Low-dispersion sampling-based parameter importance measure analysis for anti-ice piccolo structure of anti-resonance design

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Abstract

In this paper, low-dispersion sampling-based importance measure analysis method is carried out for the anti-ice piccolo structure of anti-resonance design. First, the finite element model of anti-ice piccolo structure is established and the modal analysis is performed. Then based on the deterministic analysis, the importance measure model between structural size parameters and the natural frequency response is established. The proposed method uses the quadratic response surface methodology to fit the fundamental function between the natural frequency and variables, and a low dispersion sampling method is used to perform the structural parameter importance measure analysis. The analysis results can provide guidance for the anti-ice piccolo structural anti-resonance design.

Keywords: anti-ice piccolo structure, natural frequency, importance measure, low-dispersion sampling, response surface

Introduction

When an aircraft flies through clouds with low temperature or high humidity, the unexpected icing phenomenon may occur. Icing at wings will increase the aircraft weight, thus affect the maneuverability and stability of the aircraft. Light icing may trigger flight failures, and severe cases can lead to plane crash [1, 2]. Therefore, anti-ice design is needed for the civil and transport aircrafts, especially at the wing leading edge and other key parts.

Anti-ice piccolo structure is the core part of a wing anti-ice system. Its location is close to aircraft engines, and it is subject to random excitations generated by the engines. Therefore, there exists the possibility of structural resonances. Once the resonance happens, it may result in structural fatigue failures [3, 4]. Therefore, anti-resonance design is needed for anti-ice piccolo structure. Importance measure analysis can find out those design parameters which significantly affect the structural dynamics performance. This can provide guidance for the anti-resonance design of anti-ice piccolo structure [5].

Among importance measure analysis methods, the Monte Carlo method has been studied extensively. The most important advantage of Monte Carlo method is that it is unrestricted by the number of random variables, the type of probability distributions and response function etc [6,7]. However, the traditional Monte Carlo method firstly draws random samples using the uniformly distributed random number generator, and then obtains the needed samples by the conversion according to the real distributions. It is pointed out that at the case of small sample size, the distribution characteristics of sample points obtained in this way are not evenly distributed. In order to improve the quality of sampling points, Hua and Wang[8] proposed low dispersion sampling method based on number theory. It is proved that the convergence rate of this method is faster and the computing cost is smaller than the random sampling methods[9]. Therefore, we adopt the low dispersion sampling method to perform parameter importance measure analysis for the anti-ice piccolo structure.

In this paper, the anti-ice piccolo structure is taken as the research target, and the low-dispersion sampling-based importance measure analysis method is applied to perform importance measure analysis for the anti-ice piccolo structure. First, the anti-ice piccolo structure finite element model is established and the modal analysis is performed. Then, considering the structural size dispersion and based on the deterministic analysis, the importance measure model between structural size parameters and the natural frequency response is established. The proposed method uses the quadratic response surface methodology to fit the fundamental function between natural frequency and variables, and a low dispersion sampling method is used to perform the structural parameter importance measure analysis. The analysis results can provide guidance for the anti-ice piccolo structural anti-resonance design.

1. The importance measure analysis model for the anti-ice piccolo structure

1.1 Modal analysis pipeline

An aircraft wing anti-ice cavity structure is shown in Fig.1. Anti-ice piccolo is in the middle of the ice chamber. Its role is to assign the hot air coming from the engine to the wing leading edge. By heating the wing surface, we can achieve the purpose of anti-ice.

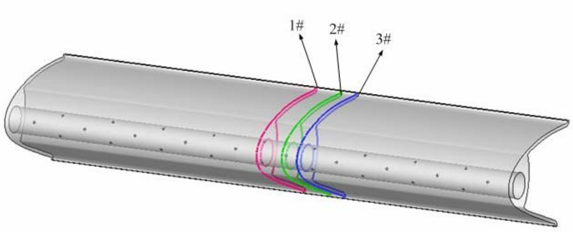


Fig.1 Geometrical model of anti-ice cavity structure

In this paper, we take one of the components of the anti-ice piccolo structure to perform the parameter importance measure analysis. The geometric dimension model of anti-ice piccolo structure is shown in Fig.2.

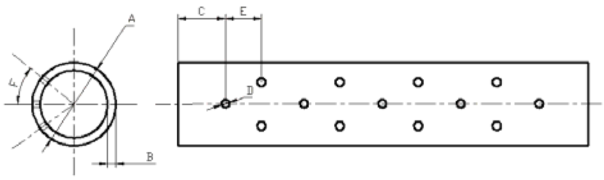


Fig.2 Geometrical model of piccolo structure

The finite element model (FEM) of anti-ice piccolo structure is shown in Fig.3, which is built with ANSYS software. The local mesh model is shown in Fig.4.

Fig.3 FEM model of piccolo structure

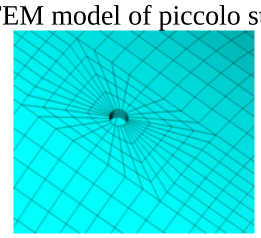


Fig.4 Local mesh model

By the modal analysis, the first four steps of nature frequencies of anti-ice piccolo structure can be obtained. The analysis results of nature frequencies are shown in Table 1 and the first four steps of natural mode shapes are shown in Fig.5.

Table 1 First four steps of nature frequencies of piccolo structure

mode step	1	2	3	4
nature frequency(Hz)	1538.4	1613.8	1819.0	1827.5

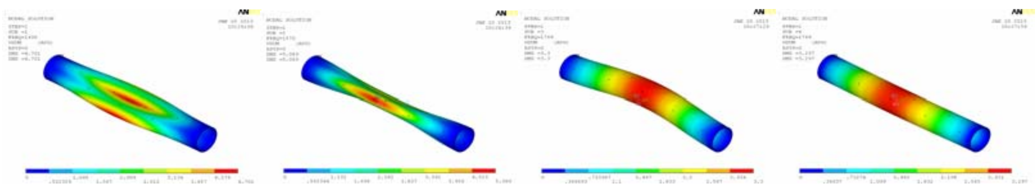


Fig.5 First four steps of natural mode shapes of piccolo structure

1.2 Importance measure analysis model of anti-ice piccolo structure

Randomness exists in the dimension parameters of piccolo structure, and these random variables are independent. The distribution types and distribution parameters are shown in table 2.

Table 2 Distribution type and parameter of the random variables

inputs	Identifier	the type of distribution	mean	standard deviation
Pipe diameter A(m)	x_1	Normal distribution	0.0558	0.001
Pipe wall thickness B(m)	x_2	Normal distribution	0.001	0.00004
Hole Side C(m)	x_3	Normal distribution	0.04	0.0008
Diameter D(m)	x_4	Normal distribution	0.0018	0.00007
Hole spacing E(m)	x_5	Normal distribution	0.04	0.0008
Hole angle F(°)	x_6	Normal distribution	45	1

Assume that the probability density functions of the variables $x_k (k=1,2,\cdots,6)$ are marked with $f_k(x_k)(k=1,2,\cdots,6)$. Then the joint probability density function of the structural parameters is $f_{\boldsymbol{x}}(\boldsymbol{x})=\prod_{k=1}^{6} f_k(x_k)$.

In the anti-ice cavity structure, the piccolo structure is subject to random excitations. As can be seen in Table 1, the external excitation frequency λ is close to the first natural frequency η of the structure. According to the stress-strength interference model, performance function of structure resonance failure can be calculated by Eq.(1):

$$g(\eta,\lambda)=|\eta-\lambda| \tag{1}$$

In engineering, a value δ is first given according to the requirement of anti-resonance design. When $g(\eta,\lambda)\le\delta$, the resonance structure fails; when $g(\eta,\lambda)>\delta$, the structure is safe. This shows that the natural frequency η is important for the anti-resonance design. The natural frequency η is a function of the basic variables $(x_1,x_2,\cdots,x_6)$. Therefore, performing importance measure analysis for the natural frequency is meaningful for anti-resonance design of the piccolo structure.

According to Refs.[10, 11], the importance measures of the structural parameters for the natural frequency are defined below

$$S_k=\frac{Var[E(\eta(\boldsymbol{x})\,|\,x_k)]}{Var[\eta(\boldsymbol{x})]}\ (k=1,2,\cdots,6) \tag{2}$$

where $Var[\eta(\boldsymbol{x})]$ is the variance of $\eta(\boldsymbol{x})$, $E(\eta(\boldsymbol{x})\,|\,x_k)$ is the mean value of $\eta(\boldsymbol{x})$.

In Eq.(2), the value of $Var[\eta(\boldsymbol{x})]$ can be estimated by the Monte Carlo method. The estimated value $\hat{V}ar[\eta(\boldsymbol{x})]$ of $Var[\eta(\boldsymbol{x})]$ is formulated as

$$\hat{V}ar[\eta(\boldsymbol{x})]=\frac{1}{N}\sum_{j=1}^{N}[\eta(\boldsymbol{x}_j)]^2-[\frac{1}{N}\sum_{j=1}^{N}\eta(\boldsymbol{x}_j)]^2 \tag{3}$$

Similarly, $Var[E(\eta(\boldsymbol{x})\,|\,x_k)]$ can be calculated by Monte Carlo method. Then substitute it into Eq.(2), the importance measures of the structural parameters can be obtained. The importance measure indicator S_k of the input parameter x_k characterizes the uncertainty of random variables x_k acting alone on the variability of the natural frequency $\eta(\boldsymbol{x})$. This variance-based importance measure can reflect the impact of input parameters variability on the variability of the natural frequency. So it can provide guidance for engineers to adjust the input design parameters for the purpose of improving the anti-resonance capability.

1.3 First natural frequency approximation based on quadratic response surface

In the process of importance measure analysis, structural natural frequency is an implicit function of input parameters, so it needs to perform the mode analysis with ANSYS software to obtain the natural frequency of anti-ice piccolo structure. If we directly use Monte Carlo method to repeatedly call the ANSYS software to implement random sample analysis, the computation cost will be very high. Therefore, in this paper, we use high-precision fitting method to fit the relationship between the natural frequency and input parameters so as to reduce the computation cost [12]. Among the response surface models, the fitting error of first-order response surface model is too large to reflect the true performance of structure. Although higher-order polynomial has higher fitting accuracy, it needs a high computational cost because a large number of items are involved. Especially in the case of multi-variable problem, the computational cost of fitting response surface will be unacceptable. For piccolo structure analysis, computational cost of second-order response surface is relatively small, and its fitting accuracy can meet the engineering application requirements [13, 14]. Therefore, this work uses quadratic response surface method to fit explicit functional relationship between the structure natural frequency and size parameters, and the formula is expressed as

$$\hat{\eta}(\boldsymbol{x})=F(x_1,\cdots,x_6)+\varepsilon \tag{4}$$

where ε is the statistical error.

The quadratic polynomial response surface model contains 28 coefficients, and $\hat{\eta}(\boldsymbol{x})$ can be calculated by Eq.(5).

$$\hat{\eta}^{(p)}(\boldsymbol{x})=c_0+\sum_{k}^{6}c_k x_k^{(p)}+\sum_{i=1}^{6}\sum_{j=i}^{6}c_{ij}x_i^{(p)}x_j^{(p)} \qquad (p=1,2,\cdots,n_s) \tag{5}$$

where n_s is the number of sample, and $\boldsymbol{c}=[c_0 \quad c_1 \quad \cdots \quad c_{27}]^T$ is the regression coefficient matrix.

Eq.(5) can be written in the form of matrices , as shown in Eq.(6)

$$\hat{\boldsymbol{\eta}}=\boldsymbol{Xc}+\boldsymbol{\varepsilon} \tag{6}$$

The regression coefficient matrix can be obtained by the least squares method.

$$\boldsymbol{c}=(\boldsymbol{x}^T\boldsymbol{x})^{-1}\boldsymbol{x}^T\hat{\boldsymbol{\eta}} \tag{7}$$

After the response surface model is obtained, we use R^2 (multiple correlation coefficient), R_a^2 (corrected multiple correlation coefficient) and *%RMSE* (root mean square deviation) to assess the adaptability of the model, which are calculated as follows

$$R^2=1-\frac{SS_E}{SS_T} \tag{8}$$

$$R_a^2=1-\frac{SS_E/(n_s-20)}{SS_T/(n_s-1)} \tag{9}$$

$$\%RMSE=100\sqrt{\frac{1}{n_s}\sum_{p=1}^{n_s}(\eta^{(p)}-\hat{\eta}^{(p)})^2}\Bigg/\left(\frac{1}{n_s}\sum_{p=1}^{n_s}\eta^{(p)}\right) \tag{10}$$

where $SS_E=\sum_{p=1}^{n_s}(\eta^{(p)}-\hat{\eta}^{(p)})^2$ is squares sum of error, $SS_T=\sum_{p=1}^{n_s}(\eta^{(p)})^2-(\sum_{p=1}^{n_s}\hat{\eta}^{(p)})^2\Big/n_s$ is the total squares sum.

2 Methods for solving importance measures of the anti-ice piccolo structural parameters

2.1 The existent method-random sampling method

The Monte Carlo-based parameter importance measure analysis method needs to produce samples following the distributions of the basic variables, which are transformed from samples of uniform distribution [0,1]. Currently, there are many methods to generate samples that follow the [0,1] uniform distribution, and the simplest method is the random sampling. The samples generating recurrence formula can be expressed as

$$\begin{cases} y_j \equiv (\lambda y_{j-1} + b)(\mathrm{mod}\, M) \\ r_j = y_j / M \end{cases} \qquad j = 1, 2, \cdots \tag{11}$$

where λ is multiplier, M is modulus, b is incremental, and y_0 is random source. All of them are pre-selected non-negative integers.

2.2 The proposed method-low dispersion sampling method

Low dispersion sampling method excludes the randomness of samples by the number theory. Thus, it can be more specific and exact to give the evenly distributed sample points. There are three low dispersion point sets. They are good lattice points set, good points set, and best uniform points set. Among these three points sets, the good lattice points set and best uniform points set are finite sets, and their sample sizes are imposed, so they are not able to add additional sample on the basis of existing sample size [9]. When using Monte Carlo method to perform importance measure analysis, the required sample size cannot be known in advance, which restricts the application of the good lattice point set and the best consistent points set on the importance measure analysis. Nevertheless, the sample size of good point set is not limited, and its computation process is simple, which makes it possible for us to use it into importance measure analysis. Hua and Wang[8] give the construction method of good points set based on low-dispersion sample. The analysis process of this method is as follows.

A six dimension good point $\boldsymbol{r} = (r_1, r_2, \cdots, r_6)$ iteration formats can be expressed as

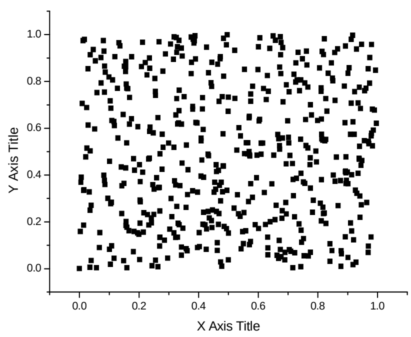
$$r_k^{(j)} = q_k j - \mathrm{int}(q_k j) \;\; (j = 1, 2, \cdots, N; k = 1, 2, \cdots, 6) \tag{12}$$

where $\mathrm{int}(\cdot)$ denotes the tail section rounding operation, and q_k can be generated through the sub-domain method, such as

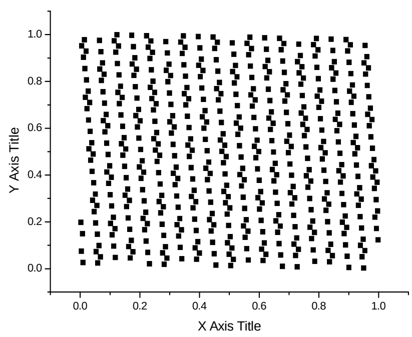
$$q_k = 2\cos\frac{2k\pi}{s} \;\; (k = 1, 2, \cdots, 6) \tag{13}$$

where s is a prime number, and $s \geq 13$.

Fig.6 shows the distribution characteristics -of 500 two-dimensional [0,1] uniformly distributed random samples that are generated by random sampling and low dispersion sampling respectively. Fig.6 clearly shows that low-dispersion sampling method under small sample size is better than random sampling method in terms of sample uniformity.



(a) random sampling



(b) low dispersion sample

Fig.6 500 Group 2D [0,1] uniformly distributed sample points

3 Importance measure analysis of the structural parameters

Based on the above analysis, we can get the process of parameter importance measure analysis for anti-ice piccolo structure, which is shown in Fig.7.

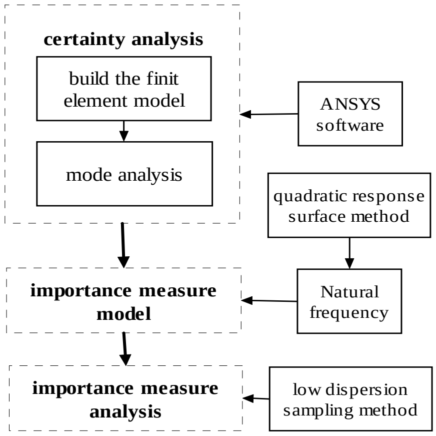


Fig 7. Flowchart of importance measure analysis for anti-icing cavity structural parameter

By using the quadratic response surface method, we obtain the relationship function $\hat{\eta}(\boldsymbol{x})$ between the first natural frequency and size parameters. After the response surface model is obtained, we use the R^2 (multiple correlation coefficient), R_a^2 (corrected multiple correlation coefficient) and % RMSE (root mean square deviation) to assess the adaptability of the model with Eqs.(8)-(10). The result shows that the multiple correlation coefficient R^2 is 3.158%, the corrected multiple correlation coefficient R_a^2 is 2.49%, and the root mean square deviation (%RMSE) is 0.98573%, which can meet the requirements of analysis accuracy.

Then low dispersion sampling method is used to perform the parameter importance measure analysis. The analysis results are shown in Table 3 and Fig.8.

Table 3 Results of importance measure analysis

variable	x_1	x_2	x_3	x_4	x_5	x_6
measure	0.566144	0.296415	0.002300	0.029427	0.070208	0.000254

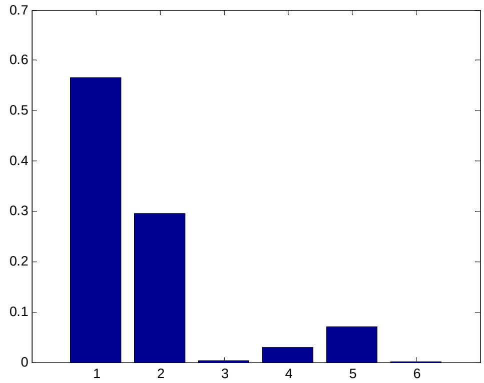


Fig.8 Diagram of random variables importance measure analysis results

As can be seen from Fig.8, pipe diameter and pipe wall thickness of the structure have greater impacts. Therefore, in optimization design they should be especially focused on.

4 Conclusions

(1) Based on mode analysis with ANSYS software, the importance measure model between structural size parameters and the natural frequency response is established. The fitting process of structural natural frequency based on quadratic response surface method is proposed.

(2) The flowchart of low-dispersion sampling-based importance measure analysis for the anti-ice piccolo structure is established.

(3) By the parameter importance measure analysis, we find that the pipe diameter and pipe wall thickness of the structure have important influences on the variability of the natural frequency. Hereby these parameters need to be paid more attention to in the anti-ice piccolo design.

Acknowledgements

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