Weak Shock Reflection from Blunt Bodies with the High-Order Numerical Simulation Using an Immersed Boundary Method

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Abstract

In this paper, we used an immersed boundary method to simulate the compressible flow field around a curvilinear body. The reflection of a moving shock wave is studied from a cylinder and other blunt bodies. WENO(weighted essentially non-oscillatory) scheme is a widely known high-order method for the simulation of compressible flow, but the implementation for an arbitrarily given boundary condition is hard to make using coordinate transformation in FDM(finite difference method) or polygonal control volume in FVM(finite volume method). Therefore, we have set a numerical code without skewness of grids by use of simple Cartesian coordinates system. At the boundary, the conservative condition with the slip velocity is guaranteed with a simple mathematical treatment called the immersed boundary condition. The reflected shock waves computed from blunt bodies are compared with experimental results.

Keywords: Euler equation, WENO scheme, Immersed boundary method

Introduction

Immersed Boundary Method(IBM) can be used for the numerical computation using the structured meshes in Cartesian coordinate for a complex geometry. Peskin[1], proposing modifications of original IBM, analyzed the blood flow around the heart valves, and Chudhuri et al.[2], using WENO scheme in conjunction with IBM, simulated the interaction of high Mach number shock waves with wedge and blunt obstacles.

In this paper, we simulate the weak shock waves reflected from blunt bodies such as a cylinder with WENO scheme[3] linked with IBM.

Governing equation

The Euler equation governing two-dimensional compressible inviscid flows can be written in conservative form as,

$$\frac{\partial Q}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=0 \tag{1}$$

$$\begin{aligned} Q &= [\rho, \rho u, \rho v, E] \\ F &= [\rho u, \rho u^2 + p, \rho uv, u(E+p)] \\ G &= [\rho v, \rho uv, \rho v^2 + p, v(E+p)] \end{aligned} \tag{2}$$

where Q is convection variables vector; F, G are inviscid flux variables vectors; ρ, u, v, p, E are density, x-direction velocity, y-direction velocity, pressure, and total energy, etc.

Immersed Boundary Method(IBM)

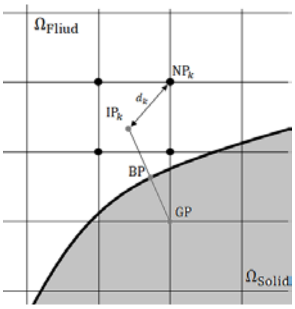

Figure 1. Immersed boundary treatment

Flow variables ϕ_{IP} in imaginary point can be evaluate as,

$$\phi_{IP} = \sum_{k=1}^{4} \delta_k \phi_{NP} \tag{3}$$

where δ_k is Inversed Distance Weight(IDW) of interpolation, and the flow variable ϕ_{BP} in the boundary point can be evaluate from Ref. [4]. In Eq. (3), the IDW is defined as

$$\delta_k = \eta_k \left(\sum_{k=1}^{4} \eta_k \right)^{-1}, \quad \eta_k = 1/d_k \tag{4}$$

where d_k is the distance between the imaginary point(IP_k) and a neighboring grid point(NP_k), which are marked in Fig. 1.

To calculate the flow variable ϕ_{GP} in ghost point using a Dirichlet boundary condition.

$$\phi_{GP} = 2\phi_{BP} + \phi_{IP} \tag{5}$$

where the ϕ_{BP} means the variable at the boundary.

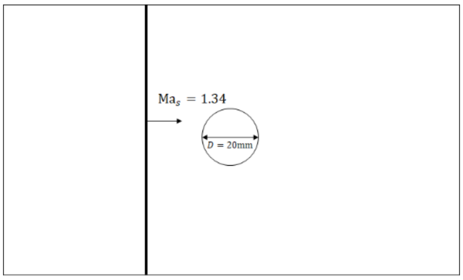

Figure 2. Schematic diagram for the weak shock wave reflected blunt body.

Numerical simulation

Fig. 2 shows the computational domain and the initial condition for the planar moving shock impinging into a cylinder, for example. The shock Mach number is set $Ma_s = 1.34$, and the pressure before and after shock can be simply computed with Rankine-Hugoniot condition. The specific heat ratio of air is fixed to 1.4.

The density contours at each time step can be visualized in Fig. 3, and the fifth-order WENO scheme is used with third-order TVD(total variation diminishing) Runge-Katta time integration for the numerical result in the present figure. In every time step, holographic interferograms[5] are compared with isopycnics, or density contours obtained from computation. The number of nodes is 258,496 and number of elements is 257,336.

Mach reflection consisting of Mach triple points and Mach stems is reflected in front of the circular cylinder. A slip line is visible as the disturbed isolines, and the Mach stems intersect each other to form a secondary Mach reflection at the aft of the cylindrical body, which is called a ‘shock-shock reflection’.

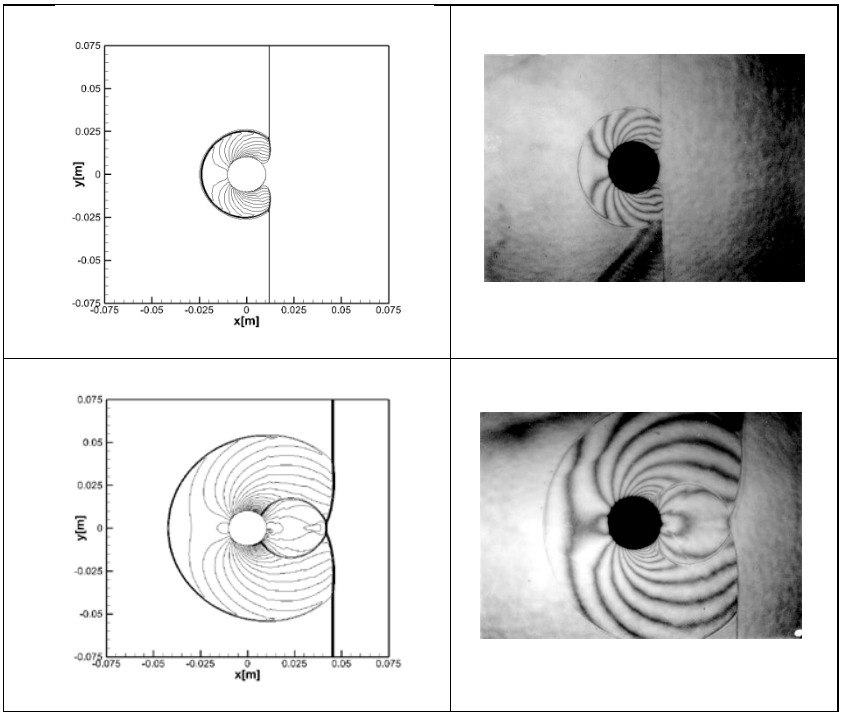

Figure 3. Numerical isopycnics using WENO with IBM(left) and experimental interferogram(right): 48, and 120μs[5]

Summary

In this paper, we have simulated two-dimensional compressible flow induced from a moving shock reflection from a circular cylinder. The incident Mach number is $Ma_s = 1.34$, and the boundary treatment from the implementation of an immersed boundary method is successfully applied for the high-order WENO method. The numerical results coincide with those of experiment, and describe delicate wave physics in detail for every time step. This method is also valid for other geometries if we just change the shape of blunt bodies.

Acknowledgement

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