An Efficient Adaptive Strategy for Acoustic Problems with the Edge-based Smoothed Point Interpolation Method (ES-PIM)

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Abstract

In this paper we carry out an H-adaptive strategy for acoustic problems with the Edge-based smoothed point interpolation method. The key features of the adaptive procedure are an error indicator, H-type refinement strategy, local critical values, Delaunay mesh generation and ES-PIM analysis. The computations are performed on meshes with three-node triangles are adapted to the solution by locally changing element sizes, taking advantage of the background mesh which is convenient to discrete and conducts numerical simulation for any complicated model. The error indicator has been performed considering the maximum values of velocity difference among the vertexes in each cell. The adaptive meshes are then obtained through global mesh regeneration using a Delaunay mesh generator. The adaptive analysis is applied to 2D acoustic frequency response analysis, especially for expansion chamber. Numerical examples are shown to illustrate the properties of the error indicator technique and the procedure of the proposed adaptive strategy. The results highlight the efficiency of adaptive analysis, which reduces computation consumption significantly, and the results also have shown the validity and efficiency of the proposed error indicator.

Keywords: Adaptive analysis; Error indicator; H-refinement; Point interpolation method; Acoustic

Introduction

Numerical computation has been widely applied for scientific research and solving practical engineering problems in many fields. A large number of research results show that the adaptive analysis is an effective way to improve the efficiency and precision of the numerical calculation[1]-[7]. Through a lot of theoretical and numerical analysis, the ES-PIM can reduce the softening effects and give a quite close-to-exact stiffness by using the edge-based strain smoothing operation [8]-[12],it is found that ES-PIM is more suitable for solving acoustic problems, and a series of innovative research results have been obtained [13]-[14]. He and Liu applied the generalized gradient smoothing technique to the field of acoustic numerical computation [15]-[16].

Adaptive analysis is a reliable way to improve the accuracy and efficiency of acoustic problems, and error estimation with high reliability is also a very important factor in the study of acoustic adaptive analysis.In summary, ES-PIM have many aforementioned features which make it become an ideal candidate for adaptive analyses.

In the following section, we describe the implementation of the adaptive analysis based on ES-PIM in acoustic problems. Our adaptive strategy contains two main issues:error indicator and refinement strategy.The proposed adaptive procedure construct an error indicator combining refinement strategy and remeshing technique with the available open source packages.

Adaptive scheme

1. Error indicator

Error indicator plays a crucial role in adaptive procedure.It is able to accurately detect the regions for mesh refinement. According to the characteristics of acoustic problems, a new error indicator has been designed considering the maximum values of velocity difference among the vertexes in each cell.Generally, the high error region is consistent with the area of steep gradient velocity.

For each three-node triangular element, we can obtain the difference value of velocity components between two different nodes, namely node 1 and node 2 by the following equation.

$$\begin{aligned}\Delta v_{xx}^{(12)} &= \left| v_{xx}^{(1)} - v_{xx}^{(2)} \right| \\ \Delta v_{yy}^{(12)} &= \left| v_{yy}^{(1)} - v_{yy}^{(2)} \right|\end{aligned} \tag{1}$$

Eq(2) means the absolute value of the difference of velocity components between any two nodes in the same element is modulo.

$$E = \sqrt{\Delta v_{xx}^2 + \Delta v_{yy}^2} \tag{2}$$

Substituting (1) into (2), we can obtain the following Eq(3).

$$E_{12} = \sqrt{\left(\Delta v_{xx}^{(12)}\right)^2 + \left(\Delta v_{yy}^{(12)}\right)^2} = \sqrt{\left| v_{xx}^{(1)} - v_{xx}^{(2)} \right|^2 + \left| v_{yy}^{(1)} - v_{yy}^{(2)} \right|^2} \tag{3}$$

In the same way, we can calculate E_{13} and E_{23} for every two nodes in a cell. Finally for every background cell, we use the maximum modulo values of all nodes as the error indicator of the three-node triangular cell. Thus the error indicator E_i for the ith cell can be obtained as following equation.

$$\left\| E_i \right\| = Max\left(E_{12}, E_{23}, E_{13}\right) \tag{4}$$

2. Refinement strategy

In this section, we will describe a very simple and rapid method to implement refinement. As shown in Fig 1, white node denotes initial node, black node denotes new node which is inserted into the high error area, three additional black nodes will be inserted at the midpoints of the three edges and the original cell will be further divided into four triangles.

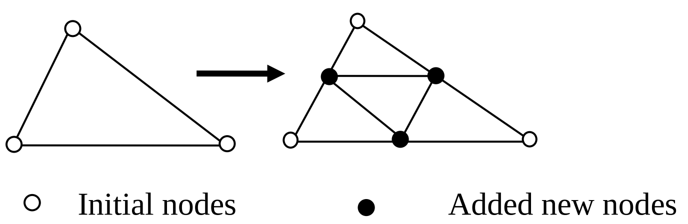


Figure 1. Illustration of the h-refinement strategy for three-node triangle

Local critical values , refinement rate and Delaunay mesh generator used in this part are similar to the counter part of adaptive ES-PIM on solid mechanics,details can be found in the previous work[2]-[5].

Numerical examples

（1）Case1: Two dimensional cavity

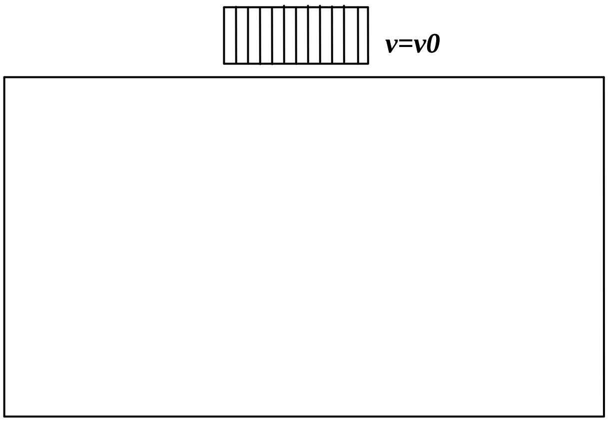


Figure 2. Geometry,boundary conditions of the two dimensional cavity

The first example is a room of length L=5 meters and width W=3 meters, which is considered as a two dimensional cavity.As shown in Fig 2, the acoustic excitation is at the top end of the room by a vibrating panel with velocity $\upsilon_0 = 1m/s$.On the other boundaries, the normal velocity is set to be zero. Fluid density in cavity is $\rho_0 = 1.225kg/m^3$, the sound travels speed in this medium is $c_0 = 340m/s$.

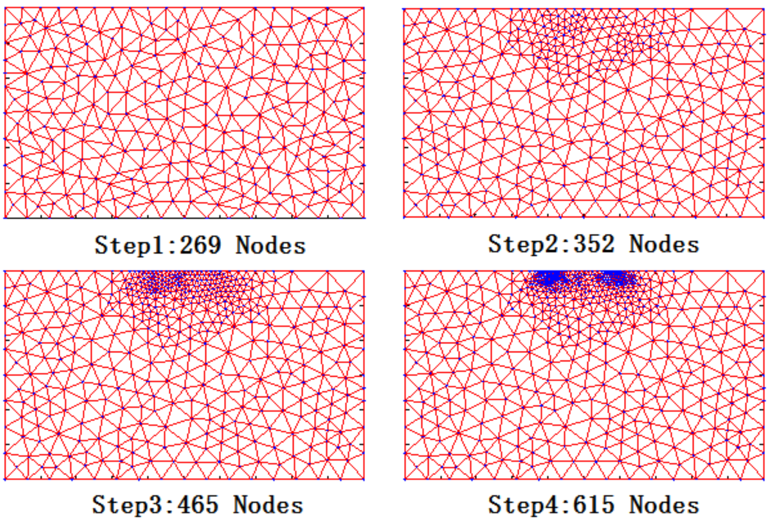


Figure 3. Nodes distribution and corresponding meshes of each adaptive step for two dimensional cavity at 40Hz

We studied this problem using 40Hz frequency, the ES-PIM adaptive analysis started from quite coarse mesh of 269 nodes and is performed for 4 steps with the refinement rate

$\eta = 0.1$.Four uniform refinement models with 269, 820, 2001 and 2910 nodes are studied respectively. The nodal distributions and corresponding meshes for 4 adaptive steps are shown in Fig. 3. The figure illustrates that our error indicator can accurately catch the steep gradient of velocity, the dense nodes are inserted near the top end of the room where the velocity gradient is high.

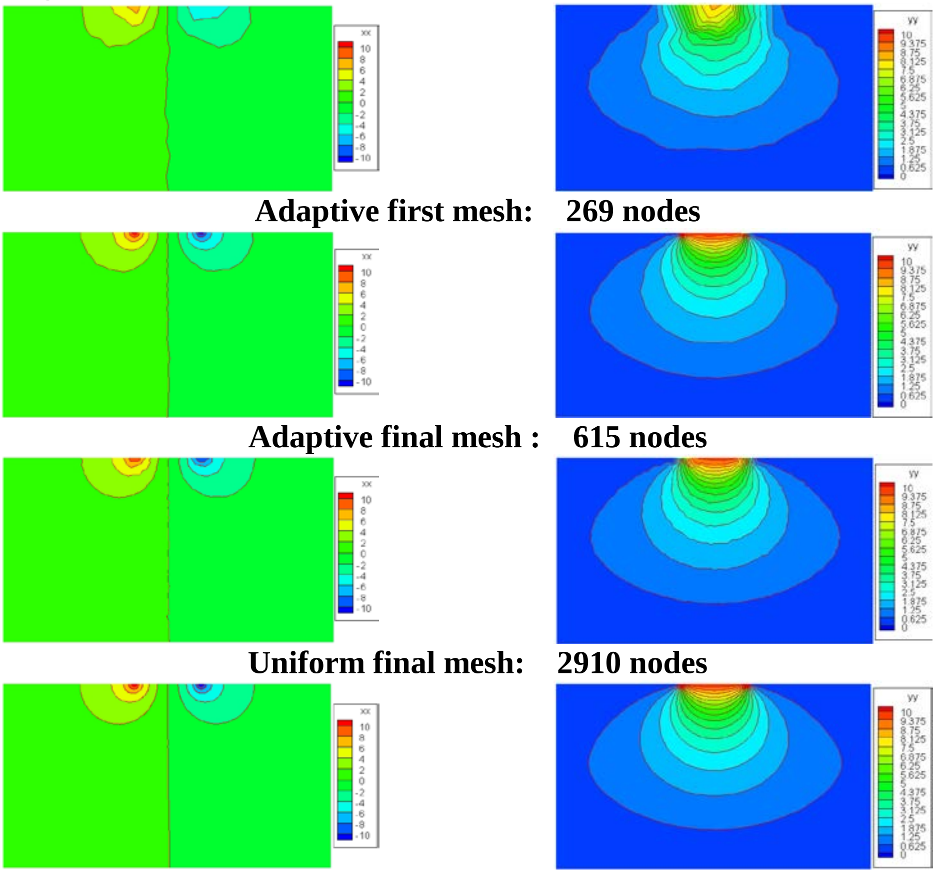


Figure 4. Comparison of velocity distributions at the first and final stage for the problem of two dimensional cavity at 40Hz

Fig. 4 compares the contours of velocity components at the first and final adaptive mesh with the final uniform mesh and FEM reference solution mesh results of 13123 uniformly distributed nodes. It clearly indicates that the velocity contours at the final stage with adaptive mesh(only 615 nodes) and uniform mesh(2910 nodes) are in good agreement with the reference solution results obtained using a very fine mesh.

（2）Case2: Expansion chamber

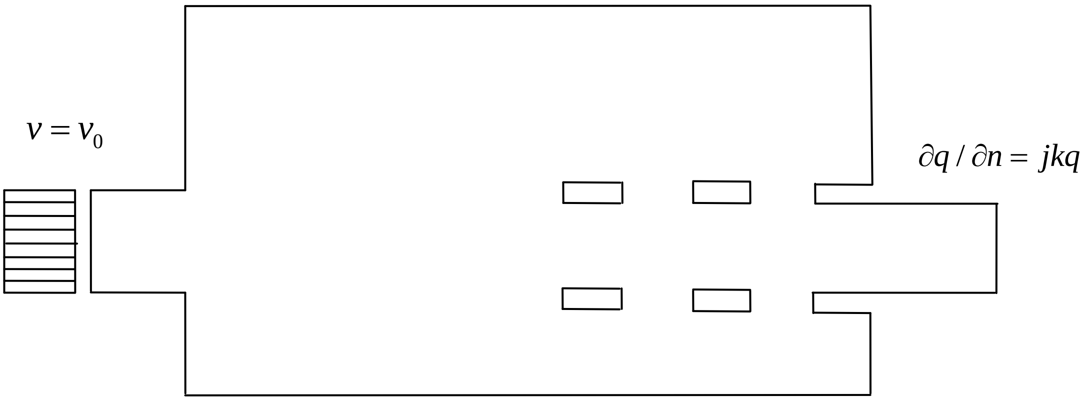


Figure 5. Geometry,boundary conditions of expansion chamber

The second problem is a 2D expansion chamber,whose geometry and boundary conditions are shown in Fig.5. Acoustic excitation is presented at the left side by a vibrating panel with velocity $\upsilon_0 = 0.1 m/s$. In order to better simulate the real situation,the boundary condition of the equation $\partial q / \partial n = jkq$ is presented at the right side, meanwhile, at the other side the normal boundary velocity is set to be zero. Fluid density in this model is $\rho_0 = 1.225 kg/m^3$,the sound travels speed in this medium is $c_0 = 340 m/s$.

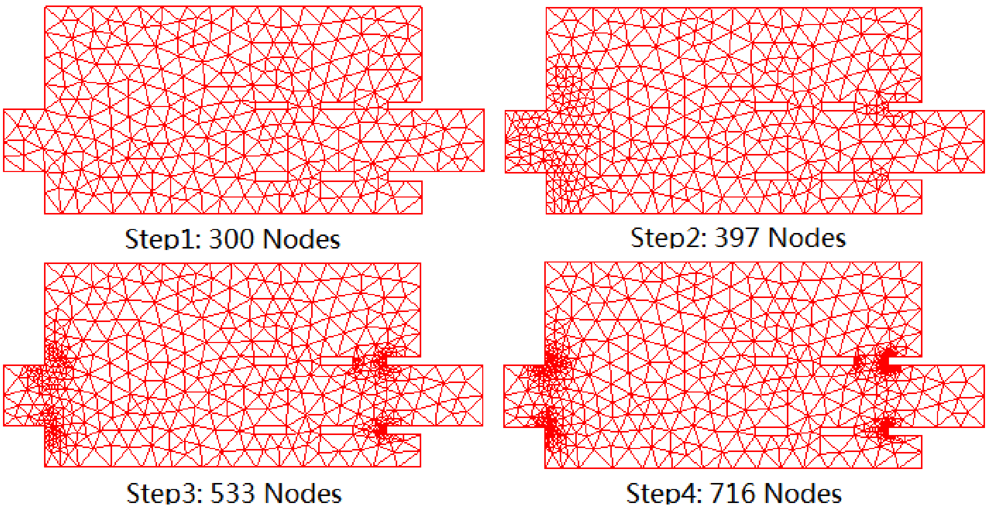
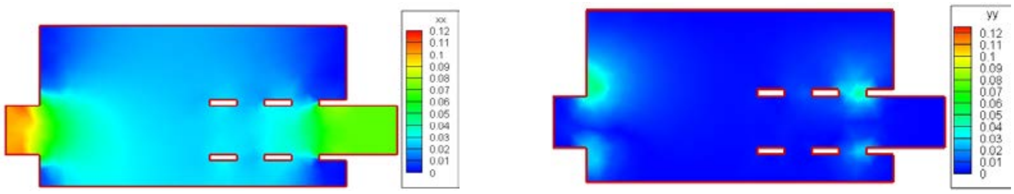


Figure 6. Nodes distributions at each adaptive step for the problem of expansion chamber

In this example, we studied expansion chamber using frequency of excitation is 200Hz. The adaptive procedure starts from an initial mesh of 300 nodes, and is performed for 4 steps with $\eta = 0.1$. The adaptive meshes for each step are shown in Fig. 6. One can notice that the proposed error indicator effectively detects all the regions for high velocity gradient and implements the refinement of nodes. For comparison, four models of uniformly refined models with 300, 1177, 2658 and 4076 nodes are also adopted to study this problem.



Adaptive first mesh: 300 nodes

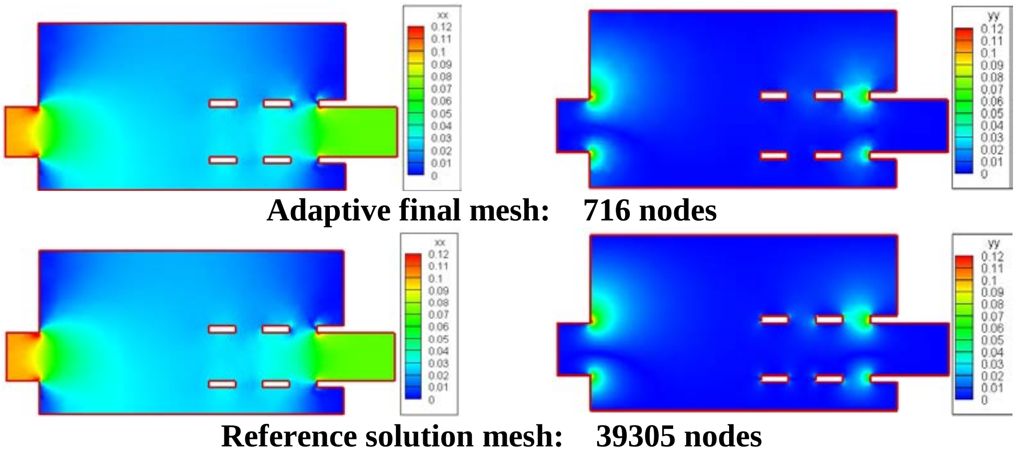


Figure 7. Comparison of velocity distributions at the first and final stage for the problem of expansion chamber at 200Hz

In Fig. 7 we compare the contours of velocity components at the first and final adaptive mesh with the FEM reference solution mesh results obtained using a very fine mesh. It can be seen that the velocity contours at the final stage with adaptive mesh (only contains 716 nodes) is in good agreement with the reference solution mesh of 39305 uniformly distributed nodes.

Conclusions

In this paper, an adaptive procedure using ES-PIM is proposed for acoustic problem. Our adaptive strategy uses initial meshes and remeshing which are implemented by the open source package TRIANGLE, FEM reference solution results using the software SYSNOISE. Numerical problems demonstrated that the proposed error indicator is able to detect the location steep gradient of velocity, h-refinement is performed by adding in new nodes. According to the practise in the research about 2D problems, which provides experience for extending to 3D problems in future.

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