Calculation of the approximate upper bound of knockdown factors for cylindrical shells based on the Willis-form finite element method

Yixiao Sun[1] and †*Zhihai Xiang[1]

[1]Department of Engineering Mechanics, Tsinghua University, China.

*Presenting author: xiangzhihai@mail.tsinghua.edu.cn
†Corresponding author: xiangzhihai@mail.tsinghua.edu.cn

Abstract

To meet the requirements of light-weight and economic efficiency for structures used in aeronautical and aerospace engineering, thin-walled structures including shells are widely adopted. As is known to all, buckling is one of the major issues for shells when evaluating their load capacity. For cylindrical shells under axial compression, linear perturbation theory gives a theoretical prediction for the value of critical compression stress [1]. However, it has been confirmed by various researches that the experimental results are significantly lower than the corresponding theoretical predictions, leading to the concept of knockdown factor, namely the ratio of the actual buckling load over the predicted value. An empirical formula established by NASA in 1965 has long been regarded as a lower bound of knockdown factors with respect to different radius to thickness ratios [2]. The main aspect that results in this severe difference is considered to be imperfections caused during manufacturing and loading processes, and many researches in early years intended to reveal the impact of imperfections [3]-[6]. This problem has lasted for nearly a hundred years, and many recent researches have paid further attention to imperfections, as well as the new concept of energy barriers [2], [7]-[11].

It has been proved that the Willis-form equations with displacement coupling terms explicitly contain the pre-stress gradients [12], and have been verified by an experiment of rotational spring [13]. Since the deformation process for buckling problems are geometrically nonlinear, there may exist inhomogeneous pre-stresses in each incremental load step. In order to take the gradient of pre-stresses into consideration, an updated Lagrangian finite element formulation has been derived based on the Willis-form equations. The corresponding finite element code has been incorporated into the Multiphysics Finite-element Analysis (MFA) programming platform from Tsinghua University. When conducting finite element analysis to cylindrical shells under axial compression with different radius to thickness ratios, it is found that the buckling load obtained is generally lower than the conventional linear value, but still higher than most existing test results. Thus, an approximate upper bound of knockdown factors has been established, and may serve as a more precise reference at the designing stage. For a certain quasi-perfect shell, the numerical result consists very well with the experimental result without introducing imperfection into the finite element model in advance [14]. Conclusively, it can be asserted that the pre-stress gradients have a significant impact on the buckling of cylindrical shells, and further investigations for other shell buckling problems remain to be continued in future studies.

Keywords: The Willis-form equations, Finite element method, Geometrical nonlinearity, Buckling, Cylindrical shells

Acknowledgment

This work was supported by the National Natural Science Foundation of China with the grant number 11672144.

References

[1] Southwell, R. V. (1910) On the general theory of elastic stability, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **213**, 187-244.

[2] Hutchinson, J. W. (2010) Knockdown factors for buckling of cylindrical and spherical shells subject to reduced biaxial membrane stress, *International Journal of Solids and Structures* **47**, 1443-1448.

[3] Koiter, W.T. (1963) The effect of axisymmetric imperfections on the buckling of cylindrical shells under axial compression, *Proceedings of the Koninklijke Nederlandse Akademic van Wetenschappen B* **66**, 265–279.

[4] Hutchinson, J.W. (1965) Axial buckling of pressurized imperfect cylindrical shells, *AIAA Journal* **3**, 1461–1465.

[5] Budiansky, B., Hutchinson, J.W. (1966) A Survey of Some Buckling Problems, *AIAA Journal* **4**, 1505-1510.

[6] Stein, M. (1968) Some Recent Advances in the Investigation of Shell Buckling, *AIAA Journal* **6**, 2339-2345.

[7] Hutchinson, J.W. (2016) Buckling of spherical shells revisited, *Proceedings of the Royal Society A* **472**, 20160577.

[8] Thompson, J. M. T., Hutchinson, J.W., Sieber, J. (2017) Probing shells against buckling: a non-destructive technique for laboratory testing, *International Journal of Bifurcation and Chaos* **27**, 1730048, 1-15.

[9] Hutchinson, J.W., Thompson, J. M. T. (2017) Nonlinear buckling behaviour of spherical shells: barriers and symmetry-breaking dimples, *Philosophical Transactions of the Royal Society A* **375**, 20160154.

[10] Hutchinson, J.W., Thompson, J. M. T. (2017) Nonlinear buckling interaction for spherical shells subject to pressure and probing forces, *Journal of Applied Mechanics* **84**, 061001, 1-11.

[11] Hutchinson, J.W., Thompson, J. M. T. (2018) Imperfections and energy barriers in shell buckling, *International Journal of Solids and Structures* **148-149**, 157-168.

[12] Xiang, Z. H., Yao, R.W. (2016). Realizing the Willis equations with pre-stresses, *Journal of the Mechanics and Physics of Solids* **87**, 1–6.

[13] Yao, R.W., Gao, H.X., Sun, Y.X., Yuan, X.D., Xiang, Z.H., (2018). An experimental verification of the one-dimensional static Willis-form equations, *International Journal of Solids and Structures* **134**, 283–292.

[14] Wang, B., Zhu, S., Hao, P., Bi, X., Du, K., Chen, B., Ma, X., Chao, Y., (2018). Buckling of quasi-perfect cylindrical shell under axial compression: a combined experimental and numerical investigation, *International Journal of Solids and Structures* **130-131**, 232–247.