Vibrations and stability of slender visco-elastic periodic beams posed on a foundation with damping. Tolerance modelling

***Jarosław Jędrysiak[1], Jakub Marczak[1]**

[1]Department of Structural Mechanics, Łódź University of Technology, Poland.

*Presenting/corresponding author: jarek@p.lodz.pl

Abstract

Slender visco-elastic beams with a periodic structure along its axis, which interact with a periodically heterogeneous foundation are considered in this note. They are made of many identical small elements, called periodicity cells, with the length l, which is treated as the microstructure parameter. Partial differential equation describes dynamic or/and stability problems of such beams. Because this governing equation has highly-oscillating, periodic and non-continuous functional coefficients in x, it is not a good tool to investigate special problems of these beams. Various simplified approaches are proposed to obtain governing equations with constant coefficients, introducing effective properties of the beam. It is necessary to distinguish those methods based on the asymptotic homogenization, cf. [5].

The literature on the problems of linear vibrations of periodic beams is extensive. The theory of Floquet-Bloch is often used in the waves analysis. This was applied in the analysis of the Euler- Bernoulli [1] beam vibrations. Certain analytical approaches and the finite element method are also applied to evaluate strength and buckling of sandwich beams having variable properties of cores, e.g. [3]. Unfortunately, the governing equations of the above models neglect usually the effect of the microstructure size on the beam behaviour.

In this note, the differential equations of periodic beams, having highly-oscillating, periodic, non-continuous functional coefficients, are replaced by equations with constant coefficients using the tolerance modelling, cf. [7]. This approach was developed for the purpose of analysis of thermomechanical problems of periodic composites in a series of papers, e.g. for dynamics of thin periodic plates on a foundation [4], for dynamics of micro-periodic beams under moving load [6], for geometrically nonlinear vibrations of periodic plates [2].

Keywords: **Periodic beams; Vibrations; Stability; Microstructure; Tolerance modelling**

Acknowledgment. This note is partly supported by the National Science Centre, Poland (Grant No. 2014/15/B/ST8/03155).

References

[1] Chen, T. (2013) Investigations on flexural wave propagation of a periodic beam using multi-reflection method, *Archive of Applied Mechanics* **83**, 315-329.

[2] Domagalski, Ł., Jędrysiak, J. (2016) Nonlinear vibrations of periodic beams, Journal of Theoretical and Applied Mechanics **54**, 1095-1108

[3] Grygorowicz, M., Magnucki, K., Malinowski, M. (2015) Elastic buckling of a sandwich beam with variable mechanical properties of the core, *Thin-Walled Structures* **87**, 127-132.

[4] Jędrysiak, J. (1999) Dynamics of thin periodic plates resting on a periodically inhomogeneous Winkler foundation, *Archive of Applied Mechanics* **69**, 345-356.

[5] Kolpakov, A.G. (1991) Calculation of the characteristics of thin elastic rods with a periodic structure, *Journal of Applied Mathematics and Mechanics* **55**, 358-365.

[6] Mazur-Śniady, K., Śniady, P. (2001) Dynamic response of a micro-periodic beam under moving load – deterministic and stochastic approach, *Journal of Theoretical and Applied Mechanics* **39**, 323-338.

[7] Woźniak, C., Michalak, B., Jędrysiak, J. (eds). (2008) Thermomechanics of microheterogeneous solids and structures. Tolerance averaging approach, Lodz Univ. Techn. Press, Lodz, Poland.