# Localized Trefftz method for solving two-dimensional Laplace, Helmholtz and biharmonic equations

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## Abstract

In this presentation, the localized Trefftz method is proposed to accurately solve two-dimensional Laplace, Helmholtz and biharmonic equations in complex domains. It is well-known that the conventional Trefftz method is a very powerful boundary-type meshless method since it is truly free from mesh generation and numerical quadrature. The numerical solution of Trefftz method is expressed as a linear combination of T-complete functions. To enforce the satisfactions of boundary condition will yield a fully-populated linear system of algebraic equations. Although the Trefftz method can acquire extremely-accurate solution, it cannot be applied to problems in complicated domains and large-scale problems owing to the resultant fully-populated matrices. In order to overcome this drawback in Trefftz method, the localized concept and the Trefftz method, in this presentation, is combined to form the localized Trefftz method, which aims to accurately solve large-scale problems. In the localized Trefftz method, both of the interior and boundary nodes are required. For each interior node, a subdomain is formed by choosing some nearby nodes. The linear algebraic equation at every node can be yielded by implementing the standard Trefftz method within every local subdomain. Similar with the finite difference method, the localized method of fundamental solution and the generalized finite difference method, a sparse system of linear algebraic equations can be yielded by enforcing the satisfaction of governing equation in every interior node and boundary condition at every boundary node. Finally, the numerical solution of the localized Trefftz method can be efficiently and accurately acquired by solving the sparse linear system. Several numerical examples of two-dimensional Laplace, Helmholtz and biharmonic equations in complex domains are provided in presentation to demonstrate the merits of the proposed localized Trefftz method. In addition, some possible extensions of the proposed localized Trefftz method will also be discussed in this talk.

**Keywords:** Localized Trefftz method, Laplace equation, Helmholtz equation, biharmonic equation, meshless method, large-scale problems.