The Problem of Static Equilibrium in Computational Methods Coupling Classical Mechanics and Peridynamics

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The presence of cracks in practically all structures of engineering relevance, in particular in aerospace engineering, is one of the main reasons that prevent to simulate the full structural life cycle. Classical Continuum Mechanics (CCM) is based on the assumption that displacements are continuous in the domain where the problem is defined, but that is not true whenever a crack is present. Therefore, an economically viable management of engineering structures requires the capability to describe fracture phenomena with computational tools.

In the last twenty years the theory of Peridynamics (PD) [1, 2] has found increasing support by the researchers because it can easily include displacement discontinuities as part of the solution, since it is not based on the use of derivatives of the displacement variables. However PD based methods are not computationally efficient, since Peridynamics is a non-local theory. Therefore, coupling of PD based computational methods to CCM based methods, such as the extremely popular displacement version of the Finite Element Method, may generate a numerical tool that possesses the advantages of both computational techniques and avoids their pitfalls. In fact coupling PD and FEM is the topic of a growing research effort [3-5].

However, when using coupled computational methods the verification of the overall structural equilibrium is not necessarily satisfied. We will illustrate the problem through simple examples of statically determinate structures, partially discretized with a PD method and partially with a classical mechanics FEM approach. The considered coupling approach was proposed by the authors themselves in [5], in our examples the magnitude of the out of balance forces is small, compared to that of the acting forces, but it cannot be assumed as a numerical error. The equilibrium equations of each single node are exactly solved, but, by doing so, a small overall equilibrium error is introduced. Our work will examine how the variation of the main features of the coupled model, such as position and size of the PD region, length of the coupling boundary, rate of change of the strains …, can affect the magnitude of the out of balance forces. Moreover, we will propose criteria to reduce the magnitude of the out of balance forces.

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