A Bézier Extraction based XIGA Approach for Vibration Analysis of Cracked FGM Plate using Simple First-Order Shear Deformation Theory

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Abstract

An extended isogeometric analysis (XIGA) approach based on Bézier extraction and simple first-order shear deformation theory (S-FSDT) is proposed for the free vibration analysis of cracked functionally graded material (FGM) plate. The XIGA relies on the concept of partition of unity to model a crack. By decomposing the NURBS basis functions into Bernstein basis functions and Bézier extraction operator, the implementation of XIGA becomes simple. The S-FSDT uses four parameters for displacement field approximation which overcomes the shear-locking and captures the shear deformation effect. The S-FSDT requires C^1 continuity which is easily achieved through non-uniform rational B-spline (NURBS) basis functions. The material properties of the FGM vary by power law along the thickness of plate. Several numerical examples are solved to validate the accuracy of the proposed approach. The effects of various parameters such as length to thickness ratio, crack length and boundary conditions are investigated on the natural frequencies and mode shapes.

Keywords: XIGA; Bézier extraction; NURBS; FGM; Vibration

1. Introduction

Functionally graded material (FGM) is a class of composite material made by mixing the two different material phases such as ceramic and metal. Unlike composite material, the material properties of the FGM vary smoothly and continuously in a certain direction and able to avoid the inter-laminar stresses and debonding phenomenon. These advantageous features of FGM are extensively used in variety of engineering applications [1]. In order to ensure the reliability of components made from functionally graded materials (FGMs), it is essential to analyze their behavior in the presence of crack, which can be done by evaluating the static and dynamic behavior, of few standard crack problems. Over the years, several researchers have performed the vibration analysis of cracked plates using different numerical techniques and plate theories. Guan-Liang *et al*. [2] employed the finite element method (FEM) to perform the free vibration analysis of cracked square plate based on the classical plate theory (CPT). Bachene *et al*. [3]

uses extended finite element method (XFEM) in context of first-order shear deformation theory (FSDT) to investigate the free vibration behavior of cracked homogenous rectangular and square plates. Further, Natarajan *et al*. [4] explored the XFEM based on FSDT to study the free vibration analysis of cracked FGM plate. Huang *et al*. [5] used the Ritz method and 3D elasticity theory to perform the free vibration analysis of cracked rectangular FGM plates.

In the present study, a simple first-order shear deformation theory (S-FSDT) is utilized for the free vibration analysis of cracked FGM plates. The S-FSDT model requires four parameters for displacement field approximation and completely overcomes the shear locking effect associated with the original FSDT model [6]. Moreover, S-FSDT model requires C^1 continuity of generalized displacement field which cannot be easily attainable using lower order Lagrangian shape functions. However, this necessity is easily attainable by the NURBS basis functions utilized by isogeometric analysis (IGA) [7]. Moreover, in order to capture the discontinuities in the domain, partition of unity (PU) enrichment functions are incorporated with IGA approximation and called as extended isogeometric analysis (XIGA) [8]. Over the years, XIGA is widely used for solving the stationary and propagating cracks in 2D [9], 3D [10], cracked plates [11, 12] and shell structures [13]. Furthermore, the implementation of XIGA can be further simplified by incorporating the Bézier extraction approach [10]. Recently, Tan *et al*. [14] employed XIGA based on Bézier extraction using refined plate theory for the free vibration analysis of cracked FGM plates. Hence, the present work aims to extend the XIGA based on Bézier extraction and S-FSDT for the free vibration analysis of cracked FGM plates. Numerous examples are solved to validate the accuracy of the proposed approach and the obtained results are compared to other published results.

2.1 Functionally Graded Plates

Let us consider a ceramic-metal functionally graded plate of uniform thickness *h*. The upper surface of the plate is assumed to be ceramic rich whereas the bottom surface is fully composed of the metal. As shown in the Fig. 1, the *x*-*y* plane is assumed as the mid-plane of the plate, and the positive *z*-axis is directed above from the mid-plane. Moreover, along the thickness direction (*z*) of the plate, Young's modulus and density are varied using power law as [6],

$$E(z) = E_m + (E_c - E_m)\left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{1}$$

$$\rho(z) = \rho_m + (\rho_c - \rho_m)\left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{2}$$

where, n refers as gradient index and subscripts m and c denote the metal and ceramic constituents, respectively.

Table 1: Material properties of FGM plate [5]

	E (GPa)	v	$\rho\left(kg/m^3\right)$
Aluminum (Al)	70	0.30	2702
Alumina (Al_2O_3)	300	0.30	3800

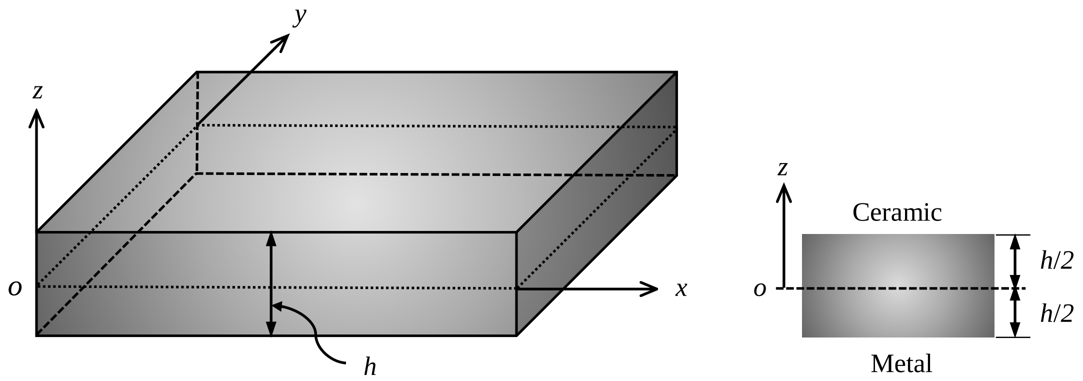

Figure 1: A schematic of cracked FGM plate

2.2 Simple First-Order Shear Deformation Plate Theory

The displacement field at any point (x, y, z) in the plate based on S-FSDT is given as [6],

$$\begin{aligned} u(x,y,z) &= u_o(x,y) - z\frac{\partial w_b(x,y)}{\partial x} \\ v(x,y,z) &= v_o(x,y) - z\frac{\partial w_b(x,y)}{\partial y} \\ w(x,y,z) &= w_b(x,y) + w_s(x,y) \end{aligned} \tag{3}$$

where, u_o and v_o represent the mid-plane displacements in x and y directions respectively; w_b and w_s represent the bending and shear components of transverse displacement (w), respectively.

Assuming the small strain condition, the non-zero strains are related with the displacement field given in Eq. (3) as,

$$\begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\gamma} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\varepsilon}_0 - z\boldsymbol{\kappa} \\ \boldsymbol{\gamma}_s \end{Bmatrix} \tag{4}$$

where,

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix},\ \boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix},\ \boldsymbol{\varepsilon}_0 = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial v_0}{\partial y} + \dfrac{\partial u_0}{\partial x} \end{Bmatrix},\ \boldsymbol{\kappa} = \begin{Bmatrix} \dfrac{\partial^2 w_b}{\partial x^2} \\ \dfrac{\partial^2 w_b}{\partial y^2} \\ 2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix} \text{ and } \boldsymbol{\gamma}_s = \begin{Bmatrix} \dfrac{\partial w_s}{\partial x} \\ \dfrac{\partial w_s}{\partial y} \end{Bmatrix}$$

The relationship between the stress and strain are related by the following equation as,

$$\boldsymbol{\sigma} = \mathbf{Q}(z)(\boldsymbol{\varepsilon}_0 - z\boldsymbol{\kappa}),\ \boldsymbol{\tau} = \mathbf{G}(z)\,\boldsymbol{\gamma} \tag{5}$$

$$\boldsymbol{\sigma} = \left\{\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{xy}\right\}^T,\ \boldsymbol{\tau} = \left\{\tau_{xz} \quad \tau_{yz}\right\}^T,\ \mathbf{Q}(z) = \frac{E(z)}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \text{ and } \mathbf{G}(z) = \frac{k\,E(z)}{2(1+\nu)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where, k is the shear correction factor (SCF). In the present work, SCF is taken as k = 5/6. Next, using the Hamilton principle, the weak form for free vibration analysis of a FGM plate can be expressed as,

$$\int_\Omega \delta\boldsymbol{\varepsilon}_b^T \mathbf{D}^b \boldsymbol{\varepsilon}_b\, d\Omega + \int_\Omega \delta\boldsymbol{\gamma}_s^T \mathbf{D}^s \boldsymbol{\gamma}_s\, d\Omega = \int_\Omega \delta\tilde{\mathbf{u}}\,\mathbf{m}\,\ddot{\tilde{\mathbf{u}}}\, d\Omega \tag{6}$$

where, $\boldsymbol{\varepsilon}_b = \begin{bmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\kappa} \end{bmatrix}$, $\mathbf{D}^b = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}$, $\mathbf{D}^s = \int_{-h/2}^{h/2} \mathbf{D}_s(z)\,dz$, $\mathbf{m} = \begin{bmatrix} I_0 & I_1 \\ I_1 & I_2 \end{bmatrix}$ and $\tilde{\mathbf{u}} = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix}$

with $\mathbf{A},\mathbf{B},\mathbf{D} = \int_{-h/2}^{h/2} (1, z, z^2)\,\mathbf{Q}(z)\,dz$, $I_0, I_1, I_2 = \int_{-h/2}^{h/2} (1, z, z^2)\,\boldsymbol{\rho}(z)\,dz$ and $\boldsymbol{\rho} = \begin{bmatrix} \rho & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \rho \end{bmatrix}$

3. Bézier Extraction of NURBS

Bézier extraction represents the NURBS basis function over each element in the form of Bernstein polynomial defined over C^0 continuous isogeometric Bézier element. Bézier element representation is given by Borden *et al*. [15] for the NURBS and further explored by Scott *et al*. [16] for T-spline. In order to decompose the NURBS basis functions in to Bernstein polynomial basis, Bézier decomposition is used. For more detail interested readers are encouraged to follow these papers [15-17].

4. Extended Isogeometric Analysis (XIGA)

The XIGA uses the merits of IGA and partition of enrichment (PU) concept for the fracture analysis of stationary and quasi-static crack growth [8, 9, 14]. In XIGA, the crack is modeled through enrichment functions added in the standard IGA approximation. At a particular point $\mathbf{x} = (x, y)$, the displacement approximation for the crack based on Bézier extraction of NURBS is written as,

$$\mathbf{u}^{\mathbf{h}}(\mathbf{x}) = \sum_{i=1}^{n_{en}} R_i(\mathbf{x})\mathbf{u}_i + \underbrace{\sum_{j=1}^{n_{cf}} R_j(\mathbf{x})\left[H(\mathbf{x}) - H(\mathbf{x}_j)\right]\mathbf{a}_j + \sum_{k=1}^{n_{ct}} R_k(\mathbf{x})\left\{\sum_{\alpha=1}^{4}\left[\beta_\alpha(\mathbf{x}) - \beta_\alpha(\mathbf{x}_k)\right]\mathbf{b}_k^\alpha\right\}}_{\text{Crack}} \tag{7}$$

where, $R_i(\mathbf{x})$ is the NURBS basis functions which is written in the terms of Bernstein polynomial basis functions and Bézier extraction, n_{en} indicates the total number of control points per element and $\mathbf{u}_i = \{u_o, v_o, w_b, w_s\}^T$ indicates the degrees of freedom (DOFs) per control point i in any NURBS element. Moreover, n_{cf} and n_{ct} represent the set of control points associated with all those elements which possess crack face and crack-tip respectively. Additionally, n_{cf} enriched with Heaviside function, $H(\mathbf{x})$ whereas n_{ct} enriched with asymptotic crack tip enrichment functions, $\beta_\alpha(\mathbf{x})$. The $\beta_\alpha(\mathbf{x})$ are taken from the Ref. [14]

Substituting Eq. (7) into Eq. (4), the strains are given as,

$$\begin{bmatrix}\boldsymbol{\varepsilon}_o & \boldsymbol{\kappa} & \boldsymbol{\gamma}_s\end{bmatrix} = \sum_{i=1}^{n_{en}} \begin{bmatrix}\mathbf{B}_i^m & \mathbf{B}_i^b & \mathbf{B}_i^s\end{bmatrix}\{\mathbf{d}_i\} \tag{8}$$

where, $\mathbf{B} = \left[\,\mathbf{B}^{std} \,\middle|\, \mathbf{B}^{enr}\,\right]$ and $\mathbf{d} = \{\mathbf{u}, \mathbf{a}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4\}$

Substituting Eq. (8) into Eq. (6), the following form is obtained,

$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right)\mathbf{d} = 0 \tag{9}$$

where, the $\mathbf{K}$ and $\mathbf{M}$ are global stiffness and mass matrix, respectively. The expression of $\mathbf{K}$ and $\mathbf{M}$ are obtained as provided in Ref. [6].

5. Results and Discussions

In this section, the free vibration analysis of cracked FGM plates using S-FSDT in the context of XIGA based on Bézier extraction approach is performed. Several rectangular and square FGM plates having center crack configuration are considered. Unless stated otherwise, ceramic-metal FGM plates whose material properties given in Table 1 are considered. Cubic NURBS basis functions are used in either direction throughout this study, as it provides faster convergence [11]. In all examples, a full integration using $(p+1)\times(q+1)$ Gauss points are used for standard (non-enriched elements) and sub-triangulation scheme for the enriched elements [12]. Moreover, three different boundary conditions are considered on the edges of plate such as; SSSS, FCFF and CFCF, where, S, F and C represent simply supported, free and clamped respectively. The simply supported boundary condition (S) used in this paper is represented as,

$$\begin{aligned} v_o = w_b = w_s = 0,\ \ x = 0, a \\ u_o = w_b = w_s = 0,\ \ y = 0, b \end{aligned} \tag{10}$$

whereas the clamped boundary condition is given as [6],

The percentage difference of normalized natural frequencies is obtained as,

$$\text{Percentage diff.} = \left(\frac{2 \times \left| \bar{\omega}^{\text{Present}} - \bar{\omega}^{\text{Reference}} \right|}{\bar{\omega}^{\text{Present}} + \bar{\omega}^{\text{Reference}}} \right) \times 100 \tag{11}$$

As shown in Fig. 2, the rectangular FGM plate with planar dimension $(a \times b)$ and uniform thickness h containing a through-thickness center crack of length d is considered. Before proceeding to the free vibration analysis of cracked FGM plates, initially a convergence study of the normalized natural frequency of cracked homogeneous plate is performed. A fully simply supported (SSSS) homogeneous rectangular plate with a/b = 1, b/h = 10, d/a = 0.3 and material properties of aluminum alloy as given in Table 1 is considered. The normalized natural frequency $\left(\text{i.e. } \bar{\omega} = \omega \times \frac{b^2}{h} \times \sqrt{\frac{\rho}{E}} \right)$ obtained using S-FSDT and XIGA based on Bézier approach is presented in Table 2. It is observed that normalized frequencies obtained using S-FSDT based XIGA match well with the 3-D elasticity results [5]. Moreover, as the number of control points increases from 32×32 to 42×40 the results converge to two significant figures. Hence, for the subsequent examples 32×32 or more number of control points will be used.

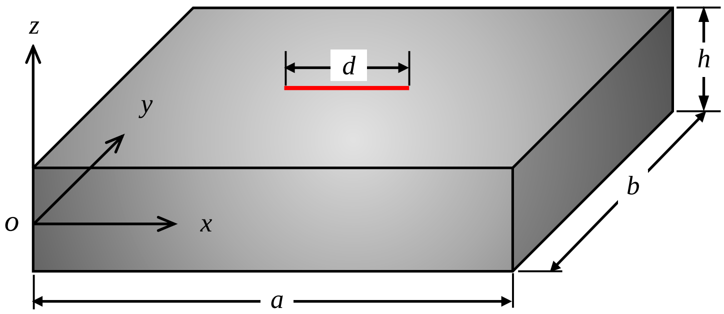

Figure 2: A rectangular FGM plate with center crack

To further illustrate the accuracy of proposed method, the normalized natural frequencies is obtained for different b/h and d/a ratios for the SSSS square homogeneous plate. The material properties are taken same as the previous example. Table 3 presents the normalized natural frequencies evaluated using S-FSDT based XIGA are compared with 3D elasticity approach [5]. It is found that the for both thick and thin plates the normalized natural frequencies obtained using present approach are in good agreement with 3D elasticity results. The maximum percentage difference between their results are within 4.21% for Mode 2 with b/h = 20 and d/a = 0.5.

Table 2: Normalized natural frequency of SSSS square homogeneous plate (b/h = 10) with center crack (d/a = 0.3)

Method	Number of control points	Mode		
		1	2	3
S-FSDT based XIGA	20×20	5.4532	13.4416	13.7265
	24×24	5.4710	13.4095	13.7282
	28×28	5.5244	13.4065	13.7373
	32×32	5.5234	13.3988	13.7369
	36×36	5.5225	13.3962	13.7368
	40×40	5.5226	13.3962	13.7369
3D elasticity [5]		5.421	13.22	13.76

Table 3: Normalized natural frequency of SSSS square homogeneous plate with center crack

d/a	*b/h*	Method	Mode				
			1	2	3	4	5
0.3	5	S-FSDT based XIGA	5.0799	11.0129	11.5279	16.6322	18.6180
		3D elasticity [5]	4.960	10.84	11.61	16.64	18.06
		% Difference	2.39	1.58	0.71	0.05	3.04
	10	S-FSDT based XIGA	5.5224	13.3887	13.7369	21.0819	23.7780
		3D elasticity [5]	5.421	13.22	13.76	20.97	23.13
		% Difference	1.85	1.27	0.17	0.53	2.76
	20	S-FSDT based XIGA	5.6573	14.3151	14.5760	23.0514	26.1400
		3D elasticity [5]	5.590	14.21	14.57	22.94	25.62
		% Difference	1.2	0.74	0.04	0.48	2.01
	100	S-FSDT based XIGA	5.7031	14.6623	14.8871	23.8391	27.1033
		3D elasticity [5]	5.701	14.65	14.89	23.82	27.11
		% Difference	0.04	0.08	0.02	0.08	0.02
0.5	5	S-FSDT based XIGA	4.8180	8.7417	11.4390	15.4412	16.7323
		3D elasticity [5]	4.633	8.764	11.43	15.97	16.89
		% Difference	3.91	0.25	0.08	3.37	0.94
	10	S-FSDT based XIGA	5.2063	11.0595	13.6018	20.7129	22.0429
		3D elasticity [5]	5.069	11.10	13.55	20.35	21.44
		% Difference	2.67	0.37	0.38	1.77	2.77
	20	S-FSDT based XIGA	5.3232	12.8075	14.4199	22.7085	24.0426
		3D elasticity [5]	5.238	12.28	14.37	22.44	23.60
		% Difference	1.61	4.21	0.35	1.19	1.86
	100	S-FSDT based XIGA	5.3628	13.1284	14.7227	23.5087	24.8437
		3D elasticity [5]	5.353	12.98	14.72	23.46	24.79
		% Difference	0.18	1.14	0.02	0.21	0.22

Next, an Al/Al_2O_3 center cracked square FGM plate is considered and the effect of various parameters such as; different boundary conditions, length to thickness ratio (b/h) and gradient index (n) on normalized natural frequencies is analyzed as presented in Table 4. In this case, the normalized natural frequency is obtained as $\bar{\omega} = \omega \times \frac{b^2}{h} \times \sqrt{\frac{\rho_c}{E_c}}$, where c represents the material properties corresponding to ceramic (Al_2O_3) in Al/Al_2O_3 FGM plate. Table 4 reveals that the normalized natural frequencies obtained using proposed method are well matched with the 3D elasticity results. However, for FCFF boundary condition the maximum percentages in normalized Mode 2 frequency is seen. Besides, the normalized frequencies increase with increasing the b/h ratios and decreases as gradient index (n) increases. It is also observed that the normalized frequencies for FCFF boundary condition is less as compared to SSSS and CFCF boundary conditions. Finally, the contour of first mode shape of square Al/Al_2O_3 FGM plate (b/h = 50) having center crack (d/a = 0.3) with CFCF and SSSS boundary conditions is shown in Fig. 3.

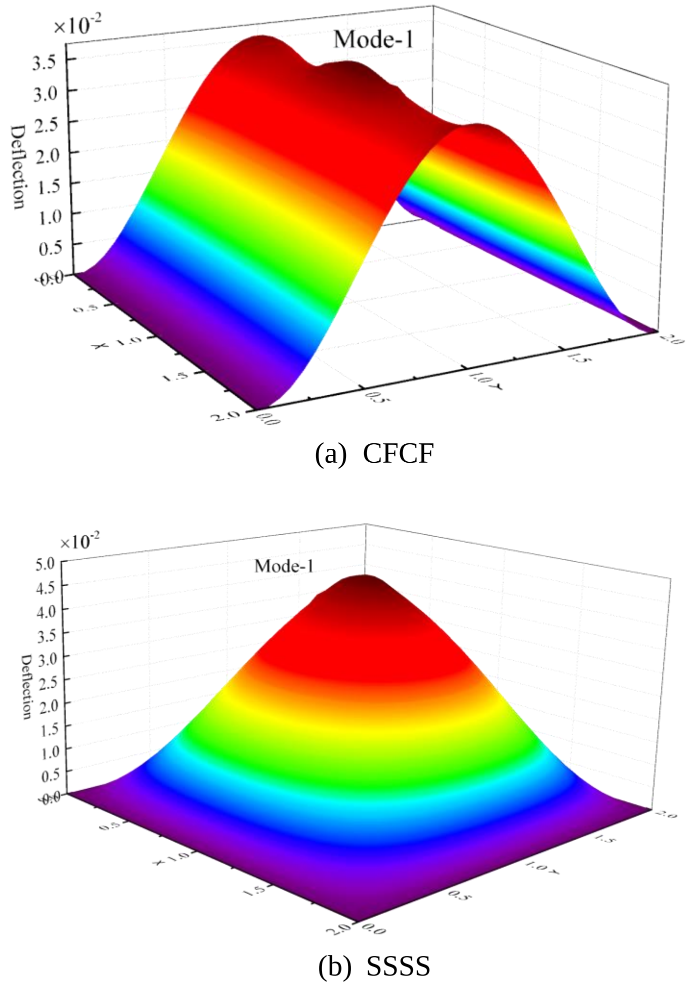

Figure 3: First mode shape of square Al/Al_2O_3 FGM plate (b/h = 50) having center crack (d/a = 0.3)

Table 4: Normalized natural frequency of Al/Al_2O_3 square FGM plate with center crack (d/a = 0.3)

BCs	*n*	*b/h*	Method	Mode				
				1	2	3	4	5
SSSS	0	5	S-FSDT based XIGA	5.0799	9.7269	9.7417	11.0129	11.5279
			3D elasticity [5]	4.959	9.728	9.742	10.84	11.60
			% Difference	2.41	0.01	0.00	1.58	0.62
		50	S-FSDT based XIGA	5.6973	14.6173	14.8468	23.7352	26.9756
			3D elasticity [5]	5.665	14.58	14.84	23.68	26.71
			% Difference	0.57	0.26	0.05	0.23	0.99
	0.2	5	S-FSDT based XIGA	4.7338	9.2672	9.2813	10.3189	10.7870
			3D elasticity [5]	4.627	9.266	9.280	10.18	10.88
			% Difference	2.28	0.01	0.01	1.36	0.86
		50	S-FSDT based XIGA	5.2876	13.5678	13.7804	22.0323	25.0406
			3D elasticity [5]	5.259	13.53	13.78	21.99	24.80
			% Difference	0.54	0.28	0.00	0.19	0.97
	5	5	S-FSDT based XIGA	3.3182	6.3156	6.3259	7.1625	7.4900
			3D elasticity [5]	3.185	6.274	6.296	6.823	7.322
			% Difference	4.10	0.66	0.47	4.86	2.27
		50	S-FSDT based XIGA	3.7498	9.6192	9.7695	15.6156	17.7484
			3D elasticity [5]	3.725	9.581	9.760	15.56	17.53
			% Difference	0.66	0.40	0.10	0.36	1.24
FCFF	0	5	S-FSDT based XIGA	1.0164	2.4343	3.2024	5.3565	6.7674
			3D elasticity [5]	1.016	2.195	3.221	5.359	6.285
			% Difference	0.04	10.3	0.58	0.05	7.39
		50	S-FSDT based XIGA	1.0494	2.5723	6.4260	7.8198	9.3068
	0.2	5	S-FSDT based XIGA	0.9445	2.2627	3.0515	5.0015	6.3027
			3D elasticity [5]	0.9441	2.049	3.069	5.010	5.869
			% Difference	0.04	9.91	0.57	0.17	7.13
		50	S-FSDT based XIGA	0.9739	2.3872	5.9641	7.2575	8.6378
	5	5	S-FSDT based XIGA	0.6674	1.5880	2.0887	3.4704	4.3767
			3D elasticity [5]	0.6633	1.406	2.098	3.394	3.992
			% Difference	0.62	12.2	0.44	2.23	9.19
		50	S-FSDT based XIGA	0.6907	1.6929	4.2288	5.1463	6.1245
CFCF	0	5	S-FSDT based XIGA	5.2039	6.6281	8.8744	10.3010	12.1163
		50	S-FSDT based XIGA	6.4333	7.9599	12.7911	18.2150	20.1967
	0.2	5	S-FSDT based XIGA	4.8705	6.1928	8.4553	9.6195	11.3993
		50	S-FSDT based XIGA	5.9712	7.3879	11.8719	16.9093	18.7481
	5	5	S-FSDT based XIGA	3.4024	4.3168	5.7698	6.6574	7.8857
		50	S-FSDT based XIGA	4.2342	5.2385	8.4171	11.9857	13.2888

6. Conclusions

In this work, the free vibration analysis of cracked FGM plates using S-FSDT in the context of XIGA based on Bézier extraction approach is successfully performed. The gradation of material properties is taken along the thickness of the plate. The bottom end of the plate possesses 100% alloy while top end possesses 100% ceramic. The material properties (i.e. Young's modulus & density) vary using power law from bottom to top end of the plate. NURBS basis functions obtained from Bézier extraction technique are used for defining the geometric description and solution approximation. The values of normalized frequencies are obtained using present method are found in good agreement with 3D elasticity solutions. Moreover, the normalized natural frequencies are significantly affected by the b/h ratios, crack aspect ratios (d/a), gradient index (n) and boundary condtions.

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