

## Review of degenerate scale in the BEM/BIEM

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It is well known that BEM is an alternative approach to deal with engineering problems. However, nonuniqueness solution may occur. First, the occurring mechanism of degenerate scale is demonstrated for the circular and elliptical domains by using degenerate kernels. Later, five treatments for the nonuniqueness of degenerate scale appearing in the BEM/BIEM are reviewed. In this talk, we examine the sufficient and necessary formulations in the boundary integral equation method for the unique solution of 2D Laplace problem subject to the Dirichlet boundary condition. Both the analytical study and the numerical implementation are addressed. For the analytical study, we employ the degenerate kernel by using the polar and elliptical coordinates to represent the fundamental solution for the circular and elliptical domain, respectively. We can prove the unique solution after using rigid body addition of fundamental solution, hypersingular equation, CHEEF method, flux equilibrium approach as well as the Fichera's formulation for any size of circle and ellipse, respectively. In the numerical implementation, the BEM program, BEPO2D, developed by NTOU/MSV group is employed to verify the formulation. The relation between the Fichera's treatment for the indirect BEM and flux equilibrium treatment for the direct BEM will be linked. Besides, an ellipse case is demonstrated by using the above five regularization techniques. Finally, an example of arbitrary shape is analytically designed by using the unit logarithmic capacity and is numerically implemented to check the validity of five regularization techniques for degenerate scale.

**Keywords:** degenerate scale, nonuniqueness solution, BEM, BIEM, Fichera's treatment