An Investigation of Matching 3D Crushed Sand Particles

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Abstract

Characterizing the fracture surfaces within a single sand particle precisely and describing it quantitatively plays an essential role in understanding the breakage behavior of sands. This paper presents two novel methods to obtain the realistic fracture surface from 3D particle fragment reassembly and the application of the point cloud registration technique. In the first phase of this research, a fracture region matching algorithm was developed to reassemble fractured sand particles by using a variety of image processing and matching techniques including the *minimum spanning tree* (*MST*), *simple chordless cycles* (*CSC*) and modified *4-points congruent set* algorithms (*4PCS*), etc. [1]. In that study, image matching was only performed between fragments and the original particle was not involved in the matching exercise. In the second phase, the target of research was set to match the individual fragments to their mother particle directly, which entailed matching the original face (if any) on the fragment surface to the mother particle and then identifying the fracture face on a fragment surface that is generated from the mechanical crushing event. That target was achieved by employing the *Standard Iterative Closest Point* (*Standard ICP*) algorithm in the 3D point cloud registration [2]. The effectiveness and efficiency of the tracking methods were demonstrated using the tomography data of 9 crushed Leighton Buzzard sand particles.

Keywords: Particle fracture; Tracking; Particle crushing; Registration

1 Introduction

Particle crushing plays an important role in the macroscopic mechanical behaviour of sands. A significant amount of research work (e.g., physical experiment and numerical simulation) has been done to investigate the crushing of sand particles. In spite of these research progress, difficulties still exist in accurately depicting the 3D fracture surfaces resulting from particle crushing in a mathematical way, and no method has been developed so far for identifying and tracking individual fragments generated during a particle crushing event.

In recent years, some efforts have been dedicated to the development of discrete particle tracking methods [3] [4]. These methods mainly rely on the use of particle volume for particle tracking. Very recently, Cheng and Wang [5] extended the method proposed by Andò et al. [4] by adopting both particle surface area and particle volume as the matching criterion for particle tracking. However, this approach suffers from the use of a small search window which may only be valid in the case of small sample deformation, and the searching efficiency could decrease dramatically when the sample is subject to a large deformation. Furthermore, it can only handle problems in which sand particles experience no or very little crushing so that the particle volume and surface area almost remain unchanged. As an alternative, Zhou et al. [6] incorporated a set of spherical harmonics invariants into the development of a novel particle tracking method, but again it will be highly complicated and challenging for the method to be employed in the matching of crushed particles. Most recently, the authors have achieved a success in reassembling fractured LBS particles by using a fracture region matching algorithm, but this method cannot be directly used to match a fragment to the mother particle from which the fragment is generated.

The objective of this paper is to propose two methods for reassembling sand particles and registering 3D point clouds that were crushed and scanned by μCT. In the first phase of this research: based on the work of Zhao et al. [7], particle fragments were separated from each other, and every fragment was converted to a point cloud. According to the magnitude of the surface curvature calculated from the second fundamental form, the points with a higher curvature was extracted. Then, we used the *minimum spanning tree* (*MST*) to connect all these points into a curve network. The *bottom-up graph pruning* algorithm was essential for removing the short branches from the tree. To improve the work efficiency, all simple chordless cycles (*CSC*s) were identified from the *MST*. After that, the *Hausdorff distance* and the modified *4PCS* algorithms were adopted to identify potential *CSC*s for matching and to match them. The registered result which was evaluated by the *ray-triangle intersection* algorithm to avoid the substantial penetration effect provides further information about the potential broken region within the sand particle.

In the second phase, the point cloud registration technique which has been widely used in many areas, including computer vision, medical diagnosis and archaeology, etc., is adopted for quantitatively characterizing the fracture surface and matching a fragment to the mother particle. The *Standard Iterative Closest Point* (*Standard ICP*) technique, proposed by Besl and McKay [8], is the most well-known algorithm among the numerous registration methods for efficiently registering 3D point clouds. The *Standard ICP* is used in this study to mathematically characterize the surfaces of fragments of 9 LBS particles that were subjected to single particle crushing tests with in-situ μCT scanning and then match these child particles to their mother particle.

2 The first phase: reassembling sand particles

A number of μCT images of a fractured particle with a resolution of about 3 μm were obtained by using the GE Phoenix v|tome|x m. The CT data visualised as a stack of μCT images, could not be utilised directly for image reconstruction because of the existing noise. Performing image processing on digital images is an essential way to gain a precise description of fracture patterns. Scripting language was compiled to separate the fragments according to the magnitude of volume for the convenience of performing image registration. It shall be noted that not all the fragments were used in the image registration due to the difficulty of finding the right feature curve networks of the fragments as they get smaller and the goal of obtaining the principal fracture surfaces.

In this study, an OBJ file which consists of point and face information was read by the MATLAB software to obtain curvature at a point on a curve. Aiming at extracting the feature curve network the points with high mean curvatures were selected based on the distribution of curvature. Then, we used the *Prim's* algorithm to build a *MST* by connecting the extracted points in the first step. *MST* is a subset of the edges of (un)directed graph that forms a tree that includes every vertex, where the total weight of all the edges of the tree is minimized. the *bottom-up graph pruning* algorithm was used to remove short branches. The purpose of data segmentation is to obtain all *CSC*s. The *CSC*, which is a set of points in which a subset cycle of those points does not exist, can be extracted. Aiming at improving the work efficiency, the *Hausdorff distance* method was adopted to identify the similarity between two *CSC*s. Then the combination with the largest degree of similarity is prioritised for matching by sorting.

2.1 Modified 4PCS algorithm

A modified *4PCS* algorithm was used to express the invariant and to obtain four-points wide bases S from chordless cycle C. It should be noted that the set of four-points wide bases S (S for chordless cycle C) is a group of four points which must satisfy the following two conditions: 1) the four points are coplanar; and 2) the quadrilateral composed by four points is the convex quadrilateral.

The main steps of the S_1 (S_1 for simple chordless cycles C_1 in fragment 1) search algorithm are: First, three points A, B, C were arbitrarily chosen from C_1. Note the fact that any three points in the space can form a plane. However, we need to eliminate the possibility that the three points selected are collinear. Then we selected the fourth point D beyond these three points in the set of *CSC* and determined whether the fourth point and the first three points were coplanar. If the four points are coplanar, we could then build a four-points wide base $X = \{A, B, C, D\}$. Next, the crossover point O of these two diagonals (AC and BD) is determined. If the following conditions are true, we can conclude it is a convex quadrilateral: A 5-dimensional descriptor vector $v_1 = \{l_1, l_2, \theta, \eta_1, \eta_2\}$ was constructed to depict the four-points wide base $X = \{A, B, C, D\}$, where l_1=$\|AC\|$, l_2=$\|BD\|$, θ is the angle between them, η_1=$\|AO\|/\|AC\|$, η_2=$\|BO\|/\|BD\|$.

The main steps of the S_2 (S_2 for simple chordless cycles C_2 in fragment 2) search algorithm are: We first determined the set of all point pairs $P_a = \{P_{a,1}, P_{a,2}, P_{a,3}, \ldots, P_{a,m}\}$ in C_2 (m is the number of point pairs), in which the distance between each pair of points was equal to l_1. Likewise, another set of point pairs $P_b = \{P_{b,1}, P_{b,2}, P_{b,3}, \ldots, P_{b,n}\}$ in C_2 (n is the number of point pairs) in which the distance between each pair of points was equal to l_2 was also determined. All the point pairs in set P_a and set P_b are alternately combined to form a set of four-points wide bases, $S_c = \{S_{c,1}, S_{c,2}, S_{c,3}, \ldots, S_{c,h}\}$, in which the number of all combinations was $h = m \times n$. Then the same algorithm used for S_1 search described above was used for S_c search. Lastly, we could construct another 5-dimensional descriptor vector v_2.

2.2 Results and discussion

In this section, we show the results of the fragment reassembly of 4 LBS particles tested by Zhao et al. [7] in which the results of fracture pattern and morphology evolution were demonstrated. The 4 particles are denoted as LBS-1, LBS-2, LBS-3 and LBS-4, where the numbering of particles follows Zhao et al. [7]. We only show the result of reassembly of LBS-1 particle, which has an initial volume of 2.18 mm^3 (Fig. 1). The crushing of LBS-1 resulted in a few hundred fragments, from which 16 fragments were successfully reassembled, making it the most successful case out of the 4 LBS particles. The volume of the reassembled LBS-1 is about 97.7% of the original volume. The smallest fragment volume matched successfully is 0.007 mm^3 and equal to 0.32% of the original particle volume. The shapes of all 16 fragments are shown in Fig. 1(e). Figs. 1(a)-1(d) indicate that LBS-1 was reassembled accurately, demonstrating again the high capacity of the proposed matching algorithm. Apparently, this fracture mode is different from those of other three particles and is resulted from the combined influence of overall less spherical shape (sphericity 0.80), unsmooth surface, little initial void and loading direction.

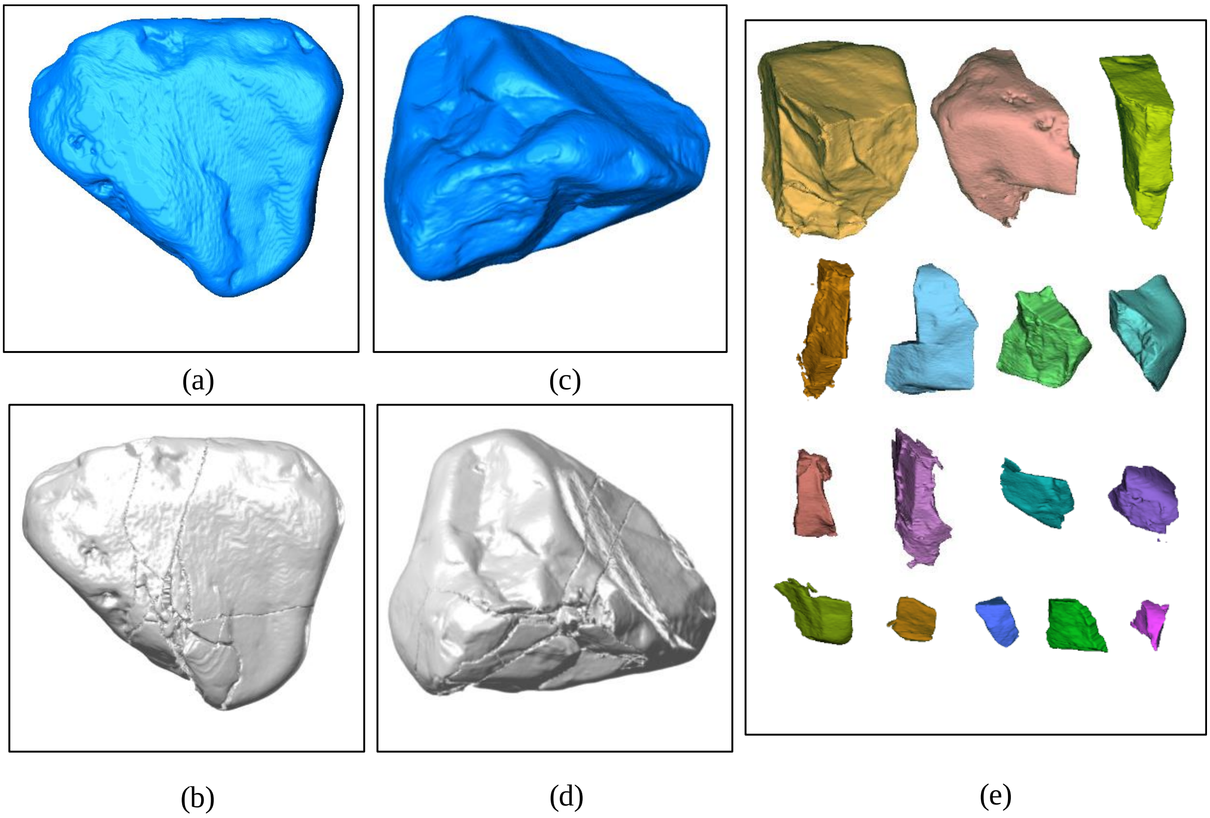

Fig. 1. LBS-1 model: (a, c) the original particle shape; (b, d) the reassembly result; (e) 3D fragment model

3 The second phase: point cloud registration

In this study, another 5 LBS particles were tested and scanned, in the same manner as Zhao et al. [7] using the µCT system (v|tome|x m, Phoenix|X-ray, General Electric Company (GE)) of Shanghai Yinghua NDT Equipment Trade Co., Ltd. The 5 LBS particles were randomly selected and had a size between 1.2 and 1.6 mm (due to the load capacity of the apparatus), which is not considered to have a significant effect on the crushing behavior. The limited CT resolution will create some difficulties for the matching algorithm, particularly for those very tiny fragments which tend to have similar morphologies. In fact, this is the main reason for choosing LBS particles for this study, which have relatively smooth surfaces and textures, reducing the difficulty level for the matching exercise. Other kinds of rough sand particles, for example, highly decomposed granite (HDG), are purposely avoided, due to its high surface roughness, which becomes intractable for the fragment identification and matching. More than 500 raw images (i.e., 2D slices) were obtained for each scan, containing a considerable amount of noises and cannot be directly used for image analysis. The voxel size of these images is 5.69 µm. The resolution of the CT scan was not changed during the scan. More details of the image processing can also be found in Zhao et al. [7].

3.1 Child particle surface segmentation

In recent years, the point cloud registration is a popular topic and has been widely used in the driverless vehicle, medical diagnose and archaeology, etc. Although the technique has received a strong development and assisted us considerably in solving graphics-related problems, shortcomings like its being sensitive to the noise and hard to recognize the useless points still exist in its working process. For this study, points consisting of the fracture face of a child particle, regarded as useless points, will affect the matching efficiency and accuracy, because there are no corresponding points to them on the mother particle surface. Therefore, a data pre-processing step, called the *contour-based mesh segmentation*, whose function is to partition the child particle surface into several faces, is necessary in the point cloud registration process. However, it shall be noted that distinguishing the fracture face from the original face in advance is highly challenging. As a result, it is necessary to try matching each face of a child particle to the mother particle surface and then use an index called the distance error to evaluate the matching degree and eliminate the wrong matching. More details of the procedure of *contour-based mesh segmentation* can also be found in Rodrigues et al. [9].

3.2 Iterative closest point algorithm

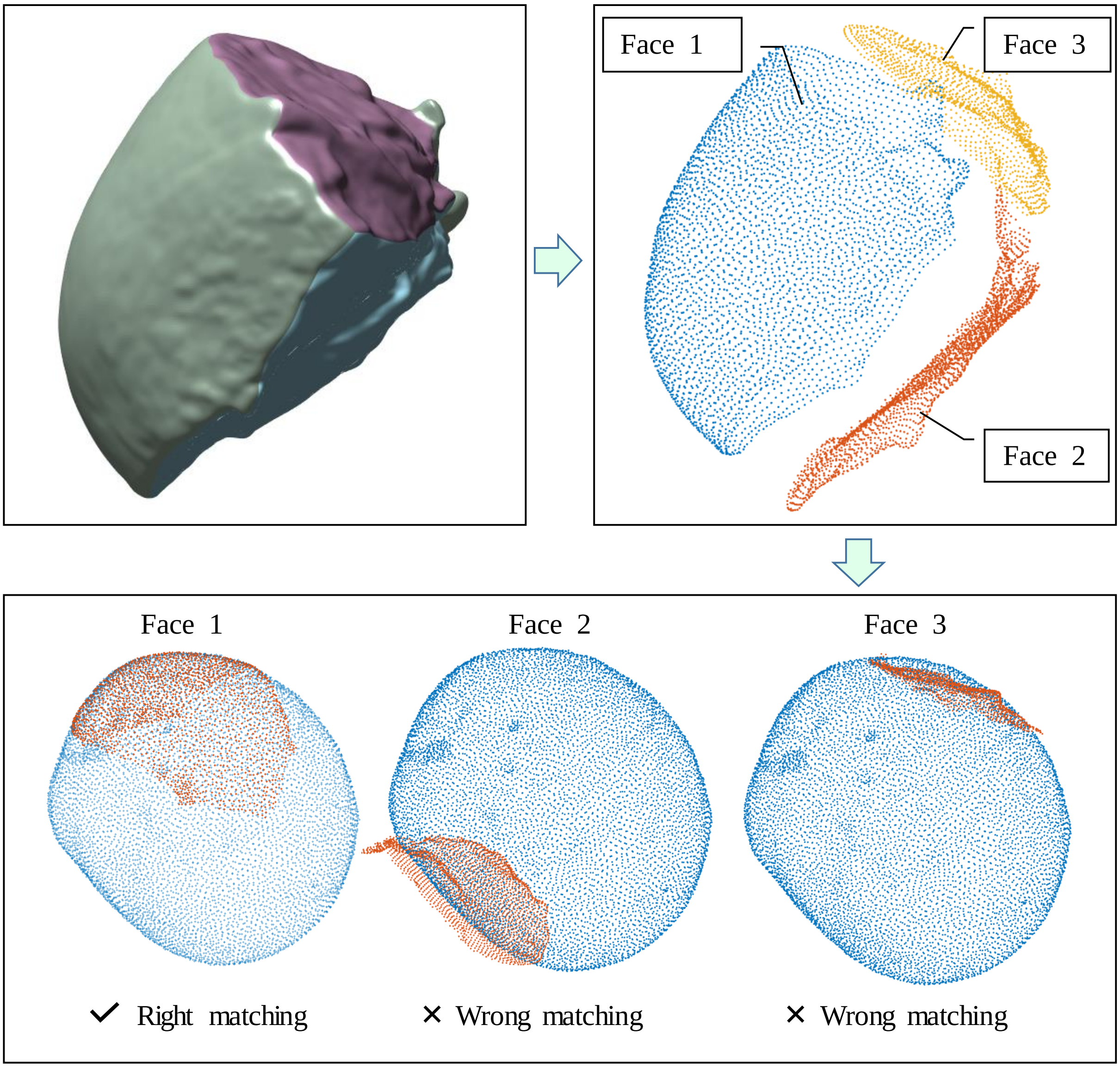

Fig. 2. The relationship between *contour-based mesh segmentation* and the *Standard ICP*

In the point cloud registration, the order of child-mother particle image matching is determined by the volume of the child particle. The face with the largest area on a given child particle is prioritized for the point cloud registration. The *Standard ICP* algorithm, which is an optimal registration method based on *Least Squares Method*, was adopted in this paper. The core idea of this algorithm is to make trial selections of corresponding points repeatedly and calculate an optimal rigid body transformation until the convergence of matching is satisfied. Accordingly, the first step in the *Standard ICP* algorithm is to find the initial corresponding points from the source cloud and the target cloud, respectively. Then in the second step, the rotation and translation matrices are calculated and applied to the source cloud, which results in the transformation of the corresponding points. In the last step, the calculation process returns to the part of adjusting and reselecting the correspondence points. These steps are iteratively performed until the termination condition is satisfied. The performance of the *Standard ICP* was tested by registering the point cloud of each face on a given child particle to the point clouds of the mother particle, as shown in Fig. 2.

3.3 Results and discussion

9 LBS particles are examined and divided into two groups, namely Group A and Group B in this section. Group A involves 4 LBS particles tested by Zhao et al. [7] in which the results of fracture pattern and morphology evolution were demonstrated. The 4 particles are denoted as LBS-1, LBS-2, LBS-3 and LBS-4, where the numbering of particles follows Zhao et al. [7] Group B includes 5 LBS particles which were tested in this study and labelled as LBS-5, LBS-6, LBS-7, LBS-8 and LBS-9. We only show the results of reassembly of LBS-1 and 5 particles (Figs. 3 and 4).

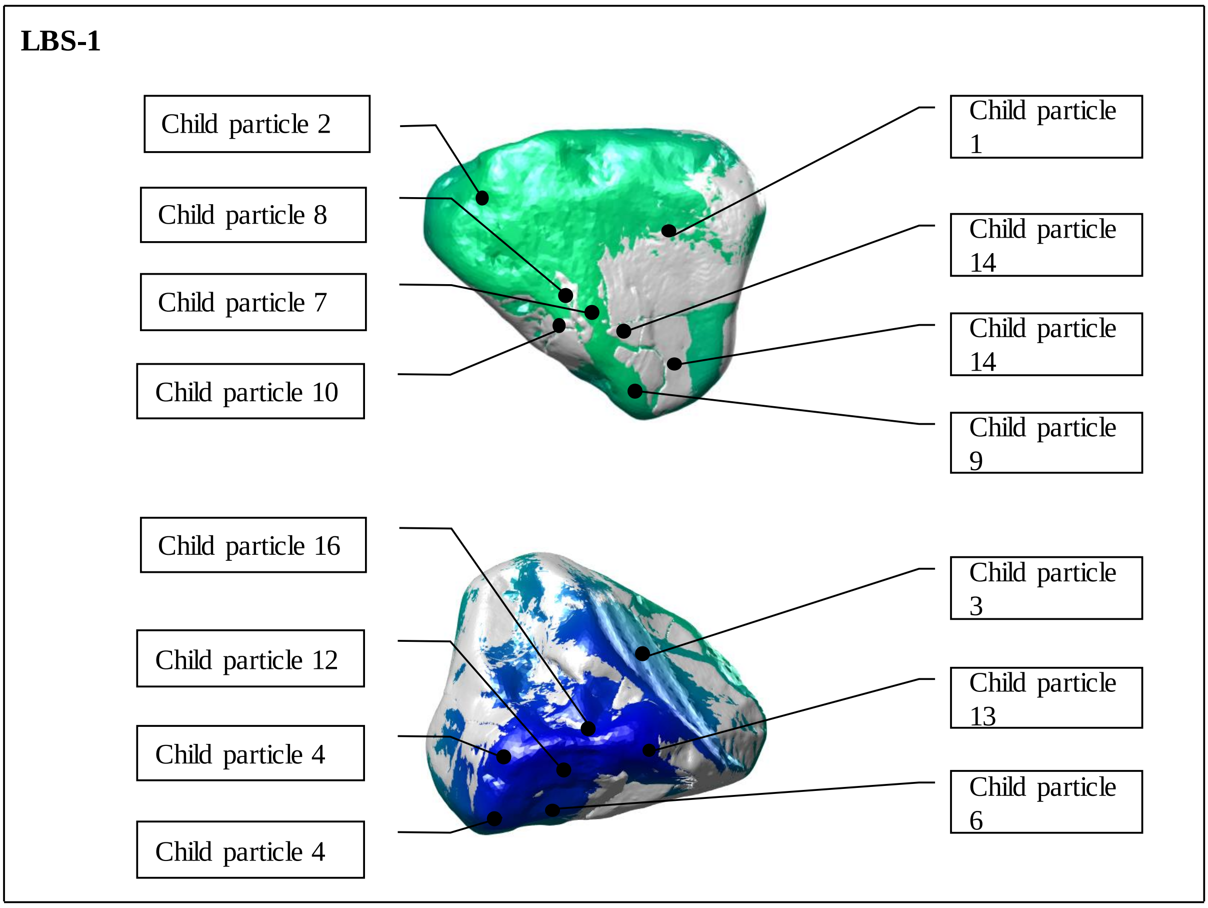

Fig. 3. Group A: the original and fracture face information for a given child particle (The grey color expresses the child particle and the mother particle is presented by the rest color.)

The point cloud registration can identify the locations of child particles in the mother particle, and upon the successful matching of more and more child particles, can restore the morphology of the mother particle. However, some limitations still exist in this algorithm. It will not be successful if there is no original face on the child particle. It will also be a challenge if the original faces for two given child particles have similar morphologies. In addition to the above backwards, calculation errors are unavoidable in this algorithm, making it difficult to match the child particle with the mother particle completely. Fig.4 shows further the matching results with colour-labeled particle surfaces.

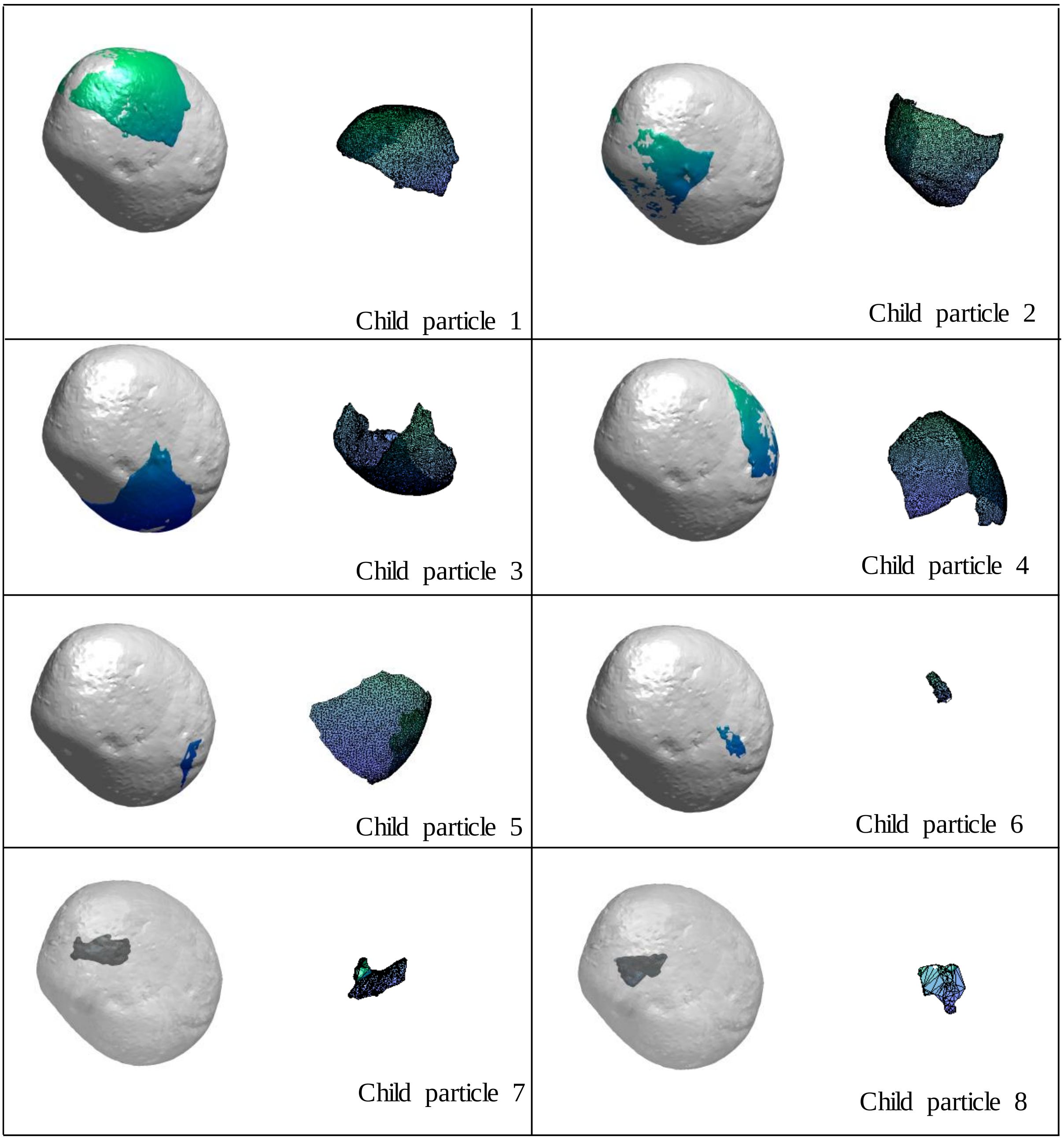

Fig. 4. The results of image matching between child particle and mother particle (The grey color expresses the mother particle and the child particle is presented with green color. The mesh expresses the region with higher point density. Translucent state exists in the child particle 7 and 8 for better observing.)

4 Concluding remarks

The main contribution of this paper is the proposal of two innovative algorithms for reassembling fractured sand particles based on the results of μCT scanning of single sand particle crushing tests.

For the first phase, the matching algorithm, which has been widely used in medical diagnosis, computer vision, archaeology, forensic investigations and other related fields, was applied to the micromechanical study of sands for the first time, making this study a pioneering one in the investigation of the 3D particle morphology restoration upon particle breakage. Although only four LBS particles were reassembled in this study, the results of particle reassembly demonstrated the high capability and robustness of the proposed algorithm, and contributed to our further understanding of the fracture pattern and morphology evolution during the sand particle crushing process. It serves as a starting point for the further investigation of the fracture mechanics of sands.

For the second phase, the point cloud registration method, which has been widely used in many areas such as image processing, computer vision, machine vision and medical diagnosis, etc., was applied for the first time to characterize fractured sand particles and match them to the original particles. This was achieved by implementing the *Standard ICP* algorithm to segment and identify the original faces and fracture faces of the child particles resulting from the crushing of LBS particles, which were subjected to the single particle crushing test and scanned by the μCT. 9 LBS particles were successfully reassembled using the above technique, demonstrating the high competence and robustness of the technique in quantifying the fragment morphologies and matching them to the original particles. Our next goal is to enhance the algorithm to enable the automatic ID-tracking of fractured particles within a loaded sand sample.

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