Equivalent Circuit Modeling and Analysis Study for Vortex-induced Aerodynamic Energy Harvesting

***Jinda Jia[1,2], Xiaobiao Shan[1], †Tao Xie[1] and and †Yaowen Yang[2]**

[1]State Key Laboratory of Robotics and System, Harbin Institute of Technology, Harbin, China

[2]School of Civil and Environmental Engineering, Nanyang Technological University, Singapore

*Presenting author: jiajinda@hit.edu.cn

†Corresponding author: xietao@hit.edu.cn; cywyang@ntu.edu.sg

Abstract

Low-speed wind energy can be captured to power wireless sensors or remote micro electro mechanical systems by utilizing vortex-induced vibration phenomenon of a cylinder tip connected to a piezoelectric cantilever. Researchers have developed several configurations to study the behavior of vortex-induced energy harvesters. However, these theoretical analysis or experimental prototypes only consider the interface circuit as a pure resistance. The interface circuit for practical usage may be more complex or the powered wireless sensor needs direct voltage input. The solve the complex interface circuit problem, this paper presents an equivalent circuit modeling and analysis study for vortex-induced energy harvesting. The mechanical parameters are substituted by standard circuit elements. The nonlinear vortex-induced aerodynamic force is represented by self-defined function sources. The total equivalent circuit is simulated by circuit simulation software. The simulation results are consistent to our previous experimental results, which can verify the accurateness of the presented equivalent circuit model.

Keywords: equivalent circuit modeling; energy harvest; vortex-induced vibration

Introduction

Wireless sensing nodes or electronic instruments have been used in structural health monitoring, medical health examination and micro electro mechanical systems, which promote the research of energy harvesting from the ambient environment or vibrations [1-3]. Different energy convention mechanisms including electrostatic [4,5], electromagnetic [6,7] and piezoelectric [8-10] mechanisms. Energy harvesting via piezoelectric mechanism has attracted much attention because of easy construction, clean and high output power density. Quite numbers of researchers focus on energy harvesting from base excitations [11-13]. However, the stable base excitation exists rarely in nature and thus limits its usage situations.

Wind exists widely in nature and has the potential to be utilized for energy harvesting to drive sensors in inaccessible mountain, canyon or seabed. In terms of energy harvesting from fluid flows, Akaydin et al. [14] proposed a piezoelectric energy harvester consisted of a cantilevered beam with attachment cylinder and the results showed a non-rigid bonding model had a better agreement with experiments than non-rigid bonding model. Wang et. al [15] studied a d31 mode piezoelectric energy harvester generating voltage from pressure oscillation in pressure chamber and the energy harvester could generate 2.2 V. Li et. al [16] tested a bio-inspired piezo-leaf architecture using flexible piezoelectric materials and a single leaf had 2 mW/cm^3 power density. Weinstein et al. [17] studied the interactions between a fin and downstream vortex shedding. The results showed the addition of the fin could make a significant improvement output power from a piezoelectric energy harvester. Gao et. al [18] presented an upright energy harvester with cylinder extension. The experimental results showed the piezoelectric energy harvester generated higher voltage in turbulent flow because of additional contribution in the lock-in region than in laminar flow. Dai [19,20] studied the output power from both vortex-induced vibration and base excitation. Among other different shapes of attachments, Liu et al. [21] proposed a Y-type three-blade bluff body and the experiment showed its superiority than a square prism. Abdelkefi et al. [22] proposed an energy harvester with an equilateral triangle cross-section and found that the minimum transverse displacement amplitude resulted in maximum harvested power. Yang et al. [23] compared the output abilities of energy harvesters with different cross sections. The experimental results demonstrated the superiority of the square-sectioned tip. However, most researchers only consider the external circuit as a pure resistance because external circuits are difficult to be modeled in mechanical systems.

Therefore, this paper puts forward a novel modeling method for vortex-induced vibration energy harvester according to the principle of equivalent circuit, which can handle complex external circuit. The mechanical parameters in practical situations are replaced by electrical circuit element. The piezoelectric coefficient parameter is replaced by an ideal transformer. The vortex-induced force is represented by oscillation circuit. The equivalent circuit model is simulated in analog software for circuits. The results of an equivalent circuit model are consistent with our previous experimental results, which can prove the validity of the modeling method.

Conventional Modeling of Vortex-induced Vibration Energy Harvesting

As Fig. 1 shows, the vortex-induced vibration piezoelectric energy harvester consists of a piezoelectric cantilever beam with cylinder extension. The cylinder attachment undergoes periodic pressure when the wind flows through it, and there occurs vibrations when the frequency is near to the natural frequency of energy harvester. The strains in the piezoelectric layer generate output voltage across the interface circuit. Here we directly use the governing equation through the nonlinear distributed parameter model by our previous work [24], as shown in Eqs. (1-3).

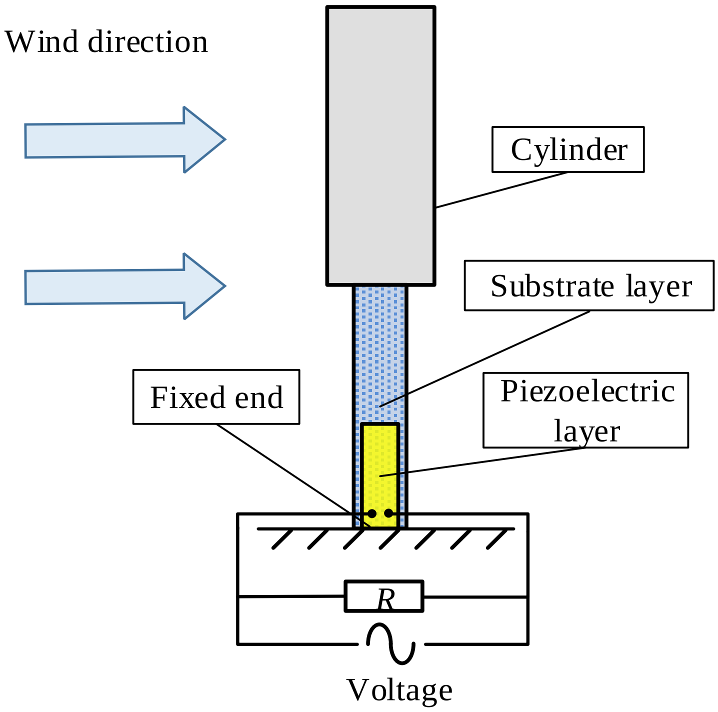

Figure 1. Composition of piezoelectric energy system [24]

$$\ddot{r}(t)+(2\zeta\omega+\eta)\dot{r}(t)+(\omega^2+\mu)r(t)+\theta V(t)=\frac{C_{L0}\rho_f DU^2}{4}\left(L_c\phi_{12}(L_s)+\frac{L_c^2}{2}\phi_{12}'(L_s)\right)q(t) \tag{1}$$

$$C_P\frac{dV_{RL}(t)}{dt}+\frac{V_{RL}(t)}{R}-\theta\dot{r}(t)=0 \tag{2}$$

$$\ddot{q}(t)+\varepsilon\omega_f\left(q^2(t)-1\right)\dot{q}(t)+\omega_f{}^2 q=\frac{A}{D}\left(\phi_{12}(L_s)+\frac{L_c}{2}\phi_{12}'(L_s)\right)\ddot{r}(t) \tag{3}$$

where $r(t)$ is model coordinate; ζ is the damping coefficient; ω is the natural frequency of energy harvester; η is fluid resistance coefficient; μ the coefficient of gravity effect; θ is piezoelectric coefficient; $V(t)$ is output voltage; C_{L0} is lift coefficient; D and L_c is diameter and length of cylinder, respectively; C_p is capacitance of piezoelectric sheet; ρ_f and U are the density and velocity of air flows, respectively; ε and A are constants; ω_f is frequency of vortex; $q(t)$ is a parameter used to describe the behavior on the near wake of the cylinder.

Equivalent Circuit Modeling

In this part, the equivalent circuit method is proposed. Equivalent electrical components for vortex-induced force and mechanical parts of energy harvesting system are listed in Table 1.

Table 1. Analogy between mechanical domain and electrical domain

Mechanical parameters	Electrical parameters
$r(t)$	Charge: $Q_1(t)$
$dr(t)/dt$	Current: $i_1(t)$
$q(t)$	Charge: $Q_2(t)$
$dq(t)/dt$	Current: $i_2(t)$
1	Inductance: L_1, L_2
$2\zeta\omega+\eta$	Resistance: R_1
$1/(\omega^2+\mu)$	Capacitance: C_1
$1/\omega_f^2$	Capacitance C_2
$\frac{C_{L0}\rho_f DU^2}{4}\left(L_c\phi_{12}(L_s)+\frac{L_c^2}{2}\phi_{12}'(L_s)\right)q(t)$	Voltage: V(A3)
$\frac{A}{D}\left(\phi_{12}(L_s)+\frac{L_c}{2}\phi_{12}'(L_s)\right)\ddot{r}(t)$	Voltage: V(A2)
$-\theta$	Turn ratio: N1

Eqs. (1-3) can be rewritten by replacing electrical parameters into mechanical parameters as Eqs. (4-6).

$$L_1\ddot{Q}_1(t)+R_1\dot{Q}_1(t)+C_1Q(t)-NV(t)=V(\mathrm{A3}) \tag{4}$$

$$C_P\frac{dV_{RL}(t)}{dt}+\frac{V_{RL}(t)}{R}+N\dot{Q}_1(t)=0 \tag{5}$$

$$L_2\ddot{Q}_2(t)+V(\mathrm{A1})/I(\mathrm{V5})\dot{Q}_2(t)+C_2Q(t)=V(\mathrm{A2}) \tag{6}$$

Model validation

In this part, the equivalent circuit model is validated [24]. The pure resistance is chosen as the external circuit for comparison with our previous experimental results. According to the analogy between the mechanical and electrical domains, the parameters of the equivalent circuit model are obtained from Table 1. Fig. 2 shows the circuit diagram of equivalent circuit when wind velocity is 4.2 m/s. Fig. 3 shows time-voltage diagram across *RL* when wind velocity is 4.2 m/s. As time goes on, the voltage fluctuates about 3.5 seconds and then forms a steady periodic fluctuation.

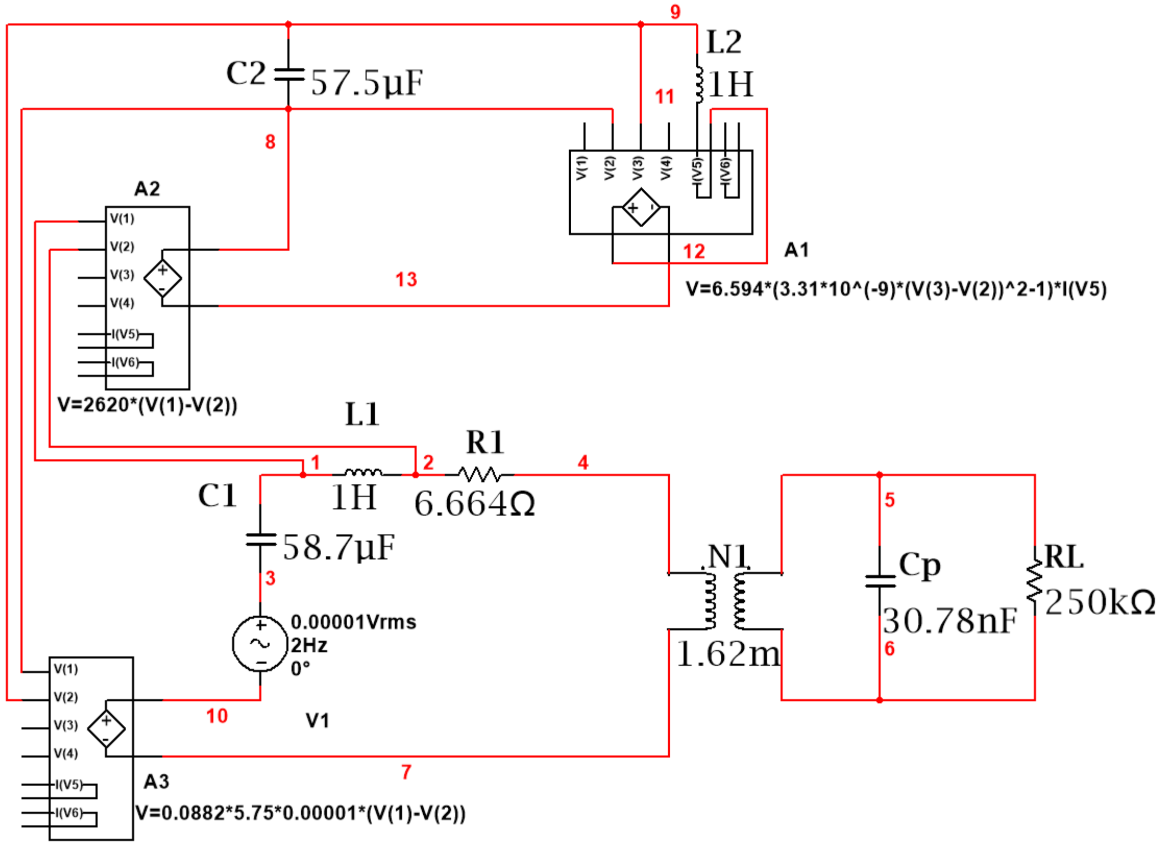

Figure 2. Circuit diagram of equivalent circuit when wind velocity is 4.2 m/s

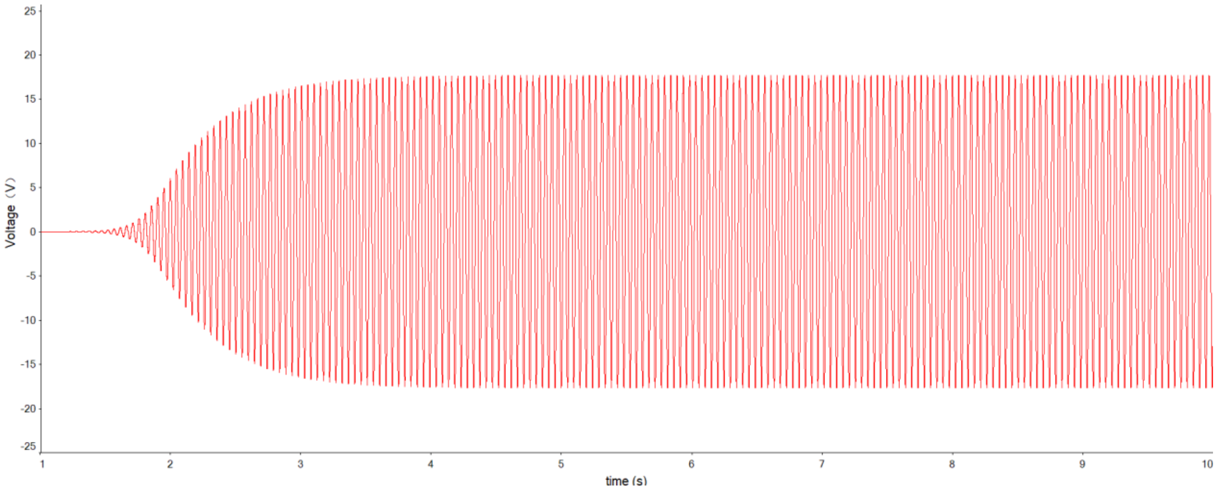

Figure 3. Time-voltage diagram

To investigate the effect of external resistance *RL* on energy harvesting ability, a serious of external resistance is chosen to calculate output power *P*. Fig. 3 shows the output power from circuit simulation and experimental versus load resistance *RL* when the wind velocity is 4.2 m/s. The circuit simulation shows that the maximum power is 626.58 μW at 250 kΩ while 605.0 μW at 250 kΩ from circuit result. The optimum resistance from experimental results is consistent with that from circuit simulation.

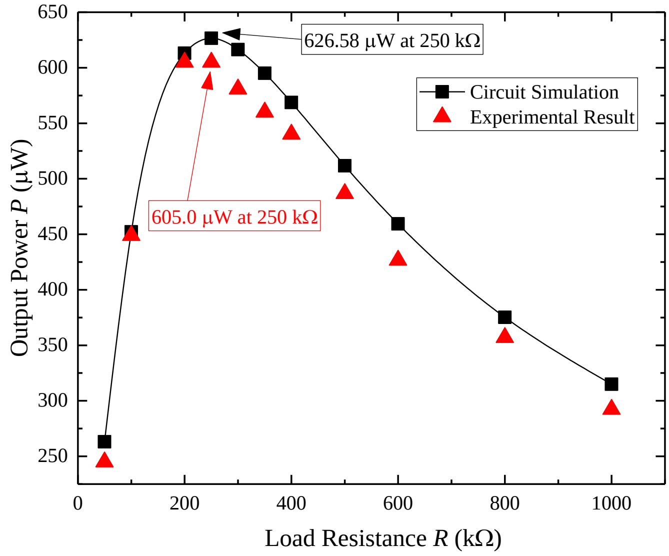

Figure 4. Output power from circuit simulation and experimental versus load resistance

For further investigating the influence of wind speed *U*, the output power of load resistance at various wind speeds are calculated through circuit simulation. Both circuit simulation and experimental resistance is chosen to 250 kΩ, which is the optimal resistance from fig. 3. Figure 4 indicates the relationship between output power and wind speed from circuit simulation and experimental result. The maximum output power *P* is 635.04 μW when the wind speed *U* is 4.2 m/s, which is obtained in the experiment. The maximum average output power calculated by circuit simulation is 628.91 μW when the wind speed *U* is 4.3 m/s. The maximum output power and optimal wind velocity from circuit simulation matches well with that from experiment.

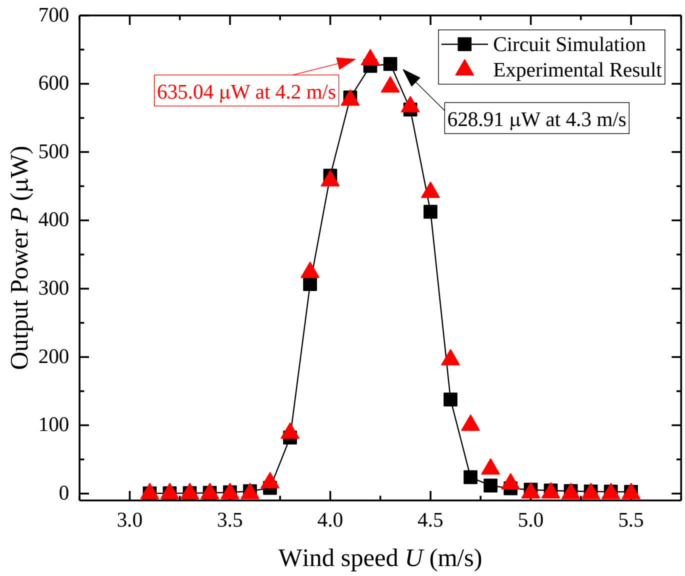

Figure 5. Time-voltage diagram

Conclusions

This paper investigates the equivalent circuit modeling method for vortex-induced piezoelectric energy harvesting. First the analogy between mechanical domain and electrical domain is presented and used to replace the aerodynamic fluid-solid-electricity governing equations by equivalent circuit equations. Next, the equivalent circuit is simulated in circuit simulation software. The output voltage becomes steady sine wave in a few seconds. Finally, the effects of external load resistance and wind velocity are studied and compared with experimental results. The comparative results illustrate that the optimal load resistance, the optical wind velocity and maximum output power from circuit simulation are consistent with that from experimental results, which validities the accurateness of the equivalent circuit simulation. This work provides basic simulation method for designing external circuit for vortex-induced piezoelectric energy harvesting.

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