A novel locking-free smoothed 4-node quadrilateral element based on the incompatible modes for near-incompressible linear elasticity

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Abstract

Due to their simple geometry, 4-node quadrilateral elements are widely used for the finite element analysis of plane problems. It is well known that the traditional FEM and CS-FEM with Q4 elements are suffering from volume-locking, therefore, which may not be suitable for the analysis of near-incompressible materials in plane strain. Besides, the traditional Q4 element yields numerical singularities in the case of an extreme distortion of the element. Furthermore, the volumetric locking can be solved by employing the reduced integration, unfortunately, spurious zero energy mode appears.

Subsequently, a formulation for the smoothed 4-node quadrilateral finite element is developed based on the principle of virtual displacements for a deformable body. Incompatible modes are introduced into the standard displacement field. Then expressions for gradient operators are obtained by using the strain smoothing technique in the physical co-ordinates. The internal degrees of freedom of the incompatible modes are eliminated on the element level, the matrix assembly completely Jacobian-free and the isoparametric mapping is not required. A series of benchmark problems are provided to demonstrate the superior performances of the proposed element. Locking effects and zero-energy modes are avoided as well as singularities of the stiffness matrix due to element distortion.

Keywords: Q4 element; Strain smoothing technique; Incompatible modes; Near-incompressible; Volume-locking.