Characteristics of MFS analysis for finite plate problems with a hole

***Wataru Fujisaki, T.Suetugu, and M. Takashima**

Department of Mechanical Engineering, Faculty of Technology, Kyusyu Sangyo University,
2-3-1 Matsukadai, Higashi-ku, Fukuoka City, Japan

*Corresponding author: fujisaki@ip.kyusan-u.ac.jp

Abstract

By using the method of fundamental solutions (MFS), an accurate stress field can be obtained in infinite problems. One of the authors has been improving the source loads of the MFS program based on the collocation method to be able to calculate the maximum stress of the notch problem correctly. It is found that the improved MFS, which uses the equally dispersed point loads (EDPL), gives the good accuracy for some infinite problems with high stress concentration factor. To confirm the accuracy, the index of the balanced forces was also proposed. Although, in case of the finite plate with a hole, the stability of accuracy mainly depends on the allocation of the collocation points and the source points. Therefore, in this study, we examine and clarify the proper allocation of the source loads in some finite plates including a center hole with internal pressure.

Keywords: method of fundamental solutions, MFS, source load, B.F-error ,stress field

Introduction

It is an important matter for the mechanical engineers to evaluate failure loads of some products including a hole. The linear notch mechanics (LNM) as shown in the work by Nisitani(1995), is a useful method to calculate the failure tensile stress and the LNM needs an accurate stress field. In this paper, the effects of source point dispositions to the accuracy of maximum stress on the finite problems are investigated using the method of fundamental solutions with the equally dispersed point load (EDPL). In this study, we calculate the stress around the hole with internal pressure of the rectangle plate as shown in Figure1. We consider a linear elastic problem, governed by the Navier equation. The fundamental solution for surface tractions on the fictitious boundary is

$$p_{lm}^{j*}(x)=\frac{-1}{4\pi(1-\nu)r^j(x)}\left\{\left[(1-2\nu)\delta_{lm}+2r^j(x)_{,l}\,r^j(x)_{,m}\right]\frac{\partial r}{\partial n}-(1-2\nu)\left(r^j(x)_{,l}\,n_m(x)-r^j(x)_{,m}\,n_1(x)\right)\right\} \tag{1}$$

where $r^j(x)=\left[\left(x_1^j-x_1\right)^2+\left(x_2^j-x_2\right)^2\right]^{1/2}$, $r,_m^j=\partial r^j/\partial x_m$, and δ_{lm} is the Kronecker delta. Equations (1) is written for the plane strain problem. In the homogeneous problem, we can obtain the known value of the variable by the sum of a series of fundamental solutions with unknown source loads $f(j),\ j=1,2N$. For point loads, following equation (2) can be expressed.

$$\sum_{j=1}^{N}\left[p_{lm}^{j*}\left(x^i\right)\cdot f(j)+p_{2m}^{j*}\left(x^i\right)\cdot f(j+N)\right]=\overline{p}_m^i \tag{2}$$

where p_{1m}^{j*} and p_{2m}^{j*} are fundamental solutions for surface tractions. For equally dispersed point loads (EDPL) by the author (Fujisaki 2005), we obtain the following equation (3) . Here, *M* is a number of EDPL, and in this study, we put *M*=10

$$\sum_{j=1}^{N}\left[\sum_{k=1}^{M}p_{1m}^{j*}\left(x_k^i\right)\cdot\frac{f(j)}{M}+\sum_{k=1}^{M}p_{2m}^{j*}\left(x_k^i\right)\cdot\frac{f(j+N)}{M}\right]=\overline{p}_m^i \tag{3}$$

Stress around a circle hole in a finite plate

We have calculated the accurate stress fields in the finite plate problem by varying the two dispositions of source loads, namely, inside the hole or outside the plate end (Fujisaki and Fujisawa 2012). Following two conclusions were obtained. (1) the index on the balanced forces is effective tool to obtain good accuracy. (2) In case of the finite plate problem, we find less good allocations to obtain good accuracy than the infinite plate problem in case of no bias (*bia=1.0*).

The stress concentration mainly occurs in a square area ($h<W$) including a hole. In this study, in order to increase the height of the plate, we divided the fictitious boundary line into the dense part and the sparse part as shown in Figure2. Here, x_{cs} is the distance between collocation point and source point, and y_{cc} is the distance between collocation points. The number of collocation points on the plate end N_e=100, and source loads inside hole are located on a fictitious arc of a circle (radius=0.7a), the number of collocation points on circle hole N_c= 40.

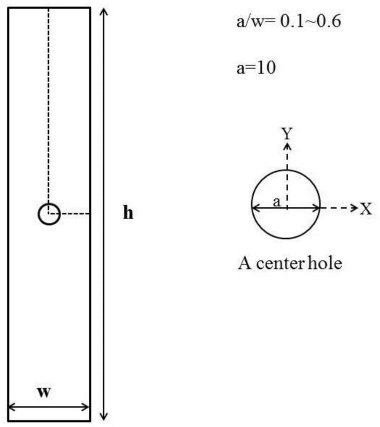

Figure 1. Calculation model

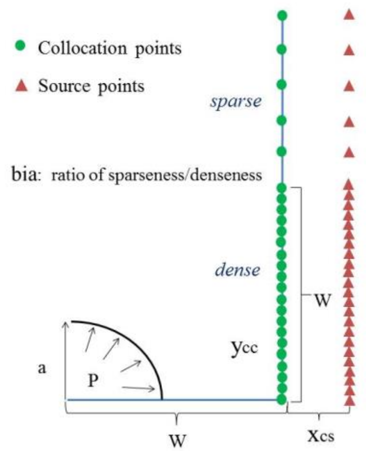

Figure 2. Discretization of plate end

We illustrate the free body diagram of a plate with a hole problem in Figure3. Here, the total external force (pa) is the summation of longitudinal force by the pressure of a quarter hole. On the other hand, the total internal force is obtained by the integral of $\sigma(x)$ dx at the ligament(unit thickness). Here if we define the index of B.F (balanced forces) as the ratio of the internal force and the external force, we can calculate the index of B.F. correctly by using some integral formula. Therefore if the index of B.F. is 1, then we can assure that the correct stress distribution is obtained. In the MFS, each point source load essentially yields a smooth curve of σ_{22}. Therefore the total superimposed curve of σ_{22} also becomes a smooth curve. If we obtain B.F-error(%) >0.0, then the calculated distribution curve exceeds the correct curve. If we obtain B.F-error(%) <0.0, the calculated curve is lower than the correct curve. This is an advantage of the MFS, and by using B.F-error(%), we can ensure the accuracy of maximum stress as shown in the work by the author (Fujisaki 2010).

In Figure4, we show the proper combinations of x_{cs} and y_{cc} to obtain B.F-error(%) <1.0 with a/w=0.6. In case of equal division *(bia*=1.0), the number N_{pc} of proper combinations is 30, but when we apply *bia*=2.0, we obtain N_{pc} =69. When we put *bia*=3.0, we can obtain N_{pc}= 92, therefore *bia*=3.0 is recommended to use. In Figure5, we summarize the results of proper combinations to obtain B.F-error(%)<0.1. In case of *bia*=1.0, N_{pc} is only 1, but when *bia*=2.0 or

3.0, we obtain Npc =9. In Figure6, we show the circumferential stress around the hole by the two conditions (A) (B) in Figure5, namely (A) xcs=3.5, ycc=5.2, and (B) xcs=14.0, ycc=2.2. We compare

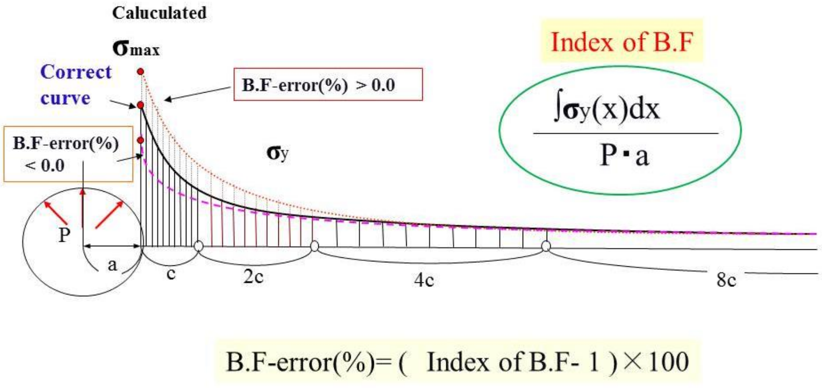

Figure 3. Comparison of circumferential stress

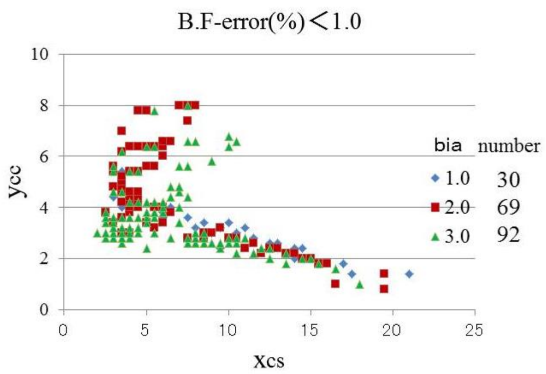

Figure4. Proper combinations- 1

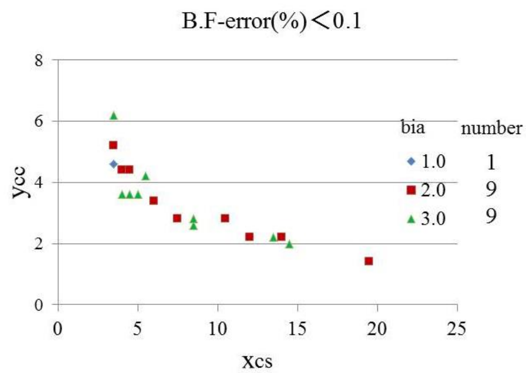

Figure 5. Proper combinations- 2

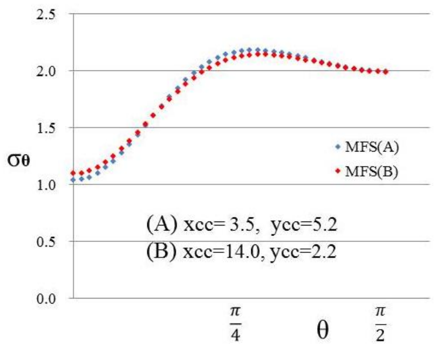

Figure 6. Comparison of circumferential stress

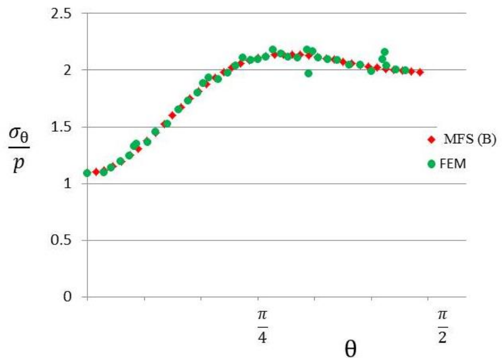

Figure 7. Comparison of MFS/FEM

those two cases with FEM (MS-Marc,8-nodes element, 2190 elements) ,and it is found that the case (B) is very consistent with FEM as shown in Figure7.

Characteristics of source loads

In Figure8, we compare the shape of source loads inside the hole in case of small width (a/w=0.6) with the three combinations ((A); x_{cs} =3.5, y_{cc}=5.2, B.F-error(%)=0.046, (B); x_{cs}=14.0, y_{cc}=2.2, B.F-error(%)=-0.077, (C); x_{cs}=8.8, y_{cc}=7.4, B.F-error(%)=20.1). In cases of (A),(B),we obtain good B.F-errors and the shapes of the source loads are similar. High source loads appear from Θ=0.1 π to 0.25 π . In case of (C), we obtain a bad B.F-error and the shape of the source loads is not so similar. High source loads appear from Θ=0.2π to 0.35π.

We show the three shapes of source loads outside the plate end (a/w=0.6) in Figure9. In case of (A), because of low x_{cs}, the maximum source load is small and appears near the ligament (h<w).

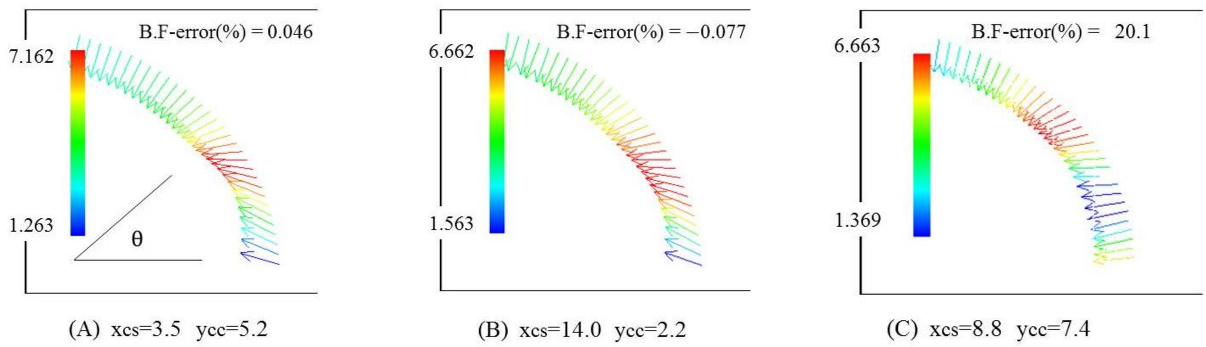

Figure 8. Comparison of source load shape inside hole

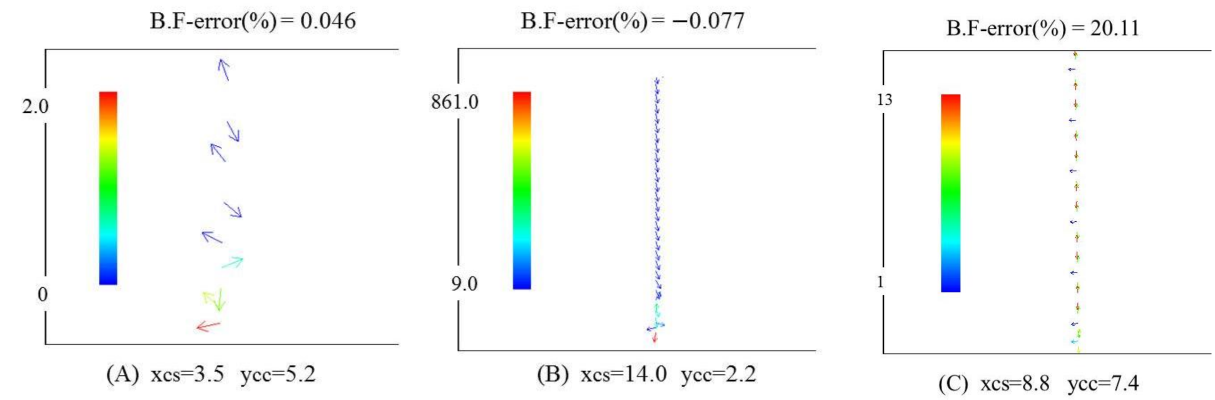

Figure 9. Comparison of source load shape outside plate end

In case of (B), because of large x_{cs}, the maximum source load is high and appears near the ligament ($h<w$) to satisfy the boundary conditions. And also because of small y_{cc}, we can see that monotonous pattern occurs in $h>w$. These source loads create the accurate stress field in the square area ($h<w$) and the remote area ($h>w$). In case of (C), B.F-error is very high, the directions of source load are horizontal or vertical and the large source loads appear everywhere not only near the ligament.

Conclusions

In order to obtain an accurate stress field of notched plate problem by the MFS, we applied the B.F-error and the sparse/dense allocation area to the plate end. Finally, the following results are mainly obtained.

(1) An accurate stress field can be obtained by minimizing B.F-error based on balanced forces.
(2) Combination of x_{cs} and y_{cc} is important to make an accurate stress field near and far the ligament. The most proper x_{cs} is nearly 0.8w, and y_{cc} is around 0.2w in case of a/w=0.6.
(3) We can allocate the collocation points and the source points sufficiently in longitudinal direction by using the sparse /dense allocation technique.

References

Nisitani, H. (1995), Linear Crack Mechanics and Linear Notch Mechanics in Fracture Problems, *Acta Mechanica Solida Sinica*, 8, pp. 114-117.

Fujisaki,W.(2005), Characteristics of Source Loads and Accuracy on Elliptical Hole by the MFS, *Advances in Computational & Experimental Engineering & Science 2005*, *Tech Science Press*, pp. 332-337,.

Fujisaki,W. (2010) ,Assurance of Accuracy on High SCF of Double Holes Problems by the Method of Fundamental Solutions, *Recent Studies in Meshless & Other Novel Computational Methods*, *Tech Science Press*, pp. 57-69.

Fujisaki,W. and Fujisawa,T. (2012) ,Stress around a center hole in a finite Plate Subjected to Internal Pressure, *European Congress on Computational Methods in Applied Science and Engineering*, *Vienna*, No.875,pp. 1-7.