

# Structure-Preserving Algorithms for Multi-Phase Flow in Fractured Porous Media

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Abstract:

Modeling and simulation of multiphase flow in porous media have been a major effort in reservoir engineering and in environmental study. Petroleum engineers use reservoir simulation models to manage existing petroleum fields and to develop new oil and gas reservoirs, while environmental scientists use subsurface flow and transport models to investigate and compare for example various schemes to inject and store CO<sub>2</sub> in subsurface geological formations, such as depleted reservoirs and deep saline aquifers. One basic requirement for accurate modeling and simulation of multiphase flow is to have the predicted physical quantities sit within a physically meaningful range. For example, the predicted saturation should sit between 0 and 1 while the predicted molar concentration should sit between 0 and the maximum value allowed by the equation of state. Unfortunately, popular simulation methods used in petroleum industries do not preserve physical bounds. A commonly used fix to this problem is to simply apply a cut-off operator (say, to the computed saturation) at each time step, i.e., to set the saturation to be zero whenever it becomes negative, and to set it to one whenever it becomes larger than one. However, this cut-off practice does not only destroy the local mass conservation but it also damages the global mass conservation, which seriously ruins the numerical accuracy and physical interpretability of the simulation results. This violation of preservation on the solution physical range (in general the violation of preservation on the solution structure) is more pronounced and gets more serious consequence in fractured porous media, as compared to porous media without fractures.

In the talk, we will present our recent work on **bound-preserving** discretization and solvers for subsurface flow models based on a **fully implicit framework**. We reformulated a few subsurface flow models using variational inequalities that naturally ensure the physical feasibility of the physical quantities including saturations and concentrations. We applied a mixed finite element method to discretize the model equations for the spatial terms, and the

implicit backward Euler scheme with adaptive time stepping for the temporal integration. The resultant nonlinear system arising at each time step was then solved in a monolithic way by using a Newton–Krylov type method, where the resultant nonlinear system was solved by a generalized Newton method, i.e., active-set reduced-space method, and then the ill-conditioned linear Jacobian systems were solved with an effective preconditioned Krylov subspace method. The used **nonlinear preconditioner** was built by applying **overlapping additive Schwarz type domain decomposition** and **nonlinear elimination**. Numerical results will be presented to examine the performance of the newly developed algorithm on parallel computers. It was observed from numerical tests that our nonlinear solver overcomes the severe limits on the time step associated with conventional methods, and it results in superior convergence performance, often reducing the total computing time by more than one order of magnitude.

This presentation reports a research study extending the joint work [1-3] with Haijian Yang (Hunan University), Chao Yang (Beijing University), and Yiteng Li (KAUST).

[1] H. Yang, S. Sun, Y. Li, and C. Yang, “A scalable fully implicit framework for reservoir simulation on parallel computers”, *Computer Methods in Applied Mechanics and Engineering*, Volume 330, Pages 334-350, 1 March 2018.

[2] Yang, S. Sun, and C. Yang, “Nonlinearly preconditioned semismooth Newton methods for variational inequality solution of two-phase flow in porous media,” *Journal of Computational Physics*, Volume 332, Pages 1-20, 1 March 2017.

[3] H. Yang, C. Yang, and S. Sun, “Active-set reduced-space methods with nonlinear elimination for two-phase flow problems in porous media”, *SIAM Journal on Scientific Computing*, 38(4), B593–B618. (26 pages), 2016.