Analytical solution of stiffness and damping for finite length vertical journal bearing

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Abstract

Stiffness and damping are key parameters of journal bearings and are widely used in stability prediction of rotor-bearing systems. However, a nonlinear model and short bearing assumption are presently widely used in vertical rotor-bearing analysis. And the linear model of finite length vertical bearing is still lacking. In this paper, an analytical solution of stiffness and damping are given for a vertical finite journal bearing under half-Sommerfeld boundary condition. The results are also compared with results from both the infinite long and infinite short bearing models. The results show that, the infinite short bearing model bring in large errors (larger than 10%) when length-to-diameter ratio larger than 0.5 and the error of infinite longer bearing model is always larger than 10% when length-to-diameter smaller than 11. It further indicates that, a finite length journal bearing model should be used in practical especially when the length-to-diameter ratio ranges in (0.5,11).

Keywords: finite journal bearing, vertical journal bearing, dynamic coefficients, analytical solutions

Theoretical background

The Reynolds equation and infinitesimal perturbation (IP) method are used to obtain the dynamic coefficients of journal bearing [1]-[3]. The equations are shown in Eq.(1).

$$\begin{cases} \frac{\partial}{\partial\varphi}\left(H^3\frac{\partial(P_0)}{\partial\varphi}\right)+\left(\frac{d}{l}\right)^2\frac{\partial}{\partial\lambda}\left(H^3\frac{\partial(P_0)}{\partial\lambda}\right)=3\frac{\partial H}{\partial\varphi} \\ \frac{\partial}{\partial\varphi}\left(H^3\frac{\partial(\partial P/\partial x)}{\partial\varphi}\right)+\left(\frac{d}{l}\right)^2\frac{\partial}{\partial\lambda}\left(H^3\frac{\partial(\partial P/\partial x)}{\partial\lambda}\right)=3\cos\varphi-\frac{9}{H}\sin\varphi\frac{\partial H}{\partial\varphi} \\ \qquad +3H\left[\left(\sin\varphi\frac{\partial H}{\partial\varphi}-\cos\varphi\, H\right)\frac{\partial P}{\partial\varphi}+\left(\frac{d}{l}\right)^2\sin\varphi\frac{\partial H}{\partial\lambda}\frac{\partial P}{\partial\lambda}\right] \\ \frac{\partial}{\partial\varphi}\left(H^3\frac{\partial(\partial P/\partial y)}{\partial\varphi}\right)+\left(\frac{d}{l}\right)^2\frac{\partial}{\partial\lambda}\left(H^3\frac{\partial(\partial P/\partial y)}{\partial\lambda}\right)=-3\sin\varphi-\frac{9}{H}\cos\varphi\frac{\partial H}{\partial\varphi} \\ \qquad +3H\left[\left(\cos\varphi\frac{\partial H}{\partial\varphi}+\sin\varphi\, H\right)\frac{\partial P}{\partial\varphi}+\left(\frac{d}{l}\right)^2\cos\varphi\frac{\partial H}{\partial\lambda}\frac{\partial P}{\partial\lambda}\right] \\ \frac{\partial}{\partial\varphi}\left(H^3\frac{\partial(\partial P/\partial v_x)}{\partial\varphi}\right)+\left(\frac{d}{l}\right)^2\frac{\partial}{\partial\lambda}\left(H^3\frac{\partial(\partial P/\partial v_x)}{\partial\lambda}\right)=6\sin\varphi \\ \frac{\partial}{\partial\varphi}\left(H^3\frac{\partial(\partial P/\partial v_y)}{\partial\varphi}\right)+\left(\frac{d}{l}\right)^2\frac{\partial}{\partial\lambda}\left(H^3\frac{\partial(\partial P/\partial v_y)}{\partial\lambda}\right)=6\cos\varphi \end{cases} \tag{1}$$

where $H=1+\varepsilon\cos(\varphi-\theta)$ is the dimensionless film thickness, and $\varepsilon=e/c$ denotes the dimensionless eccentricity of journal bearing, e and c are the eccentric distance and radial clearance separately. P_0 is the dimensionless static oil film pressure, which is defined as

$P = 2pc^2/(\Omega\mu d^2)$; d and l denote the diameter and length of journal bearings; λ (defined as $\lambda = z/(l/2)$)is the dimensionless coordinate in the axial direction; Ω is the rotational speed; μ is the dynamic viscosity of lubricant; x and y denote the displacements on horizon and vertical directions; v_x and v_y denote the velocity of journal on horizon and vertical directions; φ and z are the coordinates in the circumferential and axial directions, shown in Fig.1.

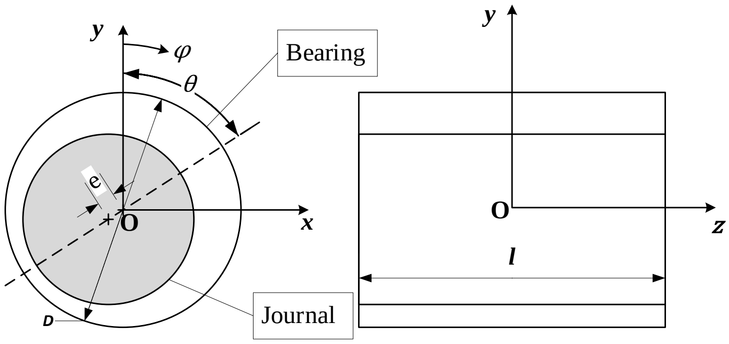

Figure 1. The sketch of journal bearing

For a vertical journal bearing, the static load vanishes, and the eccentric ratio equals 0 at the equilibrium position [4]-[5]. Eq.(1) can be further simplified as

$$\begin{cases} \frac{\partial^2}{\partial\varphi^2}(P_0) + \left(\frac{d}{l}\right)^2 \frac{\partial^2}{\partial\lambda^2}(P_0) = 0 \\ \frac{\partial^2}{\partial\varphi^2}\left(\frac{\partial P}{\partial x}\right) + \left(\frac{d}{l}\right)^2 \frac{\partial^2}{\partial\lambda^2}\left(\frac{\partial P}{\partial x}\right) = 3\cos\varphi \\ \frac{\partial^2}{\partial\varphi^2}\left(\frac{\partial P}{\partial y}\right) + \left(\frac{d}{l}\right)^2 \frac{\partial^2}{\partial\lambda^2}\left(\frac{\partial P}{\partial y}\right) = -3\sin\varphi \\ \frac{\partial^2}{\partial\varphi^2}\left(\frac{\partial P}{\partial v_x}\right) + \left(\frac{d}{l}\right)^2 \frac{\partial^2}{\partial\lambda^2}\left(\frac{\partial P}{\partial v_x}\right) = 6\sin\varphi \\ \frac{\partial^2}{\partial\varphi^2}\left(\frac{\partial P}{\partial v_y}\right) + \left(\frac{d}{l}\right)^2 \frac{\partial^2}{\partial\lambda^2}\left(\frac{\partial P}{\partial v_y}\right) = 6\cos\varphi \end{cases} \tag{2}$$

The boundary condition for Eq.(2) are that , P_0 and $\partial P/\partial\delta$ are zero at the edge of journal bearing ($z = \pm L/2$). And π-film assumption is used in circumference direction. Eq.(2) can be easily solved via the method of separation of variables, and the solutions are:

$$\begin{cases} P_0 = 0 \\ \frac{\partial P}{\partial v_y} = 2\frac{\partial P}{\partial x} = 6\cos\varphi\left(\frac{e^{l\lambda/d}+e^{-l\lambda/d}}{e^{l/d}+e^{-l/d}} - 1\right) \\ \frac{\partial P}{\partial v_x} = -2\frac{\partial P}{\partial y} = 6\sin\varphi\left(\frac{e^{l\lambda/d}+e^{-l\lambda/d}}{e^{l/d}+e^{-l/d}} - 1\right) \end{cases} \tag{3}$$

With the help of Eq.(3), the dimensionless stiffness and damping can be easily obtained analytically, which are shown in Eq.(4) and Eq.(5) .

$$\begin{cases} K_{xx} = \frac{\partial F_x}{\partial x}\Big|_{x=x_0,y=y_0} = -\int_{-1}^{1}\int_{\varphi_1}^{\varphi_2}\frac{\partial P}{\partial x}\sin\varphi\, d\varphi d\lambda = 0 \\ K_{xy} = \frac{\partial F_x}{\partial y}\Big|_{x=x_0,y=y_0} = -\int_{-1}^{1}\int_{\varphi_1}^{\varphi_2}\frac{\partial P}{\partial y}\sin\varphi\, d\varphi d\lambda = 3\pi\left(\frac{d}{l}\frac{e^{l/d}-e^{-l/d}}{e^{l/d}+e^{-l/d}} - 1\right) \\ K_{yx} = \frac{\partial F_y}{\partial x}\Big|_{x=x_0,y=y_0} = -\int_{-1}^{1}\int_{\varphi_1}^{\varphi_2}\frac{\partial P}{\partial x}\cos\varphi\, d\varphi d\lambda = -3\pi\left(\frac{d}{l}\frac{e^{l/d}-e^{-l/d}}{e^{l/d}+e^{-l/d}} - 1\right) \\ K_{yy} = \frac{\partial F_y}{\partial y}\Big|_{x=x_0,y=y_0} = -\int_{-1}^{1}\int_{\varphi_1}^{\varphi_2}\frac{\partial P}{\partial y}\cos\varphi\, d\varphi d\lambda = 0 \end{cases} \tag{4}$$

$$
\begin{cases}
C_{xx} = \left.\dfrac{\partial F_x}{\partial v_x}\right|_{x=x_0, y=y_0} = -\int_{-1}^{1}\int_{\varphi_1}^{\varphi_2} \dfrac{\partial P}{\partial v_x}\sin\varphi \, d\varphi d\lambda = -6\pi\left(\dfrac{d}{l}\dfrac{e^{l/d}-e^{-l/d}}{e^{l/d}+e^{-l/d}} - 1\right) \\
C_{xy} = \left.\dfrac{\partial F_x}{\partial v_y}\right|_{x=x_0, y=y_0} = -\int_{-1}^{1}\int_{\varphi_1}^{\varphi_2} \dfrac{\partial P}{\partial v_y}\sin\varphi \, d\varphi d\lambda = 0 \\
C_{yx} = \left.\dfrac{\partial F_y}{\partial v_x}\right|_{x=x_0, y=y_0} = -\int_{-1}^{1}\int_{\varphi_1}^{\varphi_2} \dfrac{\partial P}{\partial v_x}\cos\varphi \, d\varphi d\lambda = 0 \\
C_{yy} = \left.\dfrac{\partial F_y}{\partial v_y}\right|_{x=x_0, y=y_0} = -\int_{-1}^{1}\int_{\varphi_1}^{\varphi_2} \dfrac{\partial P}{\partial v_y}\cos\varphi \, d\varphi d\lambda = -6\pi\left(\dfrac{d}{l}\dfrac{e^{l/d}-e^{-l/d}}{e^{l/d}+e^{-l/d}} - 1\right)
\end{cases} \tag{5}
$$

where $K_{i,j}$ and $C_{i,j}$ are the dimensionless stiffness and damping, which respectively are defined as $K_{i,j} = k_{i,j}\, c^3/(\mu\Omega l r^3)$ and $C_{i,j} = c_{i,j}\, c^3/(\mu l r^3)$. φ_1 and φ_2 are the leading edge and trailing edge of oil film zone. $k_{i,j}$ and $c_{i,j}$ are dimensional stiffness and damping.

Results and discussion

The dimensionless stiffness and damping of a vertical journal bearing without static load is mainly affected by length-to-diameter ratio of journal bearing. It can be easily concluded that from Eq.(4) and Eq.(5) that $K_{xx} = K_{yy} = C_{xy} = C_{yx} = 0$ and $C_{xx} = C_{yy} = 2K_{yx} = -2K_{xy}$. Therefore, K_{xy} is mainly discussed in the following. Fig.2 shows the comparation and error analysis with the results from infinite long and short bearing models.

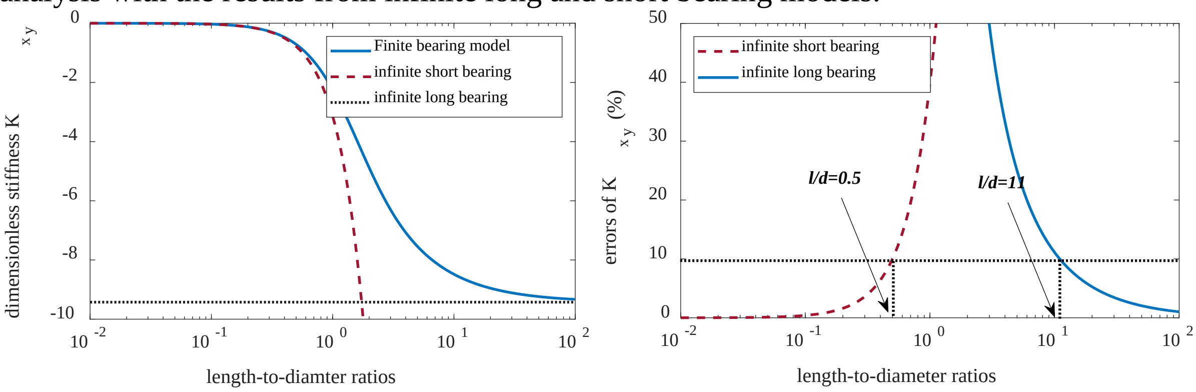

Figure 2. The comparation of K_{xy} with that of infinite long and short bearing models

Fig.2 indicates that K_{xy} close to that of short bearings while $l/d \le 0.5$ and close to results from long bearing model when $l/d \ge 11$. For this situation, the relative error smaller than 10% as shown in the figure. However, for a bearing with l/d within (0.5,11), both the short bearing model and long bearing model will bring in large errors and a finite length bearing model should be used in analysis.

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