An Enriched Scaled Boundary Finite Element Method for 3D Stress Singularities in Anisotropic Multi-Material Configurations

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Abstract

A direct enrichment approach for the Scaled Boundary Finite Element Method (SBFEM) is proposed.The SBFEM has proven its effectiveness in the treatment of 2D boundary value problems with stress singularities. Here, the high stress gradients towards the singularity can be represented analytically (in the scaling coordinate ξ) while the smooth stress functions in the circumferential direction are approximated using a finite element discretization (in the boundary coordinate η).

Unfortunately, when applied to 3D problems with line singularities (e.g. along a crack front), the SBFEM suffers from reduced accuracy and slow convergence. This is because, in the presence of line singularities, it can no longer be avoided that singularities also occur in the discretized boundary coordinates η_1 and η_2 of the SBFEM.

It is shown that appropriately enriching the SBFEM's characteristic separation of variables approach with the 2D asymptotic near-fields of the line singularities recovers the optimal convergence rates of the method:

$$\tilde{u}_i(\boldsymbol{x}) = \sum_{k\in\mathcal{N}} N_k(\boldsymbol{x_\eta})\,\tilde{u}_{ik}(\xi) \quad + \sum_{l\in\mathcal{P}} N_l(\boldsymbol{x_\eta}) \left[\sum_{s=1}^{n_S}\sum_{m=1}^{n_F}\left(F_{ism}(\boldsymbol{x_{r\varphi}}) - \sum_{k\in\mathcal{N}} N_k(\boldsymbol{x_\eta})\,F_{ism}(\boldsymbol{x_{r\varphi}}_k)\right) a_{sm}(\xi)\right]$$

with shape functions *N* and enrichment functions *F*. In this context, the required 2D asymptotic near-field functions can be obtained using an efficient and very general 2D-SBFEM approach for generalized plane strain conditions.

The presented approach is uniquely DOF-efficient. E.g., when the 3 classical crack modes shall be considered for the enrichment, the number of additionally required degrees of freedom is only 6 (two times the number of crack modes). Moreover, the direct enrichment approach is very general and can also be applied to anisotropic multi-material configurations with accordingly non-classical stress singularities.

As examples, two crack configurations in a cross-ply laminate set-up are considered and the advantages of the proposed Enriched Scaled Boundary Finite Element Method (enrSBFEM) are highlighted.