Uncertainty Quantification and Propagation Analysis Method Based on Contour Multi-ellipsoid Convex Model

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Abstract

Uncertainty quantification and propagation (UQP) analysis plays a crucial role in the research of uncertainty fields. At present, for many uncertainty problems with the limited samples, some methods presented in the non-probabilistic framework are very effective and applicable. However, the non-probabilistic UQP analysis methods with implying the hypothesis of the uniform distribution are inaccurate and unreliable for some problems with non-uniform distribution, such as Gauss distribution, extreme value distribution, which need to be further improved and expanded to make them more reasonable and appropriate for different distribution types. Therefore, this paper proposes a contour multi-ellipsoid convex model for UQP analysis by combining the areas of contour ellipsoids and their average entropy. First, based on regression analysis, the principal axis and attitude angle of ellipsoid are determined, meanwhile, the lengths of half axes are calculated. According to these geometric parameters, the covariance matrix obtained by using mathematical formula is used to construct the outermost ellipsoid. Subsequently, several contour multi-ellipsoids are draw by scaling the outermost ellipsoid in terms of different distribution types. Then, the average entropy of each ellipsoid is defined by calculating the distances of sample points and their mean to estimate the weight of each ellipsoid. Thus, the uncertainty of parameters can be quantified through the combination of the regions surrounded by contour multi-ellipsoids and their corresponding weight efficient. In uncertainty propagation process, to conveniently calculate the area of each region, all of the ellipsoids are transformed into spheres based on spatial transforms. Some regions may be divided into two domains by the response curve. Consequently, the volume ratio of the divided domain and the corresponding whole regions is calculated. The sum of the product of all the volume ratios and the weight efficient of the corresponding regions is defined as a pseudo-probability to objectively estimate the relatively possibility of each response value. Finally, through the comparisons of the proposed method and traditional method in numerical and engineering examples, it demonstrates that the proposed contour multi-ellipsoid convex model is much accurate and reasonable for different types of distributions. It also indicates that the establishment of multi-ellipsoids is easier than the traditional method based on optimization strategy and the analysis results using the proposed contour multi-ellipsoid convex model are stable and not affected by abnormal samples.

Keywords: Uncertainty quantification and propagation, Non-probabilistic framework, Contour multi-ellipsoid convex model, Average entropy, Ellipsoid weight efficient, Pseudo-probability