

**AN EXPLICIT POSITIVITY-PRESERVING FINITE-DIFFERENCE
SCHEME
FOR SIMULATING BOUNDED GROWTH OF BIOLOGICAL FILMS**

***G.F. Sun¹, M. Li¹, and G.R. Liu²**

¹College of Mathematics, Taiyuan University of Technology,
Taiyuan, Shanxi 030024, China

²School of Aerospace Systems, University of Cincinnati
2851 Woodside Dr, Cincinnati OH 45221, USA

*Corresponding author: sungaofeng@tyut.edu.cn

Key Words: *S-FEM, CS-FEM, NS-FEM, ES-FEM, FS-FEM*

In this paper, a novel explicit finite-difference method is presented to approximate the positive and bounded growth of biological films that is governed by partial differential equations (PDEs) with a nonlinear density-dependent diffusion reaction. In such a unique nonlinear equation, the diffusion operator degenerates for small biomass densities and becomes singular when approaching the biomass density bound. Our novel finite-difference scheme is designed to transfer the nonlinear terms in the PDE into linear ones, while ensuring the stability of the solution through the control of the time-step, the numerical solution is confined between zero and one. The stability analysis reveals that our method is indeed stable a wide range of grad spacing and time steps, and it behaves like an implicit scheme in terms of stability. The present scheme is applicable for solving convection-diffusion equations with variable coefficients, since it avoids to producing negative solutions. The effectiveness of our method is demonstrated in numerical examples, by comparing with existing finite-difference schemes.

Keywords: Biofilm model, Finite-difference method, Nonlinear diffusion reaction, Explicit algorithm, implicitly algorithm