Parameter-free Shape Optimization Method for Natural Vibration Design of Stiffeners on Thin-walled Structures

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Abstract

This paper presents a parameter-free shape optimization method for designing stiffeners on thin-walled structures subject to natural vibration. The design problems deal with natural frequency maximization problem and volume minimization problem, which are subject to a volume constraint and an eigenvalue constraint respectively. The boundary shapes of stiffeners are determined under the condition where the boundary is movable in the in-plane direction to the surface. The both optimization problems are formulated as distributed-parameter shape optimization problems, and the shape gradient functions are derived using the material derivative method and the adjoint variable method. The optimal free-boundary shapes of stiffeners are obtained by applying the derived shape gradient functions to the H^1 gradient method for shells, which is a parameter-free shape optimization method proposed by one of the authors. Several design examples are presented to validate the proposed method and demonstrate its practical utility of the proposed method.

Keywords: Shape optimization, Parameter-free, Stiffener, Thin-walled structure, Shell, FEM

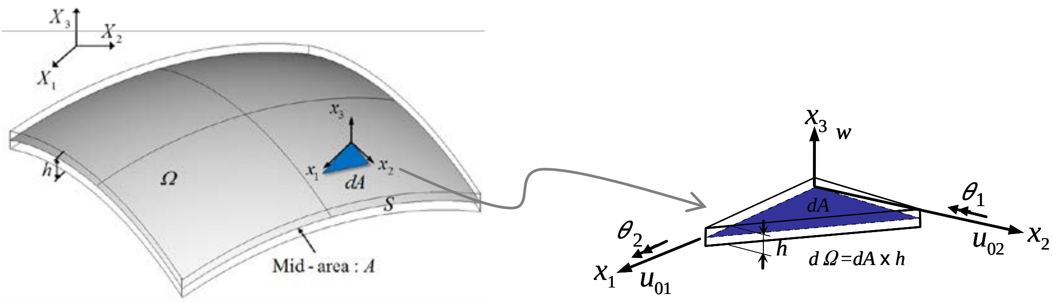
Introduction

Thin-walled or shell structures are widely used as basic structural components in various industrial products, such as car bodies, aircraft fuselages, and pressure vessels as well as in bridges and buildings. They are commonly stiffened by stiffeners to improve the bending rigidity of the basic structures. With recent enhancements of high speed, high function and weight reduction of thin-walled structures, the vibration design in consideration of the dynamic characteristics has become more important than ever. The natural frequencies (i.e., vibration eigenvalues) usually represent the dynamic characteristics of structures, especially the lower order natural frequencies are considered as an evaluation measure of the dynamic stability. The dynamic response of the structures can be reduced by increasing the lower order natural frequencies (Alejandro and Kikuchi, 1992; Ma *et al.*, 1995). Moreover, the reduction of the dynamic response of a structure generally leads to the minimum weight for the structure design (Zhao *et al.*, 1996).

For the natural vibration problems, this paper presents a shape optimization method for designing the free-boundaries of stiffeners and basic structures of thin-walled or shell structures. This method is based on the parameter-free optimization method for the boundaries of shells as mentioned above. Two kinds of natural vibration design problems are formulated here as distributed parameter shape optimization problems. One is a specified eigenvalue maximization problem subject to a volume constraint, and the other is its reciprocal volume minimization problem subject to a specified eigenvalue constraint. To eliminate difficulties caused by the "mode switching" problem (i.e., frequency crossing) (Eldred *et al.*, 1995), the Modal Assurance Criterion (MAC) (Allemang, 2003) is adopted to track the specified natural mode through changes in the eigenvalue maximization or eigenvalue constraint problem. Sensitivity functions (i.e., shape gradient functions) for the two design problems are theoretically derived using the material derivative method and the adjoint variable method. The optimal free-boundary shapes of stiffeners and the basic structure are determined by applying the derived shape gradient function to the H^1 gradient method for shells.

Variational equation for natural vibration of thin-walled structure

As shown in Fig. 1(a), a basic shell structure or stiffener with an initial bounded domain $\Omega \subset \mathbb{R}^3$ is defined by the mid-area A and the domain of thickness direction (-h/2, h/2), and the side surface S is defined by the boundary ∂A of the mid-area A.



(a) Geometry of a shell (b) D.O.F and sign convention

Figure 1. Shell geometry consisting of infinitesimal flat surfaces

The weak formed eigenvalue equation for natural vibration in terms of rth mode $(\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}) \in U$ can be expressed as Eq. (1)

$$a((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \lambda^{(r)} b((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) \;,\; (\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}) \in U, \forall (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}) \in U, \tag{1}$$

where $u_{0\alpha}$, w and θ_α express the in-plane displacement, out-of-plane displacement and rotational angle of the mid-area of the plate as shown in Fig. 1(b), respectively. $\bar{(\cdot)}$ expresses a variation and U expresses the admissible space in which the given constraint conditions of $(\boldsymbol{u}_0, w, \boldsymbol{\theta})$ is satisfied. $\lambda^{(r)}$ indicates the eigenvalue of the rth natural mode. In addition, the bilinear forms $a(\cdot, \cdot)$ and $b(\cdot, \cdot)$ are defined respectively as shown below.

$$a((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \int_A \{c^B_{\alpha\beta\gamma\delta}\theta^{(r)}_{(\gamma,\delta)}\bar{\theta}_{(\alpha,\beta)} + c^M_{\alpha\beta\gamma\delta}u^{(r)}_{0\gamma,\delta}\bar{u}_{0\alpha,\beta} + kc^S_{\alpha\beta}\gamma^{(r)}_\alpha\bar{\gamma}_\beta\} dA\,, \tag{2}$$

$$b((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \rho\int_A \{h(w^{(r)}\bar{w} + u^{(r)}_{0\alpha}\bar{u}_{0\alpha}) + I\theta^{(r)}_\alpha\bar{\theta}_\alpha\} dA\,, \tag{3}$$

where $c^B_{\alpha\beta\gamma\delta}$, $c^S_{\alpha\beta}$ and $c^M_{\alpha\beta\gamma\delta}$ express an elastic tensor with respect to bending, shearing and membrane stress, respectively. $\theta_{(\alpha,\beta)}(=\frac{1}{2}(\theta_{\alpha,\beta}+\theta_{\beta,\alpha}))$ expresses the curvatures and $\gamma_\alpha(=w_{,\alpha}-\theta_\alpha)$ expresses the transverse shear strains. Moreover, ρ and $I(=h^3/12)$ express a mass density and a second moment of area, respectively. The constant k denotes a shear correction factor, which can be used as k = 5/6 within Reissner theory of isotropic elastic plates (Reissner, 1945).

Shape optimization problem of stiffeners on the thin-walled structure

As shown in Fig. 2, the stiffened shell structure consists of a basic shell structure and stiffeners. To determine the optimal free-boundary shapes of stiffeners, the shape variations are considered as in-plane variations $\boldsymbol{V}$ in the tangential direction to the surfaces.

Eigenvalue maximization problem

Letting the eigenvalue equation in Eq. (1) and the volume be the constraint conditions and the eigenvalue of the specified rth natural mode be the objective functional to be maximized, a

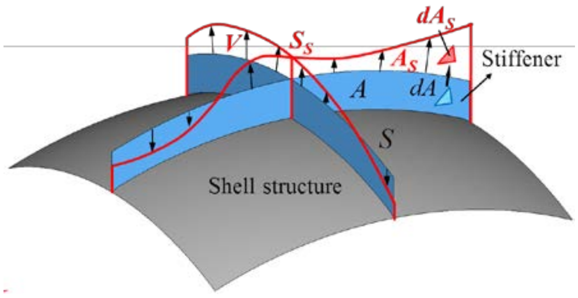


Figure 2. Shape variation of stiffeners and the basic shell by $\boldsymbol{V}$

distributed-parameter shape optimization problem for finding the optimal design velocity field of the stiffeners $\boldsymbol{V}$, or $A_s(= A + \Delta s\boldsymbol{V})$ can be formulated as shown below:

$$\text{Given} \quad A, \tag{4}$$

$$\text{find} \quad A_s \text{ (or } \boldsymbol{V}) \tag{5}$$

$$\text{minimize} \quad -\lambda^{(r)} \tag{6}$$

$$\text{subject to} \quad \text{Eq.(1) and } M \left(=\int_A hdA\right) < \hat{M} \tag{7}$$

where M and $\hat{M}$ denote the volume of the thin-walled structure with or without stiffeners and its constraint value, respectively.

For the mode switching problem as mentioned in Section 1, the Modal Assurance Criterion (MAC) [29] is used to track the specified rth natural mode of the initial shape. The mode with a maximum value of MAC in all natural modes is regarded as the corresponding mode and is tracked.

$$MAC(\phi_0^{(r)}, \phi_s) = \frac{|\{\phi_0^{(r)}\}^T\{\phi_s\}|^2}{(\{\phi_0^{(r)}\}^T\{\phi_0^{(r)}\})(\{\phi_s\}^T\{\phi_s\})} \tag{8}$$

where, $\phi_0^{(r)}$ and ϕ_s indicate the vectors of the rth mode of the initial shape and the each mode of the varied shape, respectively.

Letting $(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})$ and Λ_M denote the Lagrange multipliers for the eigenvalue equation and volume constraints, respectively, the Lagrange functional L associated with this problem can be expressed as

$$\begin{aligned} L(A,(\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}}),\Lambda_M) = \\ -\lambda^{(r)} + \lambda^{(r)} b((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) - a((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) + \Lambda_M(M - \hat{M}) \end{aligned} \tag{9}$$

Then, the material derivative $\dot{L}$ of the Lagrange functional can be derived as shown in Eq. (21) using the formula of material derivative (Choi and Kim, 2005).

$$\begin{aligned} \dot{L} = -a((\boldsymbol{u}_{0'}^{(r)}, w'^{(r)}, \boldsymbol{\theta}'^{(r)}),(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) + \lambda^{(r)} b((\boldsymbol{u}_{0'}^{(r)}, w'^{(r)}, \boldsymbol{\theta}'^{(r)}),(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) \\ -a((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}'}_0, \overline{w}', \overline{\boldsymbol{\theta}}')) + \lambda^{(r)} b((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}'}_0, \overline{w}', \overline{\boldsymbol{\theta}'})) \\ +\dot{\lambda}^{(r)}\{b((\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})) - 1\} + \dot{\Lambda}_M(M - \hat{M}) + \langle G\,\boldsymbol{n}, \boldsymbol{V}\rangle, \boldsymbol{V} \in C_{\Theta,} \end{aligned} \tag{10}$$

where,

$$\langle G\,\boldsymbol{n}, \boldsymbol{V}\rangle \equiv \int_S G\boldsymbol{n}\cdot\boldsymbol{V} dS = \int_S [-c^B_{\alpha\beta\gamma\delta}\theta^{(r)}_{(\alpha,\beta)}\overline{\theta}_{(\gamma,\delta)} \quad -kc^S_{\alpha\beta}(\theta^{(r)}_\beta - w^{(r)}_{,\beta})(\overline{\theta}_\alpha - \overline{w}_{,\alpha}) - c^M_{\alpha\beta\gamma\delta}u^{(r)}_{0\alpha,\beta}\overline{u}_{0\gamma,\delta}$$

$$+\lambda^{(r)}\rho\{h(w^{(r)}\overline{w}+u^{(r)}_{0\alpha}\overline{u}_{0\alpha)}+I\theta^{(r)}_{\alpha}\overline{\theta}_{\alpha}\}+\Lambda_M]\boldsymbol{n}\cdot\boldsymbol{V}dS \qquad (11)$$

The notation $\boldsymbol{n}$ in Eq. (11) is defined as an in-plane outward unit normal vector on boundary ∂A. Additionally, C_Θ expresses the admissible function space that satisfies the constraints of domain variation. The notation $(\cdot)'$ and $\dot{(\cdot)}$ are the shape derivative and the material derivative with respect to the domain variation, respectively (Choi and Kim, 2005).

When the optimality conditions with respect to the state variable $(\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)})$, the adjoint variable $(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})$ and Λ_{M} are satisfied, Eq. (10) becomes

$$\dot{L}=\left\langle G\boldsymbol{n},\boldsymbol{V}\right\rangle, \boldsymbol{V}\in C_\Theta. \qquad (12)$$

The sensitivity density function (i.e., the shape gradient density function) for this problem is derived as Eq. (13) by considering the quasi self-adjoint relationship as shown in (14).

$$G=-c^B_{\alpha\beta\gamma\delta}\theta^{(r)}_{(\alpha,\beta)}\overline{\theta}_{(\gamma,\delta)}-kc^S_{\alpha\beta}(\theta^{(r)}_\beta-w^{(r)}_{,\beta})(\overline{\theta}_\alpha-\overline{w}_{,\alpha})-c^M_{\alpha\beta\gamma\delta}u^{(r)}_{0\alpha,\beta}\overline{u}_{0\gamma,\delta}\ +\lambda^{(r)}\rho\{h(w^{(r)}\overline{w}+u^{(r)}_{0\alpha}\overline{u}_{0\alpha)}+I\theta^{(r)}_\alpha\overline{\theta}_\alpha\}+\Lambda \qquad (13)$$

$$(\overline{\boldsymbol{u}}_0,\overline{w},\overline{\boldsymbol{\theta}})=\frac{(\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)})}{b((\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)}),(\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)}))} \qquad (14)$$

Volume minimization problem

With the aim of designing the lightweight of stiffened thin-walled structures, we formulate the reciprocal problem of that treated in the preceding section. Letting the eigenvalue equation in Eq. (1) and the eigenvalue of the specified *r*th natural mode be the constraint conditions and the volume be the objective functional to be minimized. A distributed-parameter shape optimization problem is expressed as shown below:

$$\text{Given} \quad A, \qquad (15)$$

$$\text{find} \quad A_s\ (\text{or } \boldsymbol{V}) \qquad (16)$$

$$\text{minimize} \quad M\ (=\textstyle\int_A h dA\ ) \qquad (17)$$

$$\text{subject to} \quad \text{Eq.(1) and} \quad \lambda^{(r)}=\hat{\lambda}^{(r)} \qquad (18)$$

where $\hat{\lambda}^{(r)}$ is the constraint value of the eigenvalue of the specified *r*th natural mode. Letting $(\overline{\boldsymbol{u}}_0, \overline{w}, \overline{\boldsymbol{\theta}})$ and Λ_λ denote the Lagrange multipliers for the state equation and eigenvalue constraints, respectively, the Lagrange functional L associated with this problem can be expressed as

$$L((\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}}_0,\overline{w},\overline{\boldsymbol{\theta}}),\Lambda_\lambda)=M+\lambda^{(r)}b((\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}}_0,\overline{w},\overline{\boldsymbol{\theta}}))$$
$$-a((\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)}),(\overline{\boldsymbol{u}}_0,\overline{w},\overline{\boldsymbol{\theta}}))+\Lambda_\lambda(\lambda^{(r)}-\hat{\lambda}^{(r)})\ . \qquad (19)$$

Using the same procedure as in the case of the eigenvalue maximization problem, the shape gradient function of this problem is derived as shown in Eq. (20) by considering the quasi self-adjoint relationship in Eq. (21).

$$G=1-\Lambda_\lambda\{-c^B_{\alpha\beta\gamma\delta}\theta^{(r)}_{(\alpha,\beta)}\overline{\theta}_{(\gamma,\delta)}\ -kc^S_{\alpha\beta}(\theta^{(r)}_\beta-w^{(r)}_{,\beta})(\overline{\theta}_\alpha-\overline{w}_{,\alpha})-c^M_{\alpha\beta\gamma\delta}u^{(r)}_{0\alpha,\beta}\overline{u}_{0\gamma,\delta}\ +\lambda^{(r)}\rho\{h(w^{(r)}\overline{w}+u^{(r)}_{0\alpha}\overline{u}_{0\alpha)}+I\theta^{(r)}_\alpha\overline{\theta}_\alpha\}\} \qquad (20)$$

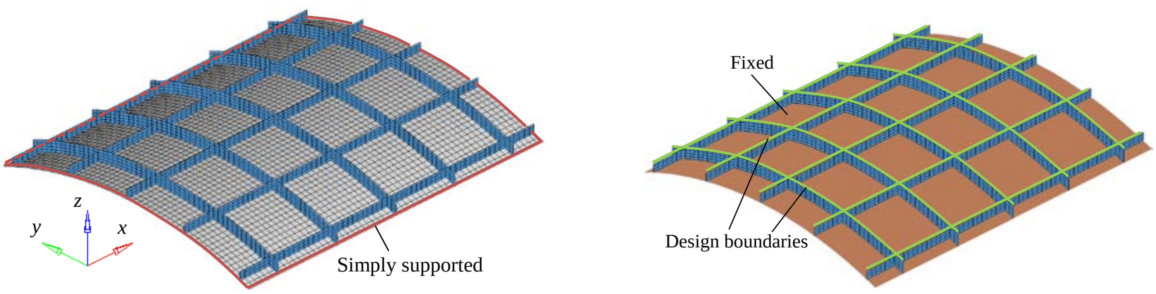
$$(\overline{\boldsymbol{u}}_0,\overline{w},\overline{\boldsymbol{\theta}})=\frac{\Lambda_\lambda(\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)})}{b((\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)}),(\boldsymbol{u}_0^{(r)},w^{(r)},\boldsymbol{\theta}^{(r)}))} \qquad (21)$$

Shape optimization method for determining the optimal free boundaries

The non-parametric shape optimization method described here for the design of the stiffened thin-walled structures is based on the H^1 gradient method, which is also called the traction method and is a type of gradient method in a Hilbert space. The original traction method was proposed by Azegami in 1994 (Azegami, 1994; Azegami *et al*., 1997). One of the authors has developed the optimization method for shell based on the original method (Shimoda and Tsuji, 2007; Shimoda *et al*., 2009; Shimoda, 2011). It is a node-based shape optimization method that can treat all nodes as design variables and does not require any shape design parametrization. This approach makes it possible to obtain the optimal boundary shapes of stiffened shell structures. The Dirichlet conditions are defined for a pseudo-elastic shell in the case of boundary shape optimization of stiffeners and the basic structure with this method. A distributed force proportional to the shape gradient function $-G\boldsymbol{n}$ is applied in the tangential direction to surfaces of the basic structure and stiffeners. The analysis for shape variation is called the velocity analysis. The shape gradient function is not applied directly to the shape variation but rather is replaced by a force, which varies shapes of stiffeners and the basic structure. This makes it possible both to reduce the objective functional and to maintain the smoothness, i.e., mesh regularity, which is the most distinctive feature of this method.

In the design problems of the eigenvalue maximization and the volume minimization, firstly, the eigenvalue analysis is done using a standard commercial FEM code and the outputs of the analysis are utilized to calculate the shape gradient function. After that, the velocity analysis is implemented, where a distributed force proportional to the negative shape gradient function $-G\boldsymbol{n}$ is applied to determine the design velocity field $\boldsymbol{V}$. Finally the shape is updated iteratively using the design velocity field $\boldsymbol{V}$. This process is repeated until the optimal shape of each design problem is obtained.

Results of numerical analysis



(a) Eigenvalue analysis (b) Velocity analysis

Figure 3. Boundary conditions of stiffened roof shell

The design example considered is a stiffener shape optimization of a roof shell stiffened by latticed stiffeners. Both eigenvalue maximization and volume minimization were carried out by the proposed method. The initial shape is shown in Fig. 3(a) along with the boundary conditions of the eigenvalue analysis, where both the round boundaries and the straight boundaries were simply supported. The constraint conditions for the velocity analysis are shown in Fig. 3(b), where the basic structure was fixed. The 1st natural mode of the initial shape obtained by the eigenvalue analysis is shown in Fig. 4. The specified 1st eigenvalue was maximized subject to the constant volume constraint, and the natural 1st mode was tracked. The optimal stiffener boundaries obtained in the eigenvalue maximization problem is shown in Fig. 5(a). According to the magnitude of the shape gradient function, the five stiffeners along the straight boundary of the basic structure were reduced, and the reduced volume shifted to the central stiffeners along the round boundary of the basic structure. Iteration histories of the compliance and the volume are shown in Fig. 6(a), in which the values have been normalized to those of the initial shape. The results show that the eigenvalue of the optimized shape increased approximately 40% while satisfying the constant volume constraint. Furthermore, the 1st eigenvalue was set as the constraint and the volume was minimized while tracking the natural 1st mode. Fig. 5(b) shows the optimal stiffener shapes obtained in the volume minimization problem. Fig. 6(b) shows iteration histories of the compliance and the volume for the volume minimization problem. It is confirmed that the volume of stiffeners decreased to 18.2% of the initial shape while satisfying the constant 1st eigenvalue constraint.

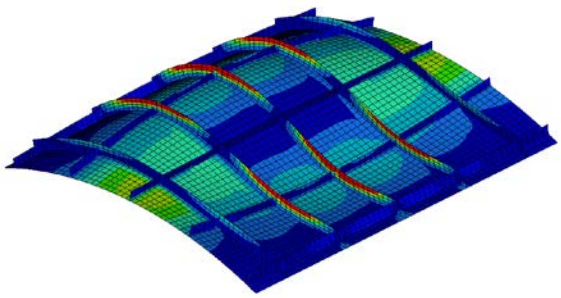
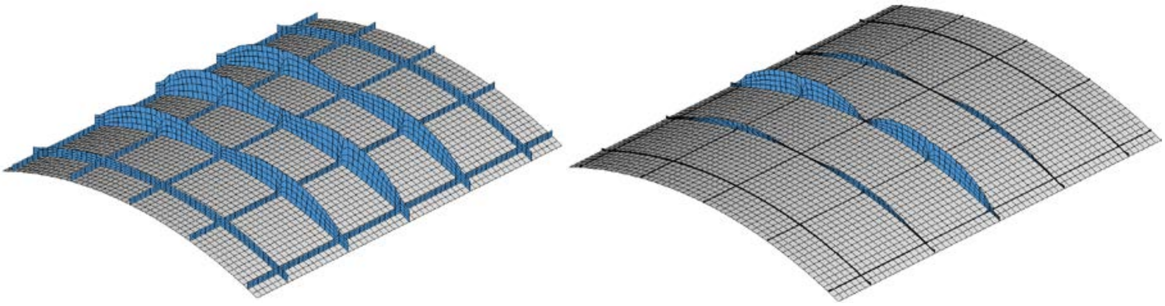
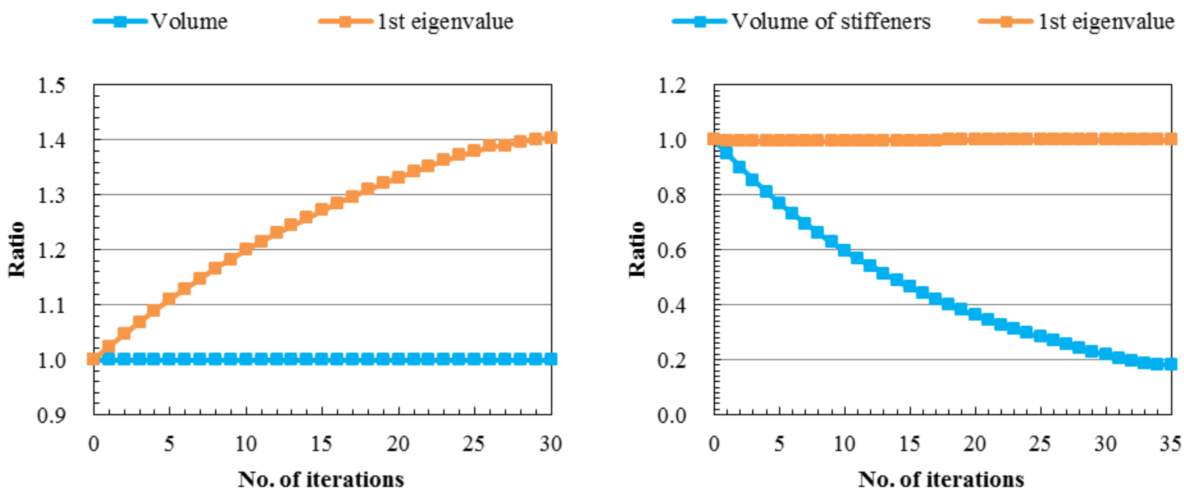


Figure 4. The 1st natural mode of stiffened roof shell



(a) Eigenvalue maximization (b) Volume minimization

Figure 5. Optimization results of stiffened roof shell



(a) Eigenvalue maximization (b) Volume minimization

Figure 6. Iteration histories of stiffened roof shell

Conclusions

This paper has proposed a parameter-free shape optimization method for designing the shapes of stiffened thin-walled or shell structures in the natural vibration problem. The optimal free boundary shapes of stiffeners can be obtained with the proposed method. A specified eigenvalue maximization subject to a volume constraint can be solved along with its reciprocal problem in which volume reduction is the objective. The proposed method has been applied to typical design problems of stiffened thin-walled structures, and the numerical results showed that smooth optimal boundary shapes were obtained in each design problem to achieve the maximum eigenvalue or lightweight structure. It has been demonstrated that the proposed method is an effective tool for designing optimal stiffeners on thin-walled or shell structures.

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