A hybrid POD-CFD approach for gust computations

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ABSTRACT

We present a numerical approach for the computation of gust flows around bodies. We are mainly interested in computing the aerodynamic coefficients under gust effects. The highest computational accuracy is thus required in a close vicinity of the body. The method we propose is an hybrid approach coupling high-fidelity computations (solving the Navier-Stokes model) around the body with lower fidelity computations based on Proper Orthogonal Decomposition (POD). In other words, the POD basis functions can be viewed as giving accurate boundary conditions to the small computation domain surrounding the body. This is obtained projecting the solution of the detailed model on the POD basis functions over a predefined overlapping region. The gust is taken into account in the POD expansion using additional basis functions as usually done for the control function method. These functions are computed as being the gust transport along the whole domain without any body. This method is highlighted on a simple however complex two dimensional flow around a circular cylinder, and around a 3D elastic airfoil for low to moderate Reynolds numbers.

Keywords: Gust, POD, CFD.

Introduction

The aerodynamic coefficients are usually computed for stationary flow configurations. However wind turbines and airfoils frequently evolve in perturbed flows where gust can have significant effects on the quantities under consideration like power, drag and lift, but also on the structural resistance of materials. These effects are expected to be predicted and computed in real time, or at least as fast as possible. A full simulation of a detailed model is thus not an answer, and we reduce the computational cost thanks to a lower fidelity model trading off accuracy for computational efficiency.

The power and the shape of gusts are unpredictable, but gust is usually mathematically defined by [3]. In this paper we chose different kind of gust as vortices traveling in the computational domain. All the details of the numerical approach are defined in the following section.

Computational method

Accurate computations of gust effects on the aerodynamic coefficients require the resolution of the detailed model in a close vicinity of the body. A lower fidelity model can be used elsewhere. We chose a Proper Orthogonal Decomposition (POD) basis representation [4, 5]. The POD basis functions give accurate boundary conditions for the detailed model around the body. This is done projecting the detailed solution on the POD basis functions. The projection is performed on a predefined overlapping region between the high and low fidelity domains. The algorithm to couple the detailed and the POD models is synthesized in figure 1. To get accurate boundary conditions for the detailed model, the POD basis should be robust to taken into account gust generated at different instant with different intensities. This kind of POD basis is generated from several resolutions of the detailed model sampling gusts with different intensities and time generation. Note that many detailed computations are necessary to compute this kind of POD basis, but all computations are done offline, and thus the gust prediction only requires detailed computations in a small domain.

For the resolution of the detailed model, *i.e.* the Navier-Stokes model, we use a paradigm based on a cartesian mesh, level set functions [6] to track the bodies and a second-order penalization [1, 2] to take into account the bodies.

Let ζ be the unknowns of the detailed model, *i.e.* the velocity $\boldsymbol{u}$ and the pressure p fields. We use the following exansion

$$\zeta(\boldsymbol{x}, t) \approx \zeta_m(\boldsymbol{x}) + \zeta_g(\boldsymbol{x}, t) + \zeta_{POD}(\boldsymbol{x}, t). \tag{1}$$

The quantity $\zeta_m(\boldsymbol{x})$ is the temporal average of the natural flow around the body, *i.e.* without gust, to fix the average flow rate. The gust is taken into account in the expansion thanks to the function $\zeta_g(\boldsymbol{x}, t)$. This is a gust transported in the computational domain without any osbtacle. This transport can be either analytic, obtained by optimal transportation or by a POD with Navier-Stokes computations. Finally, $\zeta_{POD}(\boldsymbol{x}, t) = \sum_{i=1}^{N_r} \hat{\zeta}_i(t)\Phi_i(\boldsymbol{x})$ where $\Phi_i(\boldsymbol{x})$ are POD basis functions. Several approaches can be used to compute robust basis functions $\Phi_i(\boldsymbol{x})$. Here, we start from N_g different gusts with initial condition $\{\zeta_g^i(\boldsymbol{x}, 0)\}_{i=1}^{N_g}$. We then compute N_g POD basis of the N_g flows generated with $\{\zeta_g^i(\boldsymbol{x}, 0)\}_{i=1}^{N_g}$ using snapshots of $\zeta(\boldsymbol{x}, t) - \zeta_m(\boldsymbol{x}) - \zeta_g^i(\boldsymbol{x}, t)$. Each individual POD basis is thus close to the natural basis in the region in the front of the obstacle. In that region the gust is only represented by the function $\zeta_g(\boldsymbol{x}, t)$. The natural flow is a particular flow with gust equal to zero. We finally perform a POD computation of the N_g individual POD basis to get a robust POD basis $\Phi_i(\boldsymbol{x})$.

The overall numerical approach coupling POD representation and full detailed model (figure 1) allows to predict the behavior of the flow with a particular gust $\zeta_g(\boldsymbol{x}, t)$ that is to a certain extent different with respect to those used to generate the database $\{\zeta_g^i(\boldsymbol{x}, 0)\}_{i=1}^{N_g}$.

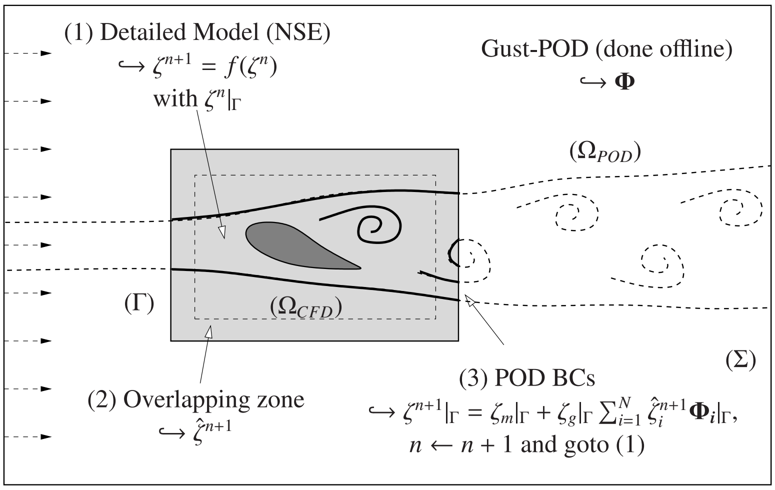

Figure 1. Sketch of the algorithm coupling detailed and POD models.

Conclusions

In the proposed presentation we will detail the numerical approach and present two-dimensional and three-dimensional test cases with fluid structure interaction. In particular we will discuss the robusteness of the low-fidelity model to parameter variations.

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