Three-dimensional simulation of unstable gravity-driven infiltration of water into a porous medium

***H. Gomez[1], L. Cueto-Felgueroso[2], R. Juanes[2]**

[1]Department of Mathematical Methods, University of A Coruña, Spain.
[2]Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, USA.

*Corresponding author: hgomez@udc.es

Infiltration of water in dry porous media is subject to a powerful gravity-driven instability. Although the phenomenon of unstable infiltration is well known, its description using continuum mathematical models has posed a significant challenge for several decades. The classical model of water flow in the unsaturated flow, the Richards equation, is unable to reproduce the instability. Here, we present a computational study of a model of unsaturated flow in porous media that extends the Richards equation and is capable of predicting the instability and captures the key features of gravity fingering quantitatively. The extended model is based on a phase-field formulation and is fourth-order in space. The new model poses a set of challenges for numerical discretizations, such as resolution of evolving interfaces, stiffness in space and time, treatment of singularly perturbed equations, and discretization of higher-order spatial partial-differential operators. We develop a numerical algorithm based on Isogeometric Analysis, a generalization of the finite element method that permits the use of globally-smooth basis functions, leading to a simple and efficient discretization of higher-order spatial operators in variational form. We illustrate the accuracy, efficiency and robustness of our method with several examples in two and three dimensions in both homogeneous and strongly heterogeneous media. We simulate, for the first time, unstable gravity-driven infiltration in three dimensions, and confirm that the new theory reproduces the fundamental features of water infiltration into a porous medium. Our results are consistent with classical experimental observations that demonstrate a transition from stable to unstable fronts depending on the infiltration flux.

Keywords: Phase-field, Water infiltration, Porous medium, Isogeometric analysis