A semi-Lagrangian meshfree method for flow problems with strong convection

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Abstract

The flow of viscous fluids is often governed by the Navier-Stokes equation. In the Eulerian framework, both convection and diffusion terms exist in the equations. Numerical difficulties often arise when the diffusion term is dominated by the convection term. In order to obtain stable results, numerical methods with up-wind property must be used. However, for the methods derived from Galerkin projection, such as finite element method and element free Galerkin method, designing a numerical scheme with up-wind property is rather difficult. On the other hand, in the Lagrangian framework, convection term no longer presents in the equations. What we confront is pure diffusion equations, and standard Galerkin methods can be directly used in solving these equations. Although there is no difficult in solving the equations, new difficulties arise from grid deformation. A smart idea is to combine the advantages from both Eulerian and Lagrangian frameworks while eliminating the disadvantages of each. That is to solve the pure diffusion equations from Lagrangian framework on a fixed grid from Eulerian framework. Such a method is named as Eulerian–Lagrangian method or semi-Lagrangian method. Historically, this kind of method is first present in the field of large scale atmospheric simulations for numerical weather prediction, and now it has been broadly used in solving the convection-diffusion equations as well as the Navier-Stokes equation. The semi-Lagrangian method involves backward time integration of a characteristic equation to find the departure point of a fluid particle arriving at an Eulerian grid point. The solution value at the departure point is then obtained by interpolation. There is no grid deformation as in Lagrangian methods because the "arrival points" employed coincide with the grid points. Note that the backward integration only gives us the departure point but tells us nothing about which grid cell this point in located. Therefore, we need to propose some algorithms of searching the elements of the grid to determine which element contains the departure point in order to interpolate the solutions at this point. For grid based methods, there may be significant interpolation cost to obtain the solution values at the "departure points", especially for unstructured grids. In this talk, we will present a semi-Lagrangian meshfree method. Because interpolations of the meshfree method use no grid, searching algorithms will not need in interpolating the values at the departure points. In addition, constructing high order interpolation functions is trivial for the meshfree method. Therefore, the proposed method is expected having better accuracy and higher efficiency. Several numerical examples in 1D and 2D are used to verify the good performance of the new method for flow problems with strong convection.

Keywords: Meshfree, semi-Lagrange, flow, convection-dominated