Continued fraction formulation for infinite acoustic fluid with uniform cross section

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Abstract

Based on the continued fraction theory and the diagonalization procedure of the scaled boundary finite element method (SBFEM) for infinite acoustic fluid with uniform cross section, the high-frequency and the doubly asymptotic continued fraction formulations were derived. These formulations were applied to analyze a transient response of infinite acoustic fluid with uniform cross section under upstream excitations. Based on the transient response, the stability and convergence of the continued fraction formulations were discussed. Numerical results showed the doubly asymptotic continued formulation converged much faster than the high-frequency continued fraction formulation to analytical solutions. Comparison of computational efficiency between the continued formulation and the dynamic mass matrix was made. Computational costs of the continued fraction formulation were much less than that of the dynamic mass matrix.

Keywords: SBFEM, Continued fraction formulation, Infinite acoustic fluid, Transient analysis.

Introduction

The infinite acoustic fluid with uniform cross section is often encountered in dam-reservoir interaction problems. To obtain the dam-reservoir interaction response, the infinite acoustic fluid was truncated into a near-field with arbitrary geometry in the vicinity of dam and a far-field with uniform cross section which extends to infinity. The near-field can be easily modeled by FEM or BEM et al, while the far-field with uniform cross section can be modeled by transmitting boundary conditions (Gogoi and Maity (2006)), BEM (Czygan and Estorff Von (2002)) and so on. An alternative to model a far field, the scaled boundary finite element method (SBFEM), was verified to model accurately and effectively unbounded medium problems (Wolf and Song (1996)).

Based on the SBFEM formulation in displacement, Lin and his co-authors developed and applied the SBFEM to solve the semi-infinite acoustic fluid with uniform cross section problems in the frequency domain (Lin et al. (2007), Lin et al. (2010)), while Li and his co-authors improved the SBFEM for frequency problems of the semi-infinite acoustic fluid with uniform cross section (Fan and Li (2008), Li et al. (2008)), based on the SBFEM formulation in dynamic stiffness. In addition, based on the dynamic mass matrix, Li (2011) also applied the SBFEM to solve the transient analysis of the semi-infinite acoustic fluid with uniform cross section. Its results were very similar to solutions from other methods, but in the author’s experience, its computational efficiency was affected greatly by convolution integrals in the SBFEM. In order to improve the computational efficiency of SBFEM, a diagonalization formulation of SBFEM and a Bessel function to evaluate the dynamic stiffness of SBFEM were proposed by Li (2009) and Li (2012), respectively. Although these methods improved the SBFEM efficiency, they still need a convolution integral evaluation which results in the nonlinear increase of computational cost with analysis step number increasing. An alternative, a continued fraction formulation, was proposed by Prempramote et al. (2009), which can avoid the evaluation of convolution integral. Based on the continued fraction formulation and the SBFEM diagonalization formulation, this research derived the continued fraction formulation in matrix form of infinite acoustic fluid with uniform cross section.

SBFEM Formulation for Infinite Acoustic Fluid with Uniform Cross Section

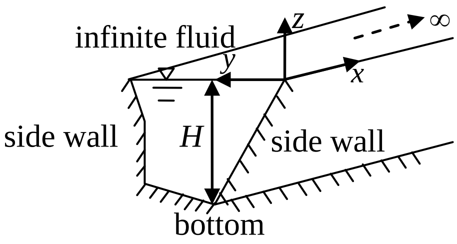


Figure 1. Infinite acoustic fluid with uniform cross section

For an infinite acoustic fluid with uniform cross section of arbitrary geometry as shown in Figure 1 only subjected to upstream excitations in x direction, the whole infinite acoustic fluid can be modeled by the SBFEM, which only needs the discretization of cross section of the infinite fluid. Its boundary conditions can refer to the reference (Li et al. (2008)). Ignoring effects of surface waves and absorption of side walls and bottom, its SBFEM formulation satisfies (Li (2009)) on the cross section

$$\mathbf{V}_{\mathrm{n}} = \sqrt{\left(\mathbf{E}^2 - \omega^2\mathbf{M}^0\right)\mathbf{E}^{0^{-1}}}\,\mathbf{E}^0\mathbf{\Phi} = \mathbf{S}^{\infty}(\omega)\mathbf{\Phi} \tag{1}$$

The symbols $\mathbf{S}^{\infty}(\omega)$, $\mathbf{\Phi}$, $\mathbf{V}_{\mathrm{n}}$ are the dynamic stiffness matrix of infinite fluid after SBFEM discretization, velocity potential vector and equivalent normal velocity vector caused by upstream excitations, respectively. ω is the excitation frequency. $\mathbf{E}^0$, $\mathbf{E}^2$, $\mathbf{M}^0$ are SBFEM coefficient matrices, which were defined by Wolf and Song (1996). $\mathbf{V}_{\mathrm{n}}$ is expressed as

$$\mathbf{V}_{\mathrm{n}} = \int_A \mathbf{N}_{\mathrm{f}}^{\mathrm{T}} v_{\mathrm{n}}\, dA \tag{2}$$

The symbol A denotes the cross section. $\mathbf{N}_{\mathrm{f}}$ is the shape function of acoustic fluid finite element. v_{n} is normal velocity. Using the diagonalization technique (Li (2009)), one has

$$\mathbf{S}_{\mathrm{d}}^{\infty}(\omega) = \mathbf{X}^{\mathrm{T}}\mathbf{S}^{\infty}(\omega)\mathbf{X} = \sqrt{\mathbf{\Lambda} - \left(\frac{\omega}{c}\right)^2\mathbf{I}} \tag{3}$$

with

$$\frac{\mathbf{E}^2}{H}\mathbf{X} = H\mathbf{E}^0\mathbf{X}\mathbf{\Lambda} \tag{4}$$

$$\mathbf{X}^{\mathrm{T}}\frac{\mathbf{E}^2}{H}\mathbf{X} = \mathbf{\Lambda} \tag{5}$$

$$\mathbf{X}^{\mathrm{T}} H\mathbf{E}^0\mathbf{X} = \mathbf{I} \tag{6}$$

where the square matrix $\mathbf{X}$ is the eigenvector matrix and the matrix $\mathbf{\Lambda}$ is the eigenvalue matrix of the eigenvalue equation (4); $\mathbf{I}$, c, H are the identity matrix, the sound speed in fluid and the height of cross section, respectively.

Continued Fraction Formulation of Dynamic Stiffness Matrix

According to the continued fraction theory (Prempramote et al. (2009)) based on the dynamic stiffness of semi-infinite layer with constant depth, the order M_{H} high-frequency continued fraction solution of Equation (3) is equivalent to

$$\mathbf{S}_{\mathrm{d}}^{\infty}(\omega) = \mathrm{i}\frac{\omega}{c}\mathbf{C}_{\infty}^{\mathrm{d}} - \left(\mathbf{Y}_{\mathrm{d}}^{(1)}(\omega)\right)^{-1}\mathbf{\Lambda} = \mathrm{i}\frac{\omega}{c}\mathbf{I} - \left(\mathbf{Y}_{\mathrm{d}}^{(1)}(\omega)\right)^{-1}\mathbf{\Lambda} \tag{7}$$

$$\mathbf{Y}_{\mathrm{d}}^{(j)}(\omega) = \mathrm{i}\frac{\omega}{c}\mathbf{Y}_1^{(j)} - \mathbf{\Lambda}\left(\mathbf{Y}_{\mathrm{d}}^{(j+1)}(\omega)\right)^{-1} \qquad (j = 1,2,\ldots,M_{\mathrm{H}}) \tag{8}$$

Substituting Equations (7, 8) into Equation (3) yields

$$\mathbf{\Lambda}-\mathrm{i}\frac{\omega}{c}2\mathbf{b}_1^{(j)}\mathbf{Y}_{\mathrm{d}}^{(j)}(\omega)-\left(\mathbf{Y}_{\mathrm{d}}^{(j)}(\omega)\right)^2=0 \qquad (j=1,2,...,M_{\mathrm{H}}) \tag{9}$$

$$\mathbf{b}_1^{(j)}=(-1)^{j+1}\mathbf{b}_1^{(1)}=(-1)^{j+1}\mathbf{I} \qquad (j=1,2,...,M_{\mathrm{H}}) \tag{10}$$

$$\mathbf{Y}_1^{(j)}(\omega)=(-1)^j 2\mathbf{b}_1^{(1)}=(-1)^j 2\mathbf{I} \qquad (j=1,2,...,M_{\mathrm{H}}) \tag{11}$$

In order to improve the accuracy of Equations (7, 8) in low frequency range, the residual term $\mathbf{Y}_{\mathrm{d}}^{(M_{\mathrm{H}}+1)}(\omega)$of Equation (8) is denoted as

$$\mathbf{Y}_{\mathrm{L}}(\omega)=\mathbf{Y}_{\mathrm{d}}^{(M_{\mathrm{H}}+1)}(\omega) \tag{12}$$

$$\mathbf{Y}_{\mathrm{L}}(\omega)=\mathbf{Y}_{\mathrm{L0}}^{(0)}+\mathrm{i}\frac{\omega}{c}\mathbf{Y}_{\mathrm{L1}}^{(0)}-\left(\mathrm{i}\frac{\omega}{c}\right)^2\left(\mathbf{Y}_{\mathrm{L}}^{(1)}(\omega)\right)^{-1} \tag{13}$$

$$\mathbf{Y}_{\mathrm{L}}^{(j)}(\omega)=\mathbf{Y}_{\mathrm{L0}}^{(j)}-\left(\mathrm{i}\frac{\omega}{c}\right)^2\left(\mathbf{Y}_{\mathrm{L}}^{(j+1)}(\omega)\right)^{-1} \quad (j=1,2,...,M_{\mathrm{L}}) \tag{14}$$

$$\mathbf{Y}_{\mathrm{L0}}^{(0)}=(-1)^{M_{\mathrm{H}}+1}\sqrt{\mathbf{\Lambda}} \tag{15}$$

$$\mathbf{Y}_{\mathrm{L1}}^{(0)}=(-1)^{M_{\mathrm{H}}+1}\mathbf{I} \tag{16}$$

$$\mathbf{Y}_{\mathrm{L0}}^{(j)}=(-1)^{M_{\mathrm{H}}+j+1}2\sqrt{\mathbf{\Lambda}} \quad (j=1,2,...,M_{\mathrm{L}}) \tag{17}$$

Note that continued fraction formulations of Equations (7-17) are expressed in a matrix form. The formulations with $M_{\mathrm{L}}=0$ and $M_{\mathrm{L}}\neq 0$ are called a high-frequency and a doubly asymptotic continued fraction formulation, respectively.

Time-Domain Formulation Based on Continued Fraction Formulation

Re-writing Equation (1) yields

$$\mathbf{V}_{\mathrm{n}}^{\mathrm{d}}=\mathbf{X}^{\mathrm{T}}\mathbf{V}_{\mathrm{n}}=\mathbf{S}_{\mathrm{d}}^{\infty}(\omega)\mathbf{X}^{-1}\mathbf{\Phi}=\mathbf{S}_{\mathrm{d}}^{\infty}(\omega)\mathbf{\Phi}^{\mathrm{d}} \tag{18}$$

Substituting Equations (7, 8) into Equation (18) yields

$$\mathbf{V}_{\mathrm{n}}^{\mathrm{d}}=\mathrm{i}\frac{\omega}{c}\mathbf{C}_{\infty}^{\mathrm{d}}\mathbf{\Phi}^{\mathrm{d}}-\sqrt{\mathbf{\Lambda}}\mathbf{\Phi}_{\mathrm{d}}^{(1)} \tag{19}$$

$$\sqrt{\mathbf{\Lambda}}\mathbf{\Phi}_{\mathrm{d}}^{(j)}=\mathbf{Y}_{\mathrm{d}}^{(j+1)}(\omega)\mathbf{\Phi}_{\mathrm{d}}^{(j+1)} \quad (j=0,1,2,...,M_{\mathrm{H}}) \tag{20}$$

$$\sqrt{\mathbf{\Lambda}}\mathbf{\Phi}_{\mathrm{d}}^{(j-1)}=\mathrm{i}\frac{\omega}{c}\mathbf{Y}_1^{(j)}\mathbf{\Phi}_{\mathrm{d}}^{(j)}-\sqrt{\mathbf{\Lambda}}\mathbf{\Phi}_{\mathrm{d}}^{(j+1)} \quad (j=1,2,...,M_{\mathrm{H}}) \tag{21}$$

where $\mathbf{\Phi}_{\mathrm{d}}^{(0)}=\mathbf{\Phi}^{\mathrm{d}}$ and $\mathbf{\Phi}_{\mathrm{d}}^{(j)}$ is an auxiliary variable. Substituting Equations (12, 13) into Equation (21), one has

$$\sqrt{\mathbf{\Lambda}}\mathbf{\Phi}_{\mathrm{d}}^{(M_{\mathrm{H}})}=\mathbf{Y}_{\mathrm{L0}}^{(0)}\mathbf{\Phi}_{\mathrm{d}}^{(M_{\mathrm{H}}+1)}+\mathrm{i}\frac{\omega}{c}\mathbf{Y}_{\mathrm{L1}}^{(0)}\mathbf{\Phi}_{\mathrm{d}}^{(M_{\mathrm{H}}+1)}-\mathrm{i}\frac{\omega}{c}\mathbf{\Phi}_{\mathrm{dL}}^{(1)} \tag{22}$$

where $\mathbf{\Phi}_{\mathrm{dL}}^{(1)}$ is an auxiliary variable and satisfies

$$\mathrm{i}\frac{\omega}{c}\mathbf{\Phi}_{\mathrm{d}}^{(M_{\mathrm{H}}+1)}=\mathrm{i}\frac{\omega}{c}\mathbf{\Phi}_{\mathrm{dL}}^{(0)}=\mathbf{Y}_{\mathrm{L}}^{(1)}(\omega)\mathbf{\Phi}_{\mathrm{dL}}^{(1)} \tag{23}$$

Substituting Equation (14) into Equation (23) leads to

$$\mathrm{i}\frac{\omega}{c}\mathbf{\Phi}_{\mathrm{dL}}^{(j)}=\mathbf{Y}_{\mathrm{L}}^{(j+1)}(\omega)\mathbf{\Phi}_{\mathrm{dL}}^{(j+1)} \qquad (j=0,1,2,...,M_{\mathrm{L}}) \tag{24}$$

$$\mathrm{i}\frac{\omega}{c}\mathbf{\Phi}_{\mathrm{dL}}^{(j-1)}=\mathbf{Y}_{\mathrm{L0}}^{(j)}\mathbf{\Phi}_{\mathrm{dL}}^{(j)}-\mathrm{i}\frac{\omega}{c}\mathbf{\Phi}_{\mathrm{dL}}^{(j+1)} \qquad (j=1,2,...,M_{\mathrm{L}}) \tag{25}$$

$$\mathbf{\Phi}_{\mathrm{dL}}^{(0)}=\mathbf{\Phi}_{\mathrm{d}}^{(M_{\mathrm{H}}+1)} \tag{26}$$

Combining Equations (18, 21, 22, 25), one has

$$\mathbf{K}_{\mathrm{h}}\mathbf{z} + \mathbf{C}_{\mathrm{h}}\dot{\mathbf{z}} = \mathbf{f} \tag{27}$$

with

$$\mathbf{K}_{\mathrm{h}} = \begin{bmatrix} \mathbf{0} & -H\mathbf{E}^0\mathbf{X}\sqrt{\mathbf{\Lambda}} & & & & & & & \\ -\sqrt{\mathbf{\Lambda}}\mathbf{X}^{\mathrm{T}}H\mathbf{E}^0 & \mathbf{0} & -\sqrt{\mathbf{\Lambda}} & & & & & & \\ & -\sqrt{\mathbf{\Lambda}} & \mathbf{0} & \ddots & & & & & \\ & & \ddots & \ddots & -\sqrt{\mathbf{\Lambda}} & & & & \\ & & & -\sqrt{\mathbf{\Lambda}} & \mathbf{0} & -\sqrt{\mathbf{\Lambda}} & & & \\ & & & & -\sqrt{\mathbf{\Lambda}} & \mathbf{Y}_{\mathrm{L0}}^{(0)} & \mathbf{0} & & \\ & & & & & \mathbf{0} & \mathbf{Y}_{\mathrm{L0}}^{(1)} & \ddots & \\ & & & & & & \ddots & \ddots & \mathbf{0} \\ & & & & & & & \mathbf{0} & \mathbf{Y}_{\mathrm{L0}}^{(M_{\mathrm{L}})} \end{bmatrix} \tag{28}$$

$$\mathbf{C}_{\mathrm{h}} = \frac{1}{c}\begin{bmatrix} H\mathbf{E}^0 & \mathbf{0} & & & & & & & \\ \mathbf{0} & \mathbf{Y}_1^{(1)} & \mathbf{0} & & & & & & \\ & \mathbf{0} & \mathbf{Y}_1^{(2)} & \ddots & & & & & \\ & & \ddots & \ddots & \mathbf{0} & & & & \\ & & & \mathbf{0} & \mathbf{Y}_1^{(M_{\mathrm{H}})} & \mathbf{0} & & & \\ & & & & \mathbf{0} & \mathbf{Y}_{\mathrm{L1}}^{(0)} & -\mathbf{I} & & \\ & & & & & -\mathbf{I} & \mathbf{0} & \ddots & \\ & & & & & & \ddots & \ddots & -\mathbf{I} \\ & & & & & & & -\mathbf{I} & \mathbf{0} \end{bmatrix} \tag{29}$$

$$\mathbf{z} = \left\{ \mathbf{\Phi}^{\mathrm{T}} \quad \mathbf{\Phi}_{\mathrm{d}}^{(1)\mathrm{T}} \quad \mathbf{\Phi}_{\mathrm{d}}^{(2)\mathrm{T}} \quad \ldots \quad \mathbf{\Phi}_{\mathrm{d}}^{(M_{\mathrm{H}})\mathrm{T}} \quad \mathbf{\Phi}_{\mathrm{d}}^{(M_{\mathrm{H}}+1)\mathrm{T}} \quad \mathbf{\Phi}_{\mathrm{dL}}^{(1)\mathrm{T}} \quad \ldots \quad \mathbf{\Phi}_{\mathrm{dL}}^{(M_{\mathrm{L}})\mathrm{T}} \right\}^{\mathrm{T}} \tag{30}$$

$$\mathbf{f} = \left\{ \mathbf{V}_{\mathrm{n}}^{\mathrm{T}} \quad \mathbf{0} \quad \mathbf{0} \quad \ldots \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \ldots \quad \mathbf{0} \right\}^{\mathrm{T}} \tag{31}$$

Equation (27) is the time-domain governing equation of infinite acoustic with uniform cross section based on the continued fraction formulation.

Note that Equations (1-31) are applicable to two- and three-dimensional problems because its derivation process is independent of problem dimension. If the SBFEM coefficient matrices $\mathbf{E}^0$, $\mathbf{E}^2$, $\mathbf{M}^0$ are from two-dimensional or three-dimensional problems, Equations (1-31) are for two-dimensional or three-dimensional problems. Therefore, these equations' accuracy is only validated by two-dimensional problems in the following section.

Numerical Examples

Consider transient responses of vertical rigid dam-reservoir system shown in Figure 2 under horizontal (upstream) excitations shown in Figure 3, where the reservoir is full of acoustic fluid with uniform cross section, and the surface wave and reservoir bottom absorption are ignored. Its aim is to validate the accuracy of Equation (27). Reservoir water height $H = 180$, water density $\rho = 1000\,\mathrm{kg/m^3}$, sound speed in water c=1438.656m/s. The reservoir was discretized by 10 three-node SBFEM elements.

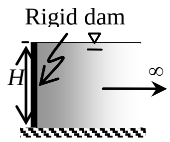


Figure 2. Rigid vertical dam-reservoir system

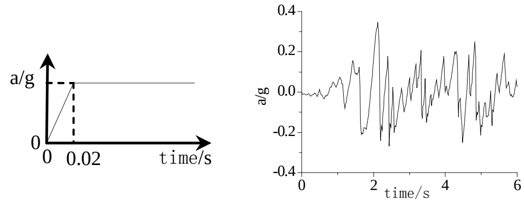


Figure 3. Horizontal accelerations (Left: Ramped; Right: El Centro)

Figure 4 shows the pressure at the heel of dam obtained by different order M_{H} and M_{L} continued fraction formulations under horizontal ramped acceleration shown in Figure 3. Time step increment is 0.005s. Results from high-frequency continued fraction formulation with $M_{\mathrm{H}} = 50$, 100 were different with that obtained from dynamic mass matrix after the time 2.5s, while results from doubly asymptotic continued fraction formulation with $M_{\mathrm{H}} = M_{\mathrm{L}} = 5$ and $M_{\mathrm{H}} = M_{\mathrm{L}} = 10$ were much more accurate, especially at late time, which showed that the doubly asymptotic continued formulation can obtain more accurate results than high-frequency continued fraction formulation. Figure 5 plots the pressure of dam's heel from the doubly asymptotic continued fraction formulation under horizontal El Centro acceleration shown in Figure 3. With the order increasing, results gradually tend to those from dynamic mass matrix. Results from dynamic mass matrix were almost similar to analytical solutions (Li (2009)). Figure 6 plots results from the doubly asymptotic continued fraction formulation with $M_{\mathrm{H}} = M_{\mathrm{L}} = 10$ using different time step increments 0.0002s and 0.005s. Results from 0.0002s are more accurate than those from 0.005s. Figures 4-6 show that results from the continued fraction formulation become more and more similar to exact solution when time step increment becomes smaller and smaller and the order of doubly asymptotic continued fraction formulation becomes higher and higher, which validates the convergence of Equation 27.

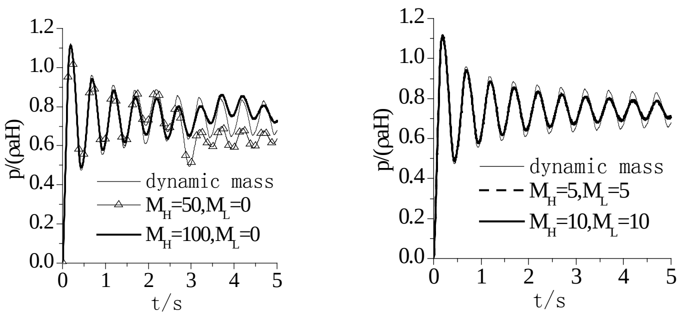


Figure 4. Pressure at the heel of rigid vertical dam under ramped acceleration

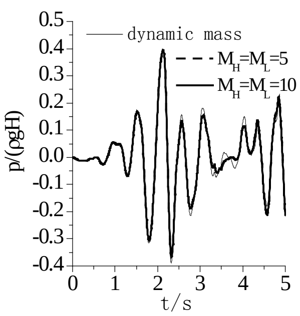
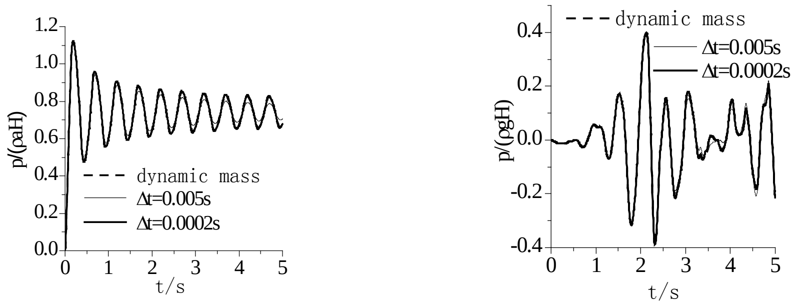


Figure 5. Pressure at the heel of rigid vertical dam under El Centro acceleration



a) Ramped acceleration **b) El Centro acceleration**

Figure 6. Pressure at the heel of rigid vertical dam under different time increment

Through solving eigenvalues of Equation (27), it can be found that real parts of eigenvalues are positive when M_{H} and M_{L} are greater than zero, which ensure Equation (27) is stable. When M_{L} is equal to zero, real parts of eigenvalues of Equation (27) are zeros, which ensure Equation (27) is not divergent. Maybe that is why results from high-frequency continued fraction formulation are not convergent to analytical solutions.

Table 1. Response computational time comparison

Time step	1000	2000	5000
Dynamic mass matrix, matrix dimension 20×20	0.35s	2.17s	25.25s
Continued fraction formulation with M_L=M_h=5, matrix dimension 240×240	0.07s	0.14s	0.34s
Continued fraction formulation with M_L=M_h=10, matrix dimension 440×440	0.28s	0.58s	1.38s

Table 1 lists the response computational cost based on the dynamic mass matrix and the continued fraction formulation. Response computational time of dynamic mass matrix is only the time cost to evaluate the response based on convolution integral, not including the dynamic mass matrix evaluation time. The response computational cost of the continued fraction formulation is the time to solve Equation (27). Table 1 shows the efficiency of the continued fraction formulation is much higher than that of the dynamic mass matrix, although the continued fraction formulation increase the dimension $N \times N$ of response solving matrix equation up to $(M_{\mathrm{H}} + M_{\mathrm{L}} + 2)N \times (M_{\mathrm{H}} + M_{\mathrm{L}} + 2)N$.

Conclusions

The continued fraction formulation of infinite acoustic fluid with uniform cross section was derived, which is applicable to two- and three-dimensional problems. The formulation can accurately model the infinite acoustic fluid and its calculation efficiency is much higher than the convolution integral efficiency of SBFEM based on dynamic mass matrix.

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