Heat transfer in periodic laminated layer – Robin boundary conditions

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Abstract

In this note, the problem of heat conduction in periodic laminated layer is considered. This layer is characterized by a microstructured composition and the microstructure is realized as a uniform distribution of the cells. The Robin boundary conditions, which are analyzed in this work, are combined with the convective heat exchange and there is an analytical solution for a homogeneous layer and this type of boundary conditions. To consider the heat conduction issue in presented laminated layer the tolerance averaging technique is used. The equations, obtained by using this technique, are solved by using finite difference method. As the results, the distributions of the temperature are obtained. The algorithm, which is created to obtain the distribution of the temperature can be verified by using the results from the analytical solution for a homogeneous structure and the Robin boundary conditions.

Keywords: Heat conduction, Robin boundary conditions, mathematical modelling, composite, microstructure

Introduction

The two-dimensional issue in periodic laminated layer is considered in this work. Every cell of this layer is made of two different materials and the proportion between the first and the second material in the cell is constant. The thickness of the cells is also constant and denoted by Δ, what is shown in Fig. 1.

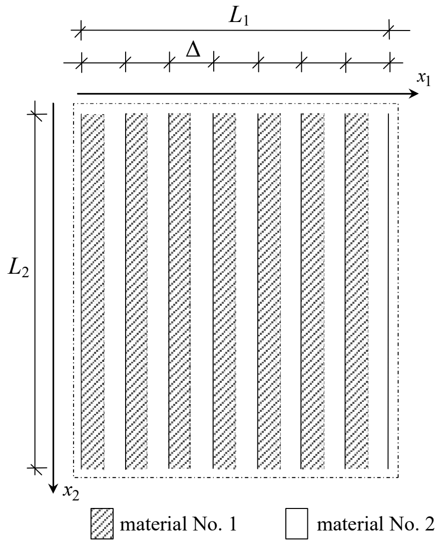

Figure 1. The cross-section of considered layer

The various issues related to this type of structures are considered in relation to micromechanical models with idealized geometry.

The Robin boundary conditions are analyzed in this note, there is an analytical solution for a homogeneous layer and this type of boundary conditions [13] and it is possible to obtain the distribution of the temperature according to the Eq. (1):

$$\theta = 2\sum_{n=1}^{\infty} \frac{\left(h^2+\alpha_n^2\right)\cos\left(\alpha_n x\right)\left\{\alpha_n \cosh\left(\alpha_n\left(L_2-y\right)\right)+h\sinh\left(\alpha_n\left(L_2-y\right)\right)\right\}}{\left[\left(h^2+\alpha_n^2\right)L_1+h\right]\left\{\alpha_n\cosh\left(\alpha_n L_2\right)+h\sinh\left(\alpha_n L_2\right)\right\}}\int_0^{L_1} f(x)\cos\left(\alpha_n x\right)dx, \quad (1)$$

where L_1, L_2 are dimensions along directions x_1, x_2, h is the quotient of heat transfer coefficient and the thermal conductivity, and α_n are the solutions of the Eq. (2):

$$\alpha\cdot \operatorname{tg}\left(\alpha L_1\right) = h. \quad (2)$$

The solution is limited to the finite number n equals 20 and shown in Fig. 2. As a material and geometry, it was assumed L_1=L_2=1 [m] and steel.

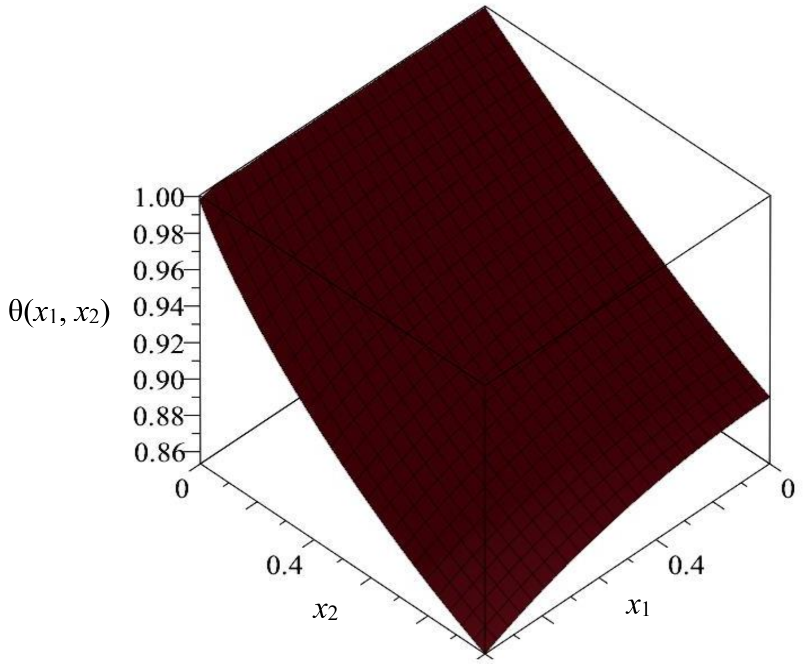

Figure 2. The temperature for homogeneous layer

To analyze the laminated layer, where the distribution function of material properties is periodic, the tolerance averaging technique is used [13]. This technique gives us a possibility to take into account the effect of the microstructure size. The tolerance modelling is expanded and applied in many publications to analyze various issues concerning both periodic and functionally graded structures. Among them are thermal issues [13]-**Błąd! Nie można odnaleźć źródła odwołania.** and dynamic problems [13]-**Błąd! Nie można odnaleźć źródła odwołania.**.

The main aim of this work is to obtain the equations of the tolerance model with the macrotemperature and the fluctuations amplitudes of the temperature as unknowns.

Modelling procedures

The stationary heat conduction issue for laminated layer can be described by Eq. (3):

$$\nabla\cdot\left(\mathbf{K}\cdot\nabla\theta\right) = 0, \quad (3)$$

where $\mathbf{K}$ is a tensor of conductivity, wherein components are denoted by k_{ij}.

The main aim of the application of the tolerance modelling is to replace the system of differential equations (3) with non-continuous coefficients, by equations, where the coefficients are slowly-varying. The basic assumption of the tolerance modelling is the micro-macro decomposition, where the temperature θ (the main unknown) can be expressed as a sum of the averaged part ϑ (the macrotemperature) and the oscillating part, according to the Eq. (4):

$$\theta(x_1,x_2) = \vartheta(x_1,x_2) + g(x_1)\psi(x_1,x_2). \tag{4}$$

On the other hand the oscillating part can be defined as a product of the know fluctuation shape function g and the fluctuation amplitudes of the temperature ψ (the new basic unknown). In this work the fluctuation shape function is assumed as a saw-type function. The second assumption of the tolerance modelling is the periodic approximation of some derivatives of function of the temperature, where some terms can be treated as negligibly small. Additionally, the tolerance averaging technique introduces some new concepts, among them the tolerance-periodic and slowly-varying function.

By using the micro-macro decomposition to the Eq. (3), using the orthogonalisation method, formulating the residuum function of temperature and the condition, which have to be fulfilled by this function, by doing appropriate averaging and transformations, the equations of the tolerance model for considered laminated layer are obtained in the form of Eqs (5):

$$\begin{aligned} &\nabla \cdot \left(\langle \mathbf{K} \rangle \cdot \nabla \vartheta + \langle \mathbf{K} \cdot \partial g \rangle \psi \right) = 0, \\ &\langle \partial g \cdot \mathbf{K} \cdot \partial g \rangle \psi + \langle \mathbf{K} \cdot \partial g \rangle \cdot \nabla \vartheta = 0. \end{aligned} \tag{5}$$

Example

Let L_1=L_2=1 [m]. The problem under consideration was a stationary heat conduction issue for laminated layer characterized by periodic structure of size Δ=L_1/20. For both sublayers the material properties (steel and aluminum) were defined and the constant distribution function of material properties was assumed (ν_1=0.5). Based on Eqs (5) and by using the assumption of the asymmetrical character of the fluctuation shape function, the equations of the tolerance model for considered issue are in the form of Eqs (6):

$$\begin{aligned} &\partial_1 \left(\langle k_{11} \rangle \partial_1 \vartheta + \langle k_{11} \partial g \rangle \psi \right) + \partial_2 \left(\langle k_{22} \rangle \partial_2 \vartheta \right) = 0, \\ &\partial_2 \left(\langle k_{22} g g \rangle \partial_2 \psi \right) - \langle k_{11} \partial g \rangle \partial_1 \vartheta - \langle k_{11} \partial g \partial g \rangle \psi = 0. \end{aligned} \tag{6}$$

The boundary conditions were assumed as follows: known temperature on the upper edge of the laminate ($\theta\big|_{x_2=0} = \theta_u$), thermally isolated left edge ($q_1\big|_{x_1=0} = q_t \Rightarrow \frac{\partial \vartheta}{\partial x_1}\big|_{x_1=0} = 0$), the Robin boundary conditions on the right edge (x_1=L_1) according to the Eq. (7) and on the bottom edge (x_2=L_2) according to the Eq. (8):

$$-k_{11} \frac{\partial \vartheta}{\partial x_1} = H\left(\vartheta - \vartheta_e\right), \tag{7}$$

$$-k_{22} \frac{\partial \vartheta}{\partial x_2} = H\left(\vartheta - \vartheta_e\right), \tag{8}$$

where ϑ_e is the external temperature and H is the heat transfer coefficient. In this note the external temperature is assumed to be equal zero. Then the boundary conditions for the fluctuation amplitudes of the temperature were assumed as: the known fluctuation amplitudes on the upper edge ($\psi\big|_{x_2=0}=0$), on the left edge ($\psi\big|_{x_1=0}=0$), on the right edge ($\psi\big|_{x_1=L_1}=0$) and the term on the bottom edge ($x_2=L_2$) following Eq. (8):

$$\frac{\partial\psi}{\partial x_2}+\frac{\langle Hgg\rangle}{\langle k_{22}gg\rangle}\cdot\psi=0. \tag{8}$$

To solve the equations of the tolerance model (Eqs (6)), the finite difference method was used. Along both directions (parallel and perpendicular to the laminas) the grid nodes distribution was uniform. By using this method the set of non-homogeneous discretized equations was obtained with the macro-temperature and the fluctuation amplitudes of the temperature as unknowns in the form of Eq. (9):

$$K\cdot X=Q, \tag{9}$$

where K is a matrix of coefficients, X is a vector of unknowns ranked alternately at individual points, and Q is a vector of free terms.

The results were shown in Fig. 3 in the form of plots of the total temperature.

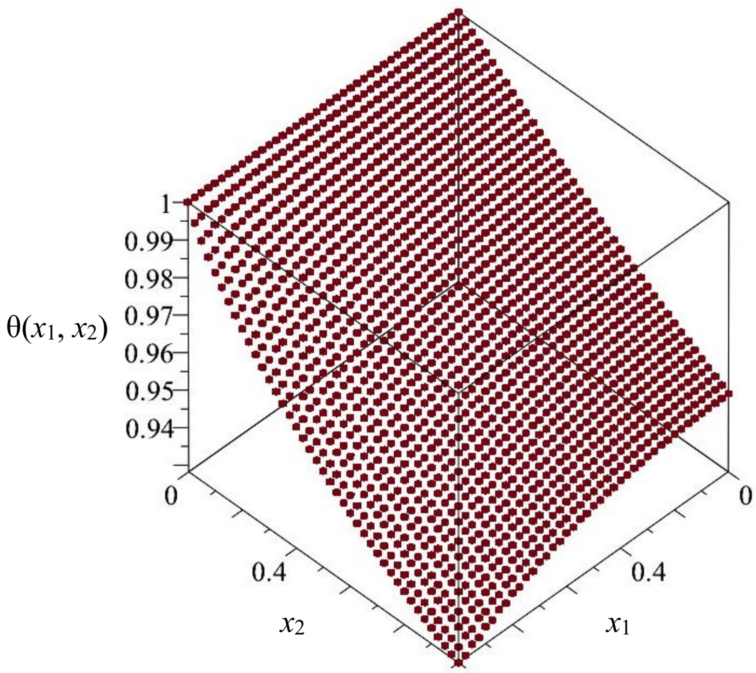

Figure 3. The total temperature

Conclusions

By using the tolerance averaging technique it is possible to replace the system of differential equations with non-continuous coefficients, by the equations where the coefficients are constant or slowly-varying. By using the equations of the tolerance model it is possible to take into account the effect of the microstructure size in thermal problems.

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