Assigning Material Properties to Finite Element Models of Bone: A New Approach Based on Dynamic Behavior

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Abstract

Finite element (FE) method is extensively employed to investigate the biomechanical behavior of bone structures. Material and morphological information of bone samples are typically provided by computed tomography (CT) scanning. Assuming that density and elasticity of bone are correlated, many studies have proposed different density-elasticity relationships to determine bone elastic constants. Herein, an innovative method for determining a single mathematical relationship between bone density and elasticity is proposed. Density distribution and morphology of a bovine bone were obtained from CT images, and the natural frequencies were measured using experimental modal analysis. The relationship between density and elasticity has a standard mathematical form with variable constants. Genetic algorithm (GA) was used to obtain the constants by minimizing the discrepancy between experimental and FE results. The relationship was then used in material properties assignment process of FE modeling and proved to be valid by predicting the natural frequencies of bone in different boundary conditions (BCs).

Keywords: Density-elasticity relationship, Modal analysis, Bone tissue, Computed tomography, Finite element method

1. Introduction

Accurate subject-specific finite element models of bone are of great importance in many state of the art research and clinical applications. FE analysis of bone provides valuable information about strain and stress fields within the tissue. Results can be used in fracture risk assessment, designing prosthetic implants and other clinical applications. Dynamic behavior and characteristics of bone such as natural frequencies, mode shapes and response to dynamic loads can also be determined by FE analysis. Obtaining the fundamental frequencies of bone, in particular, has numerous practical applications in medicine and bioengineering. It has been shown that loads with frequencies close to natural frequencies of bone can enhance bone apposition [1]. In fact, patient-specific natural frequencies of targeted bones would help physicians to optimize vibration therapies and exercise regimens and find a solution which suits the patient best [2]. In addition, resonance frequencies and mode shapes of bone provide valuable information about density-elasticity relationships [3] and orthotropic properties of long bones [4].

To generate a subject-specific model, geometry and material properties of bone are usually derived from computed tomography images. The CT images are processed to create three-dimensional (3D) geometry of bone segments. Mechanical properties of bone can also be derived from CT data using mathematical relationships, which relate CT values to material properties [5]–[9]. It has been demonstrated that the relationship between CT numbers and apparent density of bone tissue is approximately linear [10]–[12]. However, obtaining an accurate relationship between density and mechanical properties of bone, particularly elasticity,

is more challenging. Accurate determination of these relationships is important for developing precise FE models.
The relationship between Young's modulus and bone density is described by many different empirical models in the literature [13]–[22]. This relationship is generally reported in power or linear form. The complexity of experimental techniques involved in measuring mechanical properties of an anisotropic and porous material can explain the disparity in predicted values of Young's modulus in different studies. To determine the stiffness, commonly a bone specimen is loaded in a load frame. During the mechanical test, different types of error can arise which makes it difficult to obtain bone stiffness. Methods of measuring bone deformation are widely discussed in the literature [8].
To overcome the difficulties in traditional mechanical testing and improve the accuracy of the density-elasticity relationship, we have developed a new method which determines the model parameters in the general form of the density-elasticity relationship based on the results of experimental modal analysis using GA and FE methods. Unlike many reported models in the literature, this method leads to a single density-elasticity equation which is valid for all ranges of bone density.

2. Materials and methods

2.1 Experimental determination of natural frequencies

Modal analysis is a successful method to validate FE models of bone and to determine bone elastic constants [23] Simple experimental equipment, reasonably short measurement time and accuracy of measurements make modal analysis a potentially useful method for obtaining material properties.
Natural frequencies of a fresh-frozen bovine femur bone were obtained using impact hammer and shaker tests in free-free and clamped-free boundary conditions. To simulate the free-free BCs, soft elastic straps were used to suspend the sample.
The experimental setup of the shaker test is presented in Fig. 2. Computer generated random wave signals containing frequencies from 0 to 5000 Hz were used to excite the bone. Signals were amplified by a signal amplifier. Excitation and response signals were detected by accelerometers (DJB A/120/VT, DJB Co., France).
The experimental setup of the hammer test is presented in Fig. 3. The bone is excited by hitting an impact hammer equipped with a force transducer to five different points normal to the surface to excite different modes of vibration. An accelerometer is used to detect the bone response. The tests were then repeated for different positions of accelerometer. Charge amplifiers are used to condition the force and acceleration signals.
Applying a fast Fourier transform (FFT) algorithm, the frequency response of bone was analyzed considering the excitation and response signals. The resonance frequencies of different vibration modes were obtained using frequency response curves.

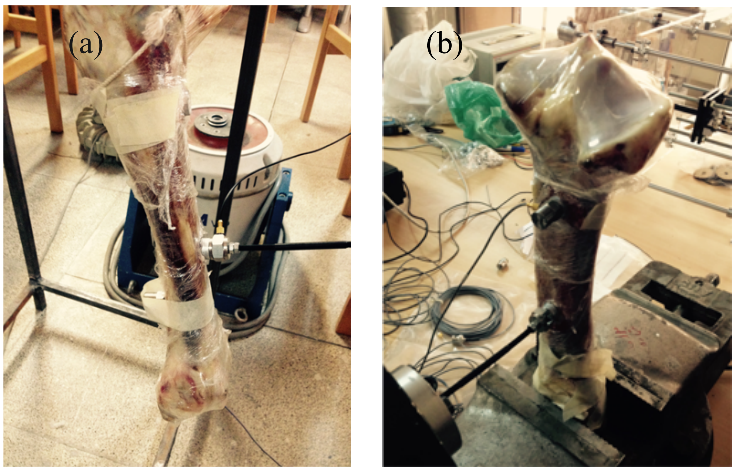

Figure 1. Shaker test setup; (a) free-free (b) clamped-free BCs

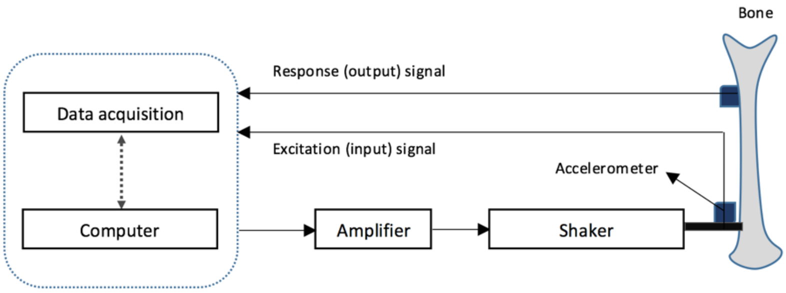

Figure 2. Measuring vibration response of bone using shaker

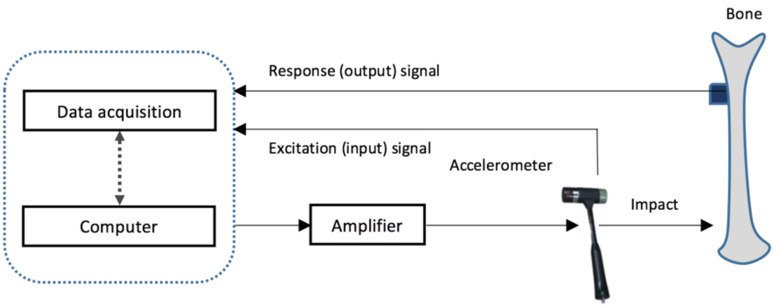

Figure 3. Measuring vibration response of bone using modal hammer

2.2 Finite element modeling

A bovine femur was CT scanned with a slice thickness of 1 mm (16 slice Siemens SOMATON emotion), and a three dimensional model of bone was created using Mimics© v17, MATERIALISE. Exporting the geometry from MIMICS to 3-Matic® v17, tetrahedral volume meshes were generated. A standard procedure (Materialise NV, Leuven, Belgium, 2010) was followed to obtain the three dimensional geometry from DICOM images and mesh the model. The acquired mesh was exported to a commercial FE software for numerical analysis.

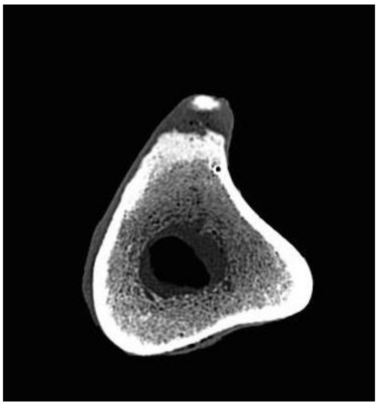

Figure 4. A CT image of the bovine bone

2.3 Material properties assignment

Based on Hounsfield gray values, Mimics can assign material properties to volumetric meshes. After bringing the mesh back to Mimics, an average Hounsfield value is calculated for each element, and the range of gray value is divided into equally sized intervals to represent different material groups. In this study, five material groups were used to model the bone.

The effective bone density and CT numbers are assumed to be linearly correlated [7], [20], [25]. The following equation was used to assign apparent density to the mesh:

$$\rho = 4.64\times10^{-4}\times HU + 1 \quad (1)$$

where ρ is the apparent density (g/cm3) and HU is the CT number (Hounsfield unit).

Considering the literature, the relationship between apparent density and Young's modulus is generally reported in the following form:

$$E = a\rho^{b} + c \quad (2)$$

where E is the Young's modulus, ρ is the apparent density (ash, wet or dry) and a, b and c are the model parameters. A Poisson ratio of 0.3 was considered for all finite elements. Here, experimental results and numerical methods were used to determine the model coefficients.

2.4 Numerical eigenfrequency analysis

The first five natural frequencies and mode shapes of the bone model were calculated using COMSOL Multyphysics v5 without considering the damping effect. The generated mesh together with the material properties were imported to COMSOL. The density-elasticity relationship and model parameters a, b and c were defined in COMSOL according to [20], as a first approximation. LiveLink ™ for MATLAB was used to apply the genetic algorithm and find the optimal coefficients.

2.5 Obtaining coefficients of density-elasticity relationship using GA

The FE model in Matlab was changed to represent a function with the coefficients a, b and c as inputs and the first five natural frequencies as outputs. Assuming that the most exact density-elasticity relationship can result in the most precise values of natural frequencies, we defined an optimization problem to obtain the coefficients in Eq. 2. The following objective function was taken to represent the discrepancy between numerical and experimental results:

$$OF = \sqrt{(f_{E1}-f_{N1})^2 + (f_{E2}-f_{N2})^2 + (f_{E3}-f_{N3})^2} \quad (3)$$

where f_{Ei} is the i^{th} natural frequency obtained from experimental modal analysis, and f_{Nj} is the j^{th} natural frequency obtained from numerical eigenfrequency analysis. The genetic algorithm toolbar in Matlab® R2014a was used to minimize the objective function. The initial population was chosen to be [a, b, c] = [2, 3, 0], and the boundary for searching the optimal answer was [0-10] for all parameters. Population size and number of generations were set to 40 and 10 respectively.

2.6 Validation

The acquired density-elasticity relationship was used to assign material properties to the FE model of bone with clamped-free BCs. The results of eigenfrequency analysis were compared with the experimental natural frequencies to assess the validity of the relationship in different BCs. Other material assignment strategies were also examined, and the results were compared.

3. Results and discussion

Both hammer and shaker tests were performed to measure natural frequencies of bovine bone in free-free boundary conditions. Accelerometers used in shaker test were only able to measure bending vibrations in the x direction.

Table 1. Natural frequencies of bone in free-free BCs; hammer and shaker tests

mode shape/direction	bending		torsion	bending	
	x	y	-	x	y
natural frequency	1st	2nd	3rd	4th	5th
hammer (Hz)	646	834	1278	1875	2342
shaker (Hz)	645	-	-	1798	-

Table 2 density of different material groups

Material number	**1**	**2**	**3**	**4**	**5**
Density (g/cm3)	711.9	1010.5	1309.0	1607.5	1906.1

Table 2 represents the apparent densities of five material groups which are calculated using Eq. 1. Many density-elasticity relationships are proposed in the literature for specific ranges of density which result in different values of elasticity.

In order to obtain more accurate Young's modulus values, genetic algorithm was applied to minimize the objective function defined in Eq. 3. Table 3 represents the results of this optimization process. The natural frequencies were determined using different density-elasticity relationships (initial value, GA and Baca et al), and the results were then compared to experimental findings.

Table 3 results of GA optimization

Study ▸		**GA**	**Initial population**	**Baca (2008)**	**experiment**
Natural frequency	1	623.3	549.3	572.7	646
	2	825.7	726.2	757.2	834
	3	1286.5	1132.0	1179.0	1278
	4	1877.5	1640.9	1706.2	1875
	5	2267.9	1981.3	2061.3	2342

Objective function	OF	25.65	205.68	145.16	-
Density-elasticity equation coefficients	a	1.26986	2	2.065	-
	b	3.81558	3	3.09	-
	c	2.62971	0	0	-

Considering the values of objective function, it is clear that genetic algorithm can be utilized to find the coefficients of density-elasticity relationship which lead to an accurate FE model. Although the first three natural frequencies were used during the optimization process, results are accurate in all modes of vibration. This fact indicates that the resultant equation predicts the real values of Young's modulus and not those which only minimize the objective function numerically.

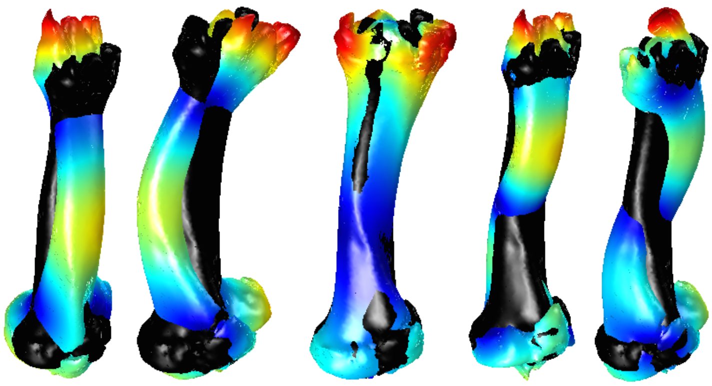

Figure 5. The first five natural modes of vibration; free-free BCs

The first five natural frequencies of bone subjected to clamped-free BCs, based on different density-elasticity relationships, are presented in Table 4. The values are compared with experimental results and the proposed GA method.

Table 4 first five natural frequencies of bone in clamped-free BCs; experimental results vs. FE

		frequency1	frequency2	frequency3	frequency4	frequency5	mean %error
Shaker test		63	-	-	556	-	-
GA method		62.3	80.8	472.4	552.4	656.6	0.879
literature	[13]	59.90	77.00	438.95	510.50	605.60	6.196
	[17]	55.68	72.20	387.25	456.60	546.85	14.417
	[20]	60.40	77.70	448.35	524.70	621.36	4.520

The suggested method results in more accurate natural frequencies not only in free-free BCs (which were used to obtain the model coefficients) but also in clamped-free BCs with totally different values of resonance frequencies. The predicted values of local Young's modulus can therefore be considered as true and reliable.

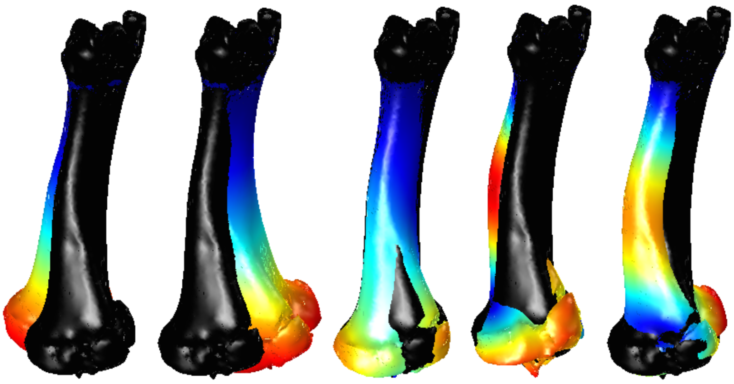

Figure 6. The first five natural modes of vibration; clamped-free BCs

A sensitivity analysis was performed to investigate the effect of changing model parameters in equation (2) on the the first five natural frequencies of the bone. In Figures 7 through 9, two parameters were kept constant while the third parameter changed around a mean value. Variation of the Poisson's ratio did not have a significant effect on the bending natural frequencies. Torsional natural frequency, however, slightly decreased with increasing Poisson's ratio values (Fig. 10).

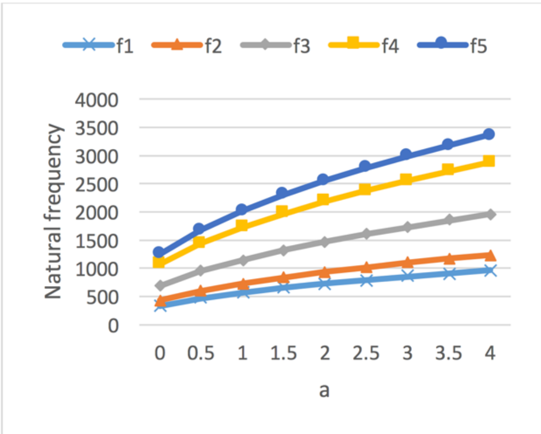

Figure 7. sensitivity analysis; parameter a

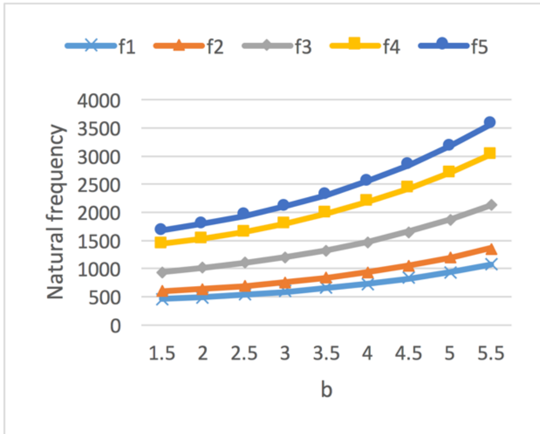

Figure 8. sensitivity analysis; parameter a

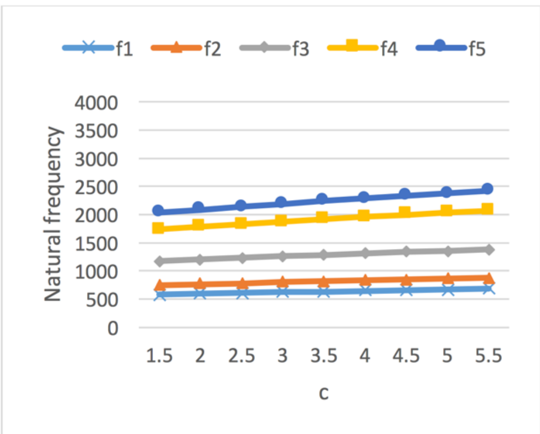

Figure 9. sensitivity analysis; parameter a

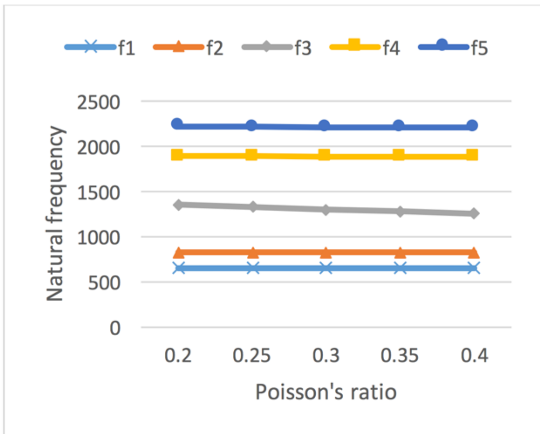

Figure 10 sensitivity analysis Poisson's ratio

Local optimal points were avoided in GA, because of mutations and the final result were closer to the global minimum. However, Genetic algorithm could be time consuming when the number of generations and population increase. To avoid this problem, number of generations and population were limited to 10 and 40, respectively.
There were several limitations associated with the FE model. Five material groups were considered to be enough to represent the distribution of the mechanical properties. Additionally, the effect of marrow on the bone response was presumed negligible, and the material behavior was assumed to be isotropic and linear elastic. A more advanced model may include more groups of materials or a continuous distribution of material properties and consider the effects of nonlinearity, anisotropy and bone marrow in the model.

4. Conclusion

In this study, the density-elasticity relationship of a bovine bone was determined by introducing and solving an optimization problem. Genetic algorithm was used to minimize the difference between natural frequencies obtained from experimental and FE modal analyses. The assumption was that the experimental and numerical results agree, if the material distribution in model approaches the real distribution.
Using the density-elasticity relationship obtained by GA, the numerical resonant frequencies were in good agreement with the experimental results in all modes of vibration with free-free and clamped-free BCs. It can be concluded that the relationship between density and elasticity of bone can be determined with a single mechanical test (experimental modal analysis) and solving an optimization problem based on FE analysis, where the results are valid for all bone density ranges.

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