Solving the Ill-posed Problems of the Helmholtz Equations Using the Boundary-Type Trefftz Collocation Method and the Equilibrated Matrix Concept

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Abstract

In this paper, the Helmholtz equations for the finite region are solved by the boundary-type Trefftz collocation method. Conventional boundary type Trefftz collocation method may encounter ill-posedness when the wave number is high or the Cauchy inverse problem is considered. To deal with the ill-posed behaviors of the resulting leading coefficient matrix, the so-called equilibrated matrix concept is adopted. The equilibrated matrix concept can rewrite the leading coefficient matrix such that the column norms of the new leading coefficient matrix become the same. Using this concept, it is found that the condition number of the leading coefficient matrix becomes smaller and numerical results show that this technique can dramatically improve the ill-posedness. After this technique is adopted, the high wave number problem such as wave number exceeds 200 m^{-1} can be easily treated. The same technique can be also applied to solve the inverse Cauchy problem in which on partial boundary the Cauchy boundary data are overprescribed and on the remaining part no information is given. This Cauchy problem is well-known for its ill-posed nature. Our proposed method is proved to be very efficient in dealing with the Cauchy inverse problems even when 5% random error exist in data. Several numerical examples are given to show the validity of the proposed method.

Keywords: boundary-type collocation Trefftz method, inverse Cauchy problem, Helmholtz equation, high wave number, ill-posedness, equilibrated matrix