Approximate calculation of certain type of statically indeterminate truss

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Abstract

The paper presents examples of approximate calculations of force values in members of a certain type of truss, which is at the same time an internal and external statically indeterminate system. Static calculations are carried out for two selected forms of trusses by the application of the two-stage method of computations of such structural systems. The two-stage method, due to the application of the principle of superposition, makes possible the calculation of such complex trusses by the means of e.g. the Cremona's method. In this two-stage method the static calculations are done in two stages. In each stage a statically determinate truss system is considered, a pattern of which is defined by removing from the basic truss a suitable number of members. The number equals the degree of statically indeterminacy of the basic truss. In the paper are presented results of calculations of statically indeterminate trusses carried out by the two-stage method, which moreover are the external statically indeterminate systems. There are also presented results of calculations of the same trusses done by means of a suitable computer software together with the comparison of outcomes obtained in two different methods.

Keywords: Statically indeterminate system, Superposition principle, Calculus of vectors, Cremona's method, Approximate solution.

Introduction

The values of forces in members of the statically indeterminate trusses have to be determined by a suitable computational method, which among others takes into consideration the stiffness differences between members connected to the truss nodes. For this purposes are applied such methods like for instance, the force method, the displacement method, the iteration methods like the method of successive approximations, and the finite elements method, etc. [1]-[8]. Mathematic concepts of these methods are adapted in modern types of various computer software [9][10]. The two-stage method has been invented during initial static analysis of a certain type of a tension-strut structure [11]-[13]. If the basic structure is overloaded then a certain number of members are excluded form process of the force transmission. Number of these members is equal to the statically indeterminacy of the basic truss system. The point of the two-stage method is to carry out static calculations in two independent stages for statically determinate trusses, shapes of which are determined by removing from area of the basic truss the number of members equal to its statically indeterminacy. In each stage it is considered an appropriate statically determinate truss, therefore values of forces in its members can be calculated by means of e.g. the Cremona's method. In the both stages are considered suitable trusses having the same clear span and construction depth like the basic truss, but they are appropriately loaded by forces having half values in comparison to values of forces applied to the basic truss. Final values of forces acting in all members of the basic truss are resultants of the force values determined in each stage for the suitable member. The calculation procedure of the two-stage method is justified by rules of calculus of vectors and principle of superposition. It is one of the approximate methods of calculation of the statically indeterminate systems [14].

Definition of research problem

The correctness of the basic theoretic assumption of the two-stage method has been verified by calculating the internal statically indeterminate trusses supported on two supports, while the one is the pivot bearing and the second one is the pivot sliding bearing. It implies that there are only three unknown bearing reactions. Two forms of trusses were computed in order to prove feasibility study that the two-stage method can be useful for calculations of the internal statically indeterminate trusses being at the same time the external statically indeterminate trusses. The first one is the vertically positioned truss loaded by the means of horizontally applied forces and supported by two pivot bearings. The second one is the horizontally located truss loaded by the means of vertically and optionally applied forces, and which is supported by two pivot bearings. All the calculations are carried out for the same geometric, structural and load conditions. Results obtained in this way are compared with outcomes gained by the application of suitable computer software for calculation of the force values in a truss having the same geometric, structural and load conditions.

First subject of static calculation and analysis

The assumed form of the basic truss is built by the usage of three square strut modules located vertically on each other, see Fig. 1. The truss is supported by two pivot bearings A and B and it is loaded by three concentrated forces F horizontally applied to nodes of its left vertical chord. All calculations have been made for the single load forces F, each of value equal to 1.00 kN, applied to nodes of a basic truss. It was assumed that the construction depth of the truss tower "H" equals 1.00 meter, while its height "L" is equal to 3.0 meters. The number of nodes was assumed to denote by symbol "w", while symbol "p" to define number of the members. The condition for the internal statically determinacy of the plane truss is determined by the following equation:

$$p = 2 \cdot w - 3 \quad (1)$$

The considered basic truss system is built by number of nodes w = 11, what implies that the statically determinate truss created by the means of this number of nodes has to be constructed by the following number of members:

$$19 = 2 \cdot 11 - 3 \quad (2)$$

The basic truss system is built by the number of members p = 22, what implies that the calculated structure is the threefold statically indeterminate system. It implies further that in order to create an appropriate statically determined system it is necessary to exclude 3 members from area of the basic truss.

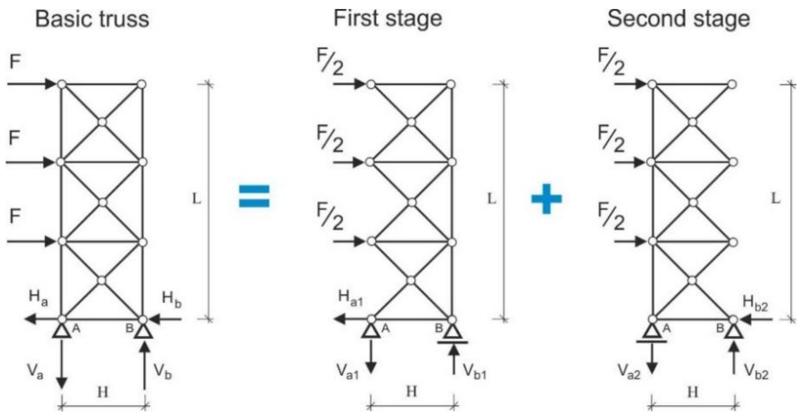

Figure 1. The concept of the two-stage method applied for calculation of a vertically positioned statically indeterminate truss

According to the rules of the two-stage method in its first stage one should remove three members, for instance from the left vertical chord of the basic truss, and then to apply load forces of half values (F/2) to suitable nodes of this chord. In the second stage it is necessary to remove three members from the right vertical cord and, like previously, to apply load forces of half values (F/2) having the same senses, like in the basic truss, to corresponding nodes of the left vertical chord. Because the basic truss is supported by two pivot bearings A and B, therefore it is also the external statically indeterminate system. That is why similar operations have to be undertaken regarding the changes of statuses of the supports. In the first stage it is proposed to keep support A as the pivot bearing and consider support B as the pivot sliding bearing, see Fig. 2. In the second stage support A is considered as the pivot sliding bearing, while support B remains the pivot bearing, see Fig. 3. Values of forces acting in members of the basic truss are resultants of forces calculated in corresponding members at each stage. The concept of the two-stage method is compatible with the rules of calculus of vectors, principle of superposition and respects the three fundamental conditions of equilibrium presented below:

$$\sum_{i=1}^{n} F_{ix} = 0 \tag{3}$$

$$\sum_{i=1}^{n} F_{iy} = 0 \tag{4}$$

$$\sum_{i=1}^{n} M_{i} = 0 \tag{5}$$

The results of the calculations of the basic truss, having a tower configuration, see Fig. 1, obtained in each stage of the two-stage method by application of the Cremona's method, are presented respectively in Fig. 2 and in Fig. 3. Final values of the forces, defined in this method, in all members of the basic truss are shown in Fig. 4a.

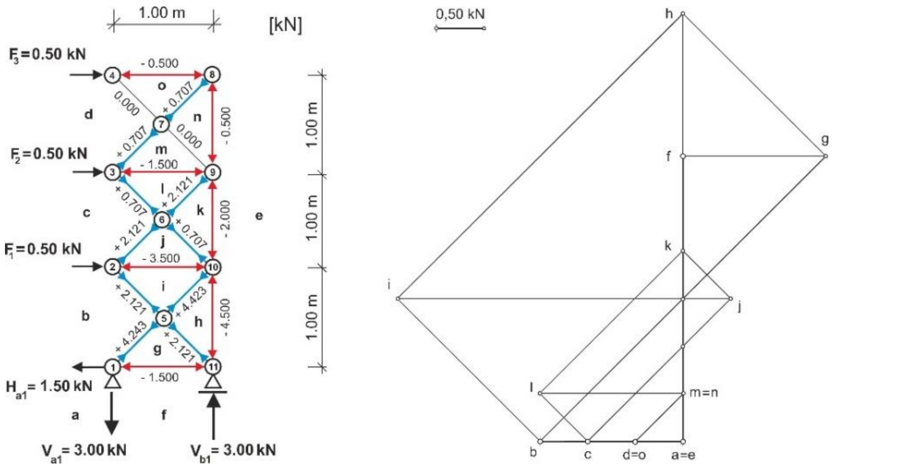

Figure 2. The values of the forces determined in the first stage of calculations for the truss of a tower configuration with suitable Cremona's polygon of forces

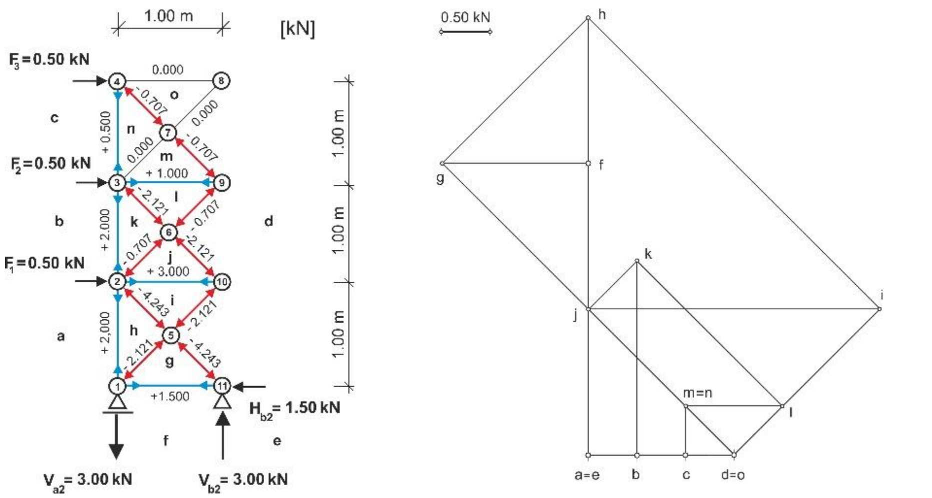

Figure 3. The values of the forces determined in the second stage of calculations for the truss of a tower configuration with suitable Cremona's polygon of forces

The same vertical, tower configuration of the basic truss has been subjected to the static calculation carried out by the application of the Autodesk Robot Structural Analysis Professional 2019. The computer software is considered to be the very precise tool for calculation of the force values acting in members of the statically indeterminate systems. Static calculations were made by the assumption that the truss consists of the steel tubular members having diameter of 30.00 mm, the thickness of the section equals to 4.00 mm and the steel material has the Young's modulus equal to 210 GPa. The results of the computer calculations of the same truss system are presented in Fig. 4b.

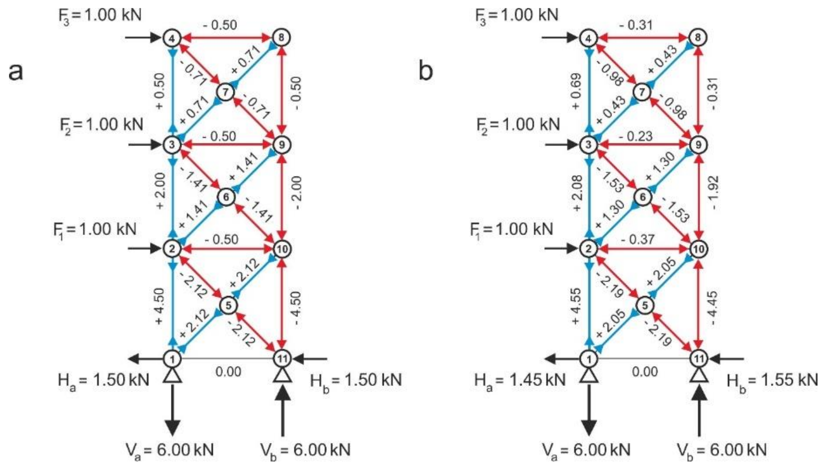

Figure 4. Comparison of the values of forces in members for the tower truss configuration calculated, a) in the two-stage method, b) by the means of computer software

In the two-stage method the final values of forces acting in particular members are calculated according to the rules of the calculus of forces and to principle of superposition. For instance the final value of a compressive force in the cross brace located between node 3 and node 6 is equal to -1.41 kN, see Fig. 4a. It is a resultant of the tensile force +0.707 kN determined in the first stage in corresponding member located between nodes of the same numbers, see Fig. 2, and the compressive force -2.121 kN determined in the second stage, see Fig. 3. In similar way the final force is defined in e.g. vertical member placed between node 3 and node 2. In the first stage of calculation this member has been rejected from the basic truss, see Fig. 2, therefore it is assumed, that in this case the appropriate force value equals 0.000 kN. In the second stage the value of tensile force defined in corresponding member is equal to +2.000 kN, see Fig. 3. Therefore the final force value in member located between nodes 2 and 3 equals +2.000 kN, see Fig. 4a.

From the comparison of the force values gained in both methods for the same truss members follows that in general the results are congruent to each other. For instance value of a compressive force determined in the member placed between node 3 and node 9 by application of the two-stage method is equal to -0.50 kN. The force value defined in the same member by application of the computer software mentioned above equals -0.23 kN. The relative difference equals up to 54% of the bigger value. One can notice a smaller differentiation between values of forces calculated in two different methods for the same member e.g. in vertical member placed between node 2 and node 1. The value of the tensile force calculated for this member in the two-stage method equals +4.50 kN, while by applying of the computer software it is equal to +4.55 kN. In this case the relative difference is equal to only ca. 1% of the bigger value. It is worthy to notice that values of all types of the suitable bearing reactions determined in both compared methods are the same or they are of very approximated values.

Second subject of static calculation and analysis

The scheme of the basic truss presented in Fig. 5 has been selected as an object of further comparative investigation in order to estimate the usefulness of the two-stage method for calculation of all types of the planar internal and external statically indeterminate systems. The basic truss is of similar structure like the previous one but it is located horizontally and supported by two supports, both being the pivot bearings. That is why four unknown bearing reactions have to be considered in these supports. It implies that the basic truss, like the previous one, is the threefold internal indeterminate system and once-fold external indeterminate system. Moreover the truss is loaded by the forces, which may be applied at any directions, what is represented by the direction of force F_1. This force is inclined at an angle of 45 degrees towards the horizontal line. According to the rules of the two-stage method presented above, in its first stage it is calculated truss, which form is obtained by removing three members of e.g. the bottom chord from the pattern of the basic truss. The shape of truss considered in the second stage is a result of the cancelation of three members of the top chord from the basic truss. In both stages the calculated truss is loaded by forces of the half values applied in appropriate way to the suitable truss system. The results of calculations obtained in the first stage are shown in Fig. 6. The values of the forces defined in the second stage of calculations are presented in Fig. 7. The final values of forces calculated in the two-stage method are presented in Fig. 8a. The results obtained by the application of suitable computer software are shown in Fig. 8b.

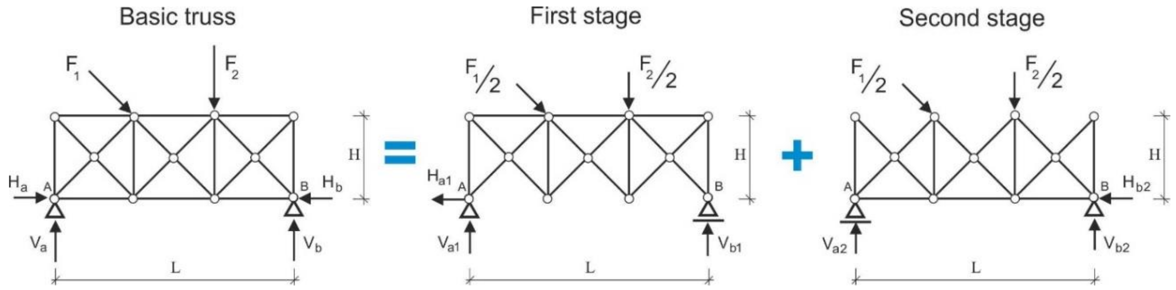

Figure 5. The concept of the two-stage method applied for the calculation of horizontally positioned statically indeterminate truss supported in two pivot bearings

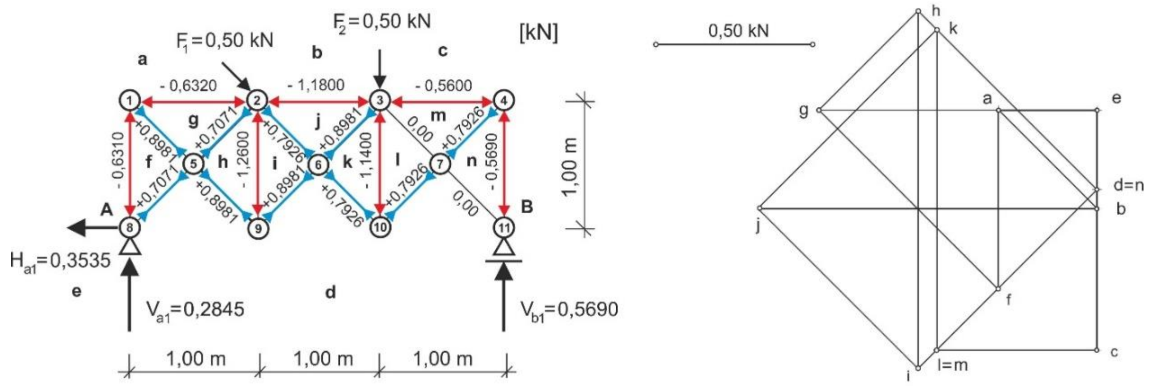

Figure 6. The values of the forces defined in the first stage of the calculations for horizontally positioned truss with suitable Cremona's polygon of forces

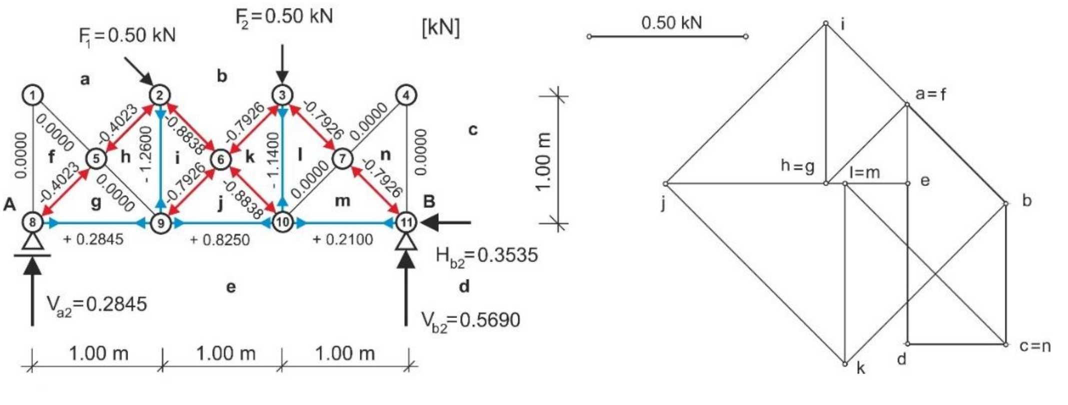

Figure 7. The values of the forces defined in the second stage of the calculations for horizontally positioned truss with suitable Cremona's polygon of forces

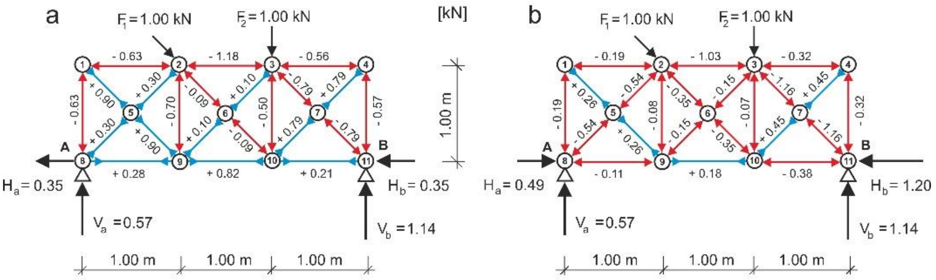

Figure 8. Comparison of the values of the forces in the members of horizontally positioned truss calculated, a) in the two-stage method, b) by application of computer software

One can notice significant differences of the values and even of senses of the force determined in the same members in two compared methods of calculations. For instance, by application of the two-stage method value of tensile force in a member located between nodes 3 and 6 equals +0.10 kN, while by applying of the computer software it was defined as a compressive force of value equal to -0.15 kN. Much smaller difference is noticed between compressive force value defined in a member placed between nodes 3 and 2 by the help of the two-stage method, which equals -1.18 kN, while the outcome of computer software defines it as a compressive force having value of -1.03 kN. One can observe the substantial differentiations in values of forces calculated by the application of compared methods in numerous members of the truss. For example, in the cross brace placed between nodes 2 and 5 in the two-stage method it is calculated as a tensile force of value equal +0.30 kN, while in the same member a force calculated by means of computer software is defined as the compression force of the value equal to -0.54 kN. The values and senses of the vertical components of the bearing reactions calculated in both methods are equal. To the most significant differences one has to count the differentiation of values and senses of the horizontal components of the bearing reactions. These reactions, defined in the two-stage method, are of equal values and both have same senses, see Fig. 8a, what is directly determined by the basic principles of this method. Because the investigated truss is also the once-fold external indeterminate system, therefore the real horizontal components of the bearing reactions are of the values and senses presented in Fig. 8b. It implies that the horizontal components of these reactions have to be calculated in another way by application of the two-stage method.

The calculation of the horizontal components of reaction in two stages

General schemes of a computation procedure proposed for the calculation of approximate force values of this type of the bearing reactions in the basic truss system are presented in Fig. 9.

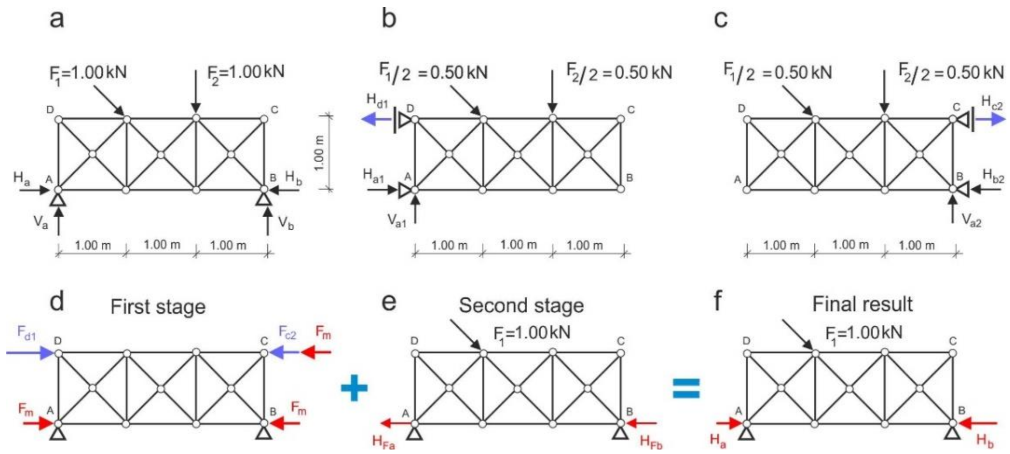

Figure 9. Schemes of the computation procedure of calculation of horizontal components of bearing reactions in two stages

Taking into account general static principles of the two-stage method it was assumed, that the basic truss, see Fig. 9a, can be considered as suitable composition of two respectively supported cantilever trusses having the same internal structure and loaded in the same way like the basic truss. In the first step the truss is considered as cantilever system supported in nodes A and D, see Fig. 9b. Because corner node D in the basic truss is not the support node therefore in the first step it is treated as the sliding pivot bearing, in which only horizontal component of reaction H_{d1} can exist. In this case the value of the horizontal reaction is computed in the following way:

$$\sum M_A = 0 \tag{6}$$

$$\frac{F1x}{2} \cdot 1.00m + \frac{F1y}{2} \cdot 1.00m + \frac{F2}{2} \cdot 2.00m - H_{d1} \cdot 1.00m = 0 \tag{7}$$

$$\frac{\sqrt{2}}{4}\, kN \cdot 1.00m + \frac{\sqrt{2}}{4}\, kN \cdot 1.00m + 0.5\, kN \cdot 2.00m - H_{d1} \cdot 1.00m = 0 \tag{8}$$

$$H_{d1} \approx 1.7071067\, kN \tag{9}$$

In the second step of the proposed procedure the truss is also considered as the cantilever system, but this time supported in nodes B and C, see Fig. 9c. Because node C in the basic truss is also not the support node, therefore in this stage, like previously, it is considered as being supported in pivot sliding bearing where only horizontal component of reaction H_{c2} can exist. Its value will be defined in the way presented below:

$$\sum M_B = 0 \tag{10}$$

$$-\frac{F1y}{2} \cdot 2.00m + \frac{F1x}{2} \cdot 1.00m - \frac{F2}{2} \cdot 1.00m + H_{c2} \cdot 1.00m = 0 \tag{11}$$

$$-\frac{\sqrt{2}}{4}\, kN \cdot 2.00m + \frac{\sqrt{2}}{4}\, kN \cdot 1.00m + 0.5\, kN \cdot 1.00m + H_{c2} \cdot 100m = 0 \tag{12}$$

$$H_{c2} \approx 0.8535533\, kN \tag{13}$$

The values of horizontal components of the bearing reactions H_{a1} and $H_{b2,}$ determined in this calculation in support A and B, have to be omitted, what is justified by the first condition of equilibrium (3). Value of the force H_{d1} is bigger than the force value H_{c2}. Difference of values of these two is called F_m and its value equals:

$$F_m = H_{d1} - H_{c2} = 1.7071067\, kN - 0.8535533\, kN \tag{14}$$

$$F_m = 0.8535534\, kN \tag{15}$$

Members located between nodes D and C in the top chord of the basic truss are subjected to act of compressive forces, compare Fig. 8. Therefore force F_{d1}, see Fig. 9d, has to have the same value like the force H_{d1} but its sense must be inversed and it is applied to the corner node D. Similar operation one should make in corner node C. Horizontal force F_{c2} is applied to this node, its value is equal to value of force H_{c2}, but its sense is oppositely directed. Force F_m has to be appropriately applied to the top chord in order to keep the force balance in this chord. From the first condition of equilibrium (3) follows, that in the first stage force F_m has to be applied to node C, having the same sense like force F_{c2}, see Fig. 9d. The condition of equilibrium of the whole considered structure justifies suitable application of two oppositely directed horizontal forces F_m to the two bearing nodes A and B. From the basis of the same condition it follows that horizontal components of the bearing reactions H_{Fa} and H_{Fb} are of the same values, as well as their senses, like it is shown in Fig. 9d. When the horizontal component of the load force $F_1 = \sqrt{2}/2$ kN ≈ 0.7071067 kN then the absolute values of the bearing reactions H_{Fa} and H_{Fb} are equal to 0.3535533 kN. Final values of horizontal components of bearing reactions acting in these supports, see Fig. 9f, will be the resultants of horizontal reactions calculated in the first stage, see Fig. 9d, and calculated in the second stage, see Fig. 9e.

$$\mathrm{H_a} = \mathrm{F_m} - \mathrm{H_{Fa}} = 0.8535534\ kN - 0.3535533\ kN \approx 0.50\ kN \tag{16}$$

$$\mathrm{H_b} = \mathrm{F_m} + \mathrm{H_{Fb}} = 0.8535534\ kN + 0.3535533\ kN \approx 1.21\ kN \tag{17}$$

The values of horizontal components of bearing reactions computed in this way are almost of the same values as if they were calculated by the application of the computer software, compare Fig. 8b, for the same type of the internal and external statically indeterminate truss. The procedure presented above applies the rules of calculus of forces as well as the principle of superposition. That is why it can be considered as an integral part of the two-stage method of calculation of statically indeterminate trusses.

Another way of calculation of the same basic statically indeterminate truss

Application of the two-stage method in a direct way gives in result not exact values and senses of horizontal bearing reactions, compare Fig. 8a. In order to recognize features of the two-stage method it has been applied for calculation of the same basic truss, see Fig. 9a, for values of horizontal bearing reactions determined in two stages described above. Intermediate results are presented in Fig. 10 and in Fig. 11. Final force values are shown in Fig. 12.

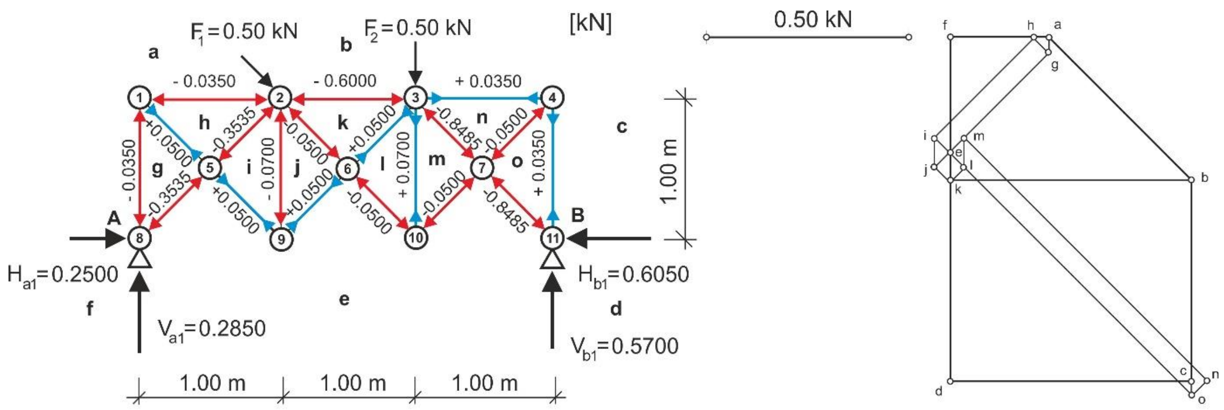

Figure 10. Force values computed in the first stage of calculations for values of bearing reactions of basic truss estimated in two stages together with Cremona's polygon of forces

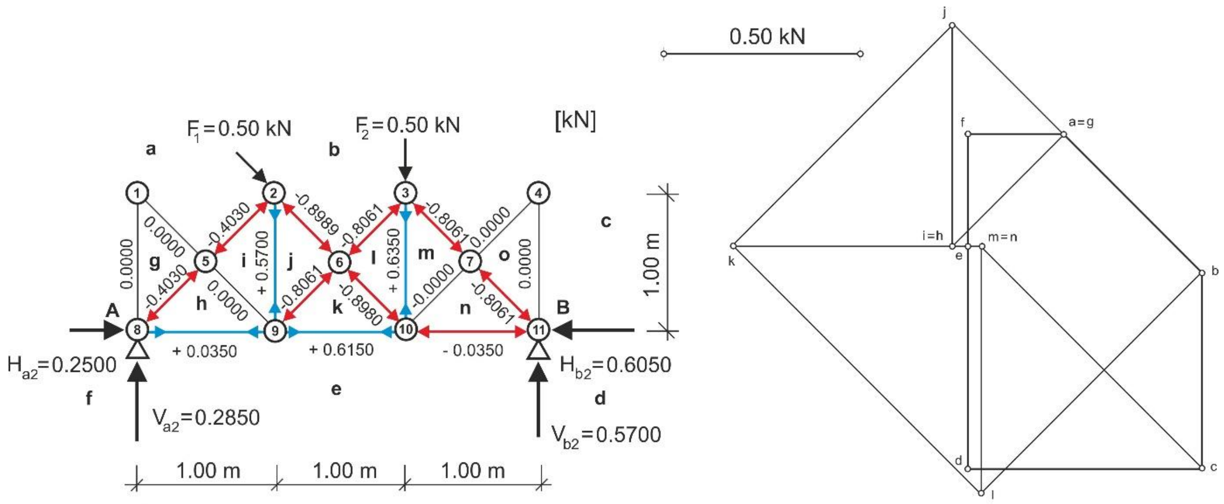

Figure 11. Force values defined in the second stage of calculations for values of bearing reactions of basic truss estimated in two stages together with Cremona's polygon of forces

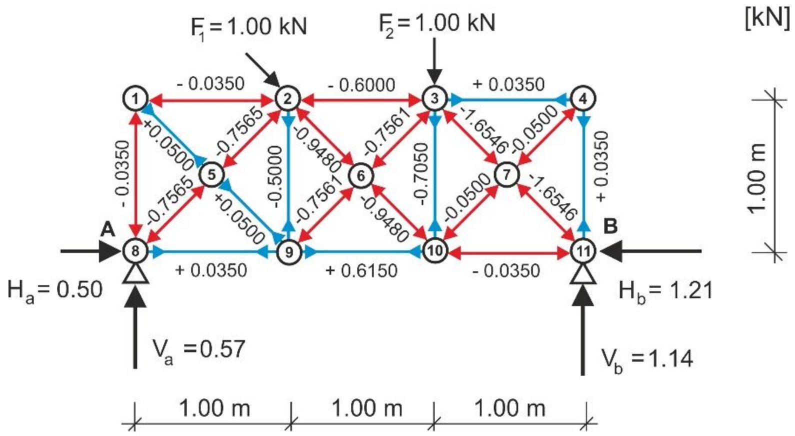

Figure 12. Values of forces determined in members of the basic truss as a result of alternative static calculations

Values of forces calculated in members of the investigated basic truss by application of the Autodesk Robot Structural Analysis Professional 2019 are considered as the exact results. In comparison with them the force values determined in the two-stage method for the same members by taking into account the approximate values of the real horizontal components of the bearing reaction, while H_a = 0.50 kN and H_b = 1.21 kN and being almost equal to the exact values, are considerably different. The degree of differentiation is even bigger than in the results of calculations carried out by the direct application of the simple rules of the two-stage method. For instance compressive force value computed by means of the computer software in the member located between nodes 2 and 3 equals -1.03 kN, see Fig. 8b. In the same member due to the application of the simple kind of two-stage method, value of compressive force is defined as equal to -1.18 kN, see Fig 8a, while due application of the two-stage method with taking into account real values of horizontal components of bearing reactions, the calculated value of compressive force equals -0.60 kN, see Fig. 12. Similar remarks refer also regarding various senses of the vector forces calculated in both compared kinds of the two-stage method. From comparative analysis of the presented results calculated for the horizontal positioned trusses loaded by forces applied at optional directions to the truss nodes follows, that somewhat better approximate force values to the exact values of forces acting in members of the truss, one can obtain by application of the simple kind of the two-stage method, see Fig. 8a. Values of the horizontal components of the bearing reactions should then be calculated separately in the way described above, the procedure of which is presented in schemes shown in Fig. 9.

Conclusions

The two-stage method is an approximate method of calculation of the statically indeterminate trusses because in both stages it applies the rules, which are appropriate for the calculation procedures of the statically determinate trusses. The degree of approximation of the obtained force values to the values of forces defined by means of the exact methods in general is good enough when the two-stage method is applied for calculation of the inner indeterminate trusses. One can notice significantly differences between the exact and appropriate force values calculated for members, where are acting the smallest forces, especially having absolute values close to zero. However one should be aware that members subjected to the act of such forces are designed mostly according to instructions of the building codes. In these cases the cross-sectional areas of such members are much larger than they are determined directly on basis of the results of static calculations. The accuracy of the force values calculated by application of the two-stage method can be significantly improved by taking into account the differences between the stiffness of members connected in the truss nodes. It can be made by defining a set of appropriate coefficients defining differences of members connected to each particular node. When the two-stage method is applied for calculation of the external and internal statically indeterminate truss and if directions of the applied load forces are parallel to the line determined by positions of two pivot bearings, then the approximate force values are almost in exact accordance with outcomes gained by usage of suitable computer software. If this method is used in the computation processes of such trusses loaded by forces applied at optional directions then one can notice quite big differences between the force values defined in this way and the values of forces calculated by application of an exact computer method. Various possible applications of the two-stage method for calculations of different types of statically indeterminate trusses are planned to be subjects of further research in order to estimate more closely the features and the practical usefulness of this method for static computations.

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