence rates. Similar trends are observed for all uncertain variables.

Statistical results

In Table 2, ω, L and R refer to the angular velocity, TE tip length and TE tip radius respectively.

Regarding each uncertain parameter, the mean value μ, the standard deviation σ, variance σ_v^2 and the coefficient of variation CoV = σ/μ of the total to static efficiency with the gPC at P = 11 are presented in Table 2. It can be seen that the blade thickness profile has the most influential effect on the turbine total-to-static efficiency closely followed by the TE tip radius R while L doesn't appear to be a critical geometric parameter in regards to the efficiency. The gPC method was also applied for coupled uncertain parameters with a lower polynomial order P=5 in order to minimize the computational cost. When parameters are coupled, the most influential coupled random variables on the total-to-static efficiency are R-ω.

Table 2. Mean, standard deviation and CoV of the total-to-static efficiency for each individual uncertain parameter for P=11 and coupled parameters for P=5.

	gPC 1D (P=11)				gPC 2D (P=5)		
Variable	***ω***	***L***	***R***	***Blade Thickness***	***R-ω***	***L-ω***	***R-L***
μ	85.09	86.72	85.65	85.5	83.27	85.34	81.68
$\sigma \times 10^{-3}$	13.409	1.720	16.611	17.400	25.360	14.971	13.102
$\sigma_v^2 \times 10^{-3}$	0.180	0.003	0.276	0.303	0.643	0.224	0.172
$\mathrm{CoV} \times 10^{-3}$	15.759	1.983	19.393	20.400	30.454	17.543	16.040

Conclusion

In this paper, a deterministic 3D CFD solver is coupled with gPC approach and successfully applied to investigate a complete 3D high-pressure ratio radial-inflow turbine. The uncertainty quantification has been applied to the performance analysis of radial turbine for the propagation of various aerodynamic and geometric uncertainties. The convergence rate for each uncertain parameter has been investigated, showing that the stochastic spectral projection decreases dramatically with the increase of polynomial order. The initial deterministic study highlighted the non-linear response of the total-to-static turbine efficiency in regard to the variations of the angular velocity, TE tip radius, TE tip length and blade thickness. From the preliminary study, for the CoV of the total to static efficiency, the most influential uncertainty is the blade thickness closely followed by the TE tip radius. When the gPC approach is applied to coupled random parameters, the most influential coupled random variables are the trailing edge tip radius with the angular velocity. In future work, other parameters of radial turbine and more dimensional gPC will be investigated, such as maximum blade thickness. Then stochastic collocation method will be applied for the uncertainty quantification analysis of the radial turbine.

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