Application of varying order B-splines discretization for accurate peeling computations

Vishal Agrawal, Saipraneeth Gouravaraju*, and Sachin Singh Gautam†

Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati 781039, India.

*Presenting author: saipraneeth@iitg.ac.in, †Corresponding author: ssg@iitg.ac.in

Abstract

Numerical analysis of peeling of an adhesive strip involves unique computational challenges that arise due to highly nonlinear adhesive forces acting in a very narrow zone. This often leads to instability and poor convergence rate in the contact computations. In order to address these issues using standard finite element based formulation, a very fine mesh is required, which in turn undesirably increases the computational cost. Motivated by the superior accuracy and stability of isogeometric analysis compared to the standard finite element, this work employs a varying order B-splines based discretization for the peeling computations. Corresponding results are then compared with the existing finite element based local enrichment strategies. It is shown that the varying order B-splines discretization technique delivers more accurate results compared to existing enriched contact finite elements.

Keywords: Peeling, adhesion, computational contact, enriched finite elements, B-splines, isogeometric analysis

Introduction

Studying the peeling behaviour is of great importance in many engineering applications, e.g. in adhesive bonding and debonding of thin films, coatings, and biological adhesive systems such as geckos and insects [1]. Numerical modelling and analysis of peeling problems poses challenges due to the presence of large stress gradient within a very narrow peeling zone. Standard finite element based simulations requires a very fine mesh to accurately capture the peeling behaviour. However, this approach is computationally expensive and becomes inefficient if a very fine mesh is utilized [2]. Sauer [2], introduced two different local contact finite element enrichment strategies based on the incorporation of higher order Lagrange and Hermite interpolation functions for peeling computations at a coarser mesh compared to standard finite element (FE) formulation. With this, more accurate and stable results are obtained compared to standard FE as higher order Lagrange polynomials are employed for the evaluation of contact integrals.

Based on the superiority of NURBS over Lagrange interpolations in terms of accuracy and stability, in the present work, the application of NURBS based isogeometric analysis technology [3, 4] is investigated for the peeling computations. From the analysis point of view, it is known that the accuracy of the contact solution is mainly governed by the interactions at the contact interface. Therefore, in this work, higher order NURBS interpolations are employed for the evaluation of contact integrals and a lower order NURBS are used for the description of the bulk domain. Adhesion between a deformable strip and a rigid substrate is simulated using these proposed methodology. Adhesion is assumed to be due to the van der Waals interactions between the molecules of the strip and the substrate. The adhesive contact is modelled using the continuum based coarse-grained contact model of Sauer and Li [5].

Adhesive contact tractions

The contact tractions $\mathbf{T}_\mathrm{c}$ due to the van der Waals adhesion between the strip and the substrate is given as [5]

$$\mathbf{T}_\mathrm{c} = \frac{A_H}{2\pi {r_o}^3}\left[\frac{1}{45}\left(\frac{r_o}{r_s}\right)^9 - \frac{1}{3}\left(\frac{r_o}{r_s}\right)^3\right]\boldsymbol{n}_p \qquad (1)$$

where A_H, r_o, and r_s denotes the Hamaker's constant, equilibrium spacing of interacting particles, and the minimum distance between contacting bodies, respectively. Further, $\boldsymbol{n}_p$ denotes the surface orientation of the substrate and is constant for flat surfaces.

Varying order B-splines

For bulk elements, p_1 and p_2 order of bivariate B-splines interpolations are employed that are defined using the tensor product of p_1 and p_2 order of univariate $B_1^{p_1}(\xi)$ and $B_2^{p_2}(\eta)$ functions along the ξ and η parametric directions [4]. In order to improve the approximation of contact responses, higher-order, i.e. $p_c > p_1$, B-splines $B_1^{p_c}(\xi)$ are used for the representation of contact interface. The bivariate B-spline basis functions for the derived contact elements are given by:

$$\begin{aligned} B_1^{p_c,p_2}(\xi,\eta) &= B_1^{p_c}(\xi)\cdot B_2^{p_2}(\eta), \\ &\vdots \\ B_{p_c+1}^{p_c,p_2}(\xi,\eta) &= B_{p_c+1}^{p_c}(\xi)\cdot B_2^{p_2}(\eta). \end{aligned} \qquad (2)$$

Problem set-up

A deformable strip having dimensions $L \times h$ (with $L = 200R_0, h = 10R_0$ with $R_0 = 1$ nm) is initially lying flat at an equilibrium distance on the flat rigid substrate (Figure 1). Neo-Hookean material model with Young's modulus $E = 2$ GPa and Poisson's ratio $\upsilon = 0.2$, under plain strain conditions is considered [2]. Adhesion is assumed to be present in the bottom 75% of the strip surface ("AE" in Figure 1). Adhesive forces are calculated using Eq. (2) with $r_0 = 0.4$ nm and $A_H = 10^{-19}$ J [2]. An external rotation angle θ is applied on the right end (CD) of the strip with a step size of $\Delta\theta = 0.1^o$. Two different mesh sizes with $n_y = 12$ and $n_x = 240$ elements in each directions are considered.

Results and Discussion

Bending moment required to rotate the right end of the strip by an angle θ remains constant after a certain rotation angle (at approx. 60°, see Figure 2(a)). However, as shown in Figure 2(b), the bending moment curves display non-physical oscillations around a mean line rather than being constant. These oscillations occur due to inaccurate capture of the contact tractions that affect the accuracy of contact responses. Thus, to improve the approximation of contact tractions, fifth-order B-splines are used. To achieve a large number of degrees of freedom across

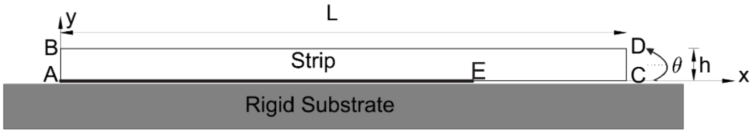
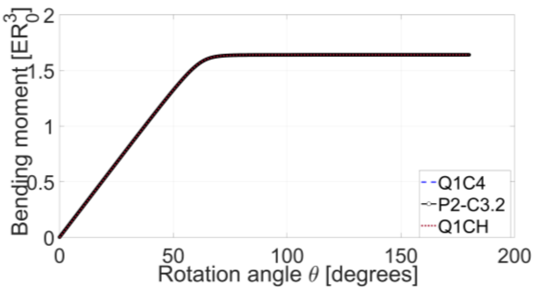


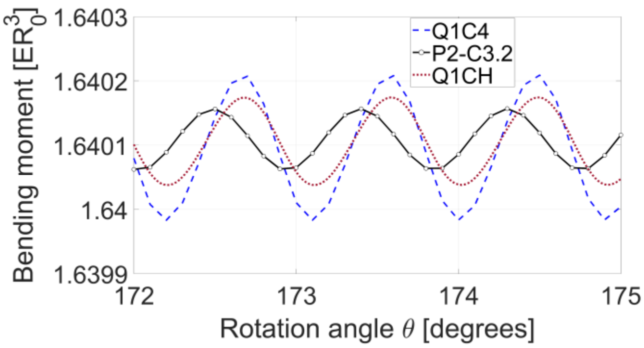
Figure 1. A deformable strip on a rigid substrate.

the contact interface with fifth order B-spline based contact curve, two steps of additional order elevation refinements over the cubic-order B-spline curves are applied. Such an application of order elevation refinement on cubic-order B-spline based contact curve is denoted as C3.2. On the other hand, in order to minimize the overall computational cost associated with this simulation, quadratic-order of B-splines are used for the bulk description, i.e. denoted by P2. Thus, the resultant discretization is accordingly denoted as P2-C3.2.

The plot of bending moment versus rotation angle is shown in Figure 2 for P2-C3.2 and FE-based Q1C4 and Q1CH enriched elements. It can be observed that P2-C3.2 element yields best results with the least oscillation error among all the elements.



(a)



(b)

Figure 2. Comparison of bending moment obtained using proposed discretization method with FE-based discretization strategies of [2].

Conclusions

It is shown that the proposed varying order B-splines based discretization within the framework of isogeometric analysis delivers improved results compared to existing FE-based surface enrichment strategies. For the same mesh size, more accurate result is obtained using the proposed method compared to FE-based enrichment strategies.

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