# Effects of Continuum Breakdown on Aerodynamics

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## Abstract

Attempt was made to analyse aerodynamics of hypersonic vehicle near space with slip NS methods. The applicability of several different slip boundary conditions was investigated. The difference between continuum and rarefied predictions for surface properties of cylinder and trapezoid wing was shown. The results show that present code was valid for predicting slip regime flow by comparing with results in reference.Type-2 slip model proposed by Gokcen had more extensive rarefied limit, and could give a best agreement with DSMC results in higher Knudsen numbers comparing with other slip models. Peak transfer rate differences range from over 1.6% for 50KM to almost 14.5% for 80KM. Pressure coefficient on the surface is little affected by rarefied gas effect, while heat transfer rate is most influenced by that.

**Keywords:** near space; hypersonic vehicle; aerodynamics; rarefied gas effect; slip model

## 0.Introduction

Future hypersonic flight vehicles[1] have many specific performance features, such as long distance and accurate attack, and maneuvering flight. Therefore, these vehicles in atmosphere must have the ability of large passage maneuvering flight, the characteristics of long-time flight and high lift-to-drag ratio aerodynamic configurations, and specific trajectories. A typical configuration property is the sharp leading edges. The flight passage suffering transitional flow effect is much larger due to the smaller characteristic length scale, and hence in flight at high altitude, the vehicles can globally or locally suffer transitional flow effect. So, it is a key and important problem that the transitional flow effects on aerodynamic force and heating for flight vehicle design. Aerodynamic force and heating for these type vehicles in rarefied flow regime are very different from that in continuum flow regime. These difference are never been paid much attention for traditional blunted vehicles due to more redundancy design used. However, future hypersonic flight vehicles allow only to taking less redundancy, and thus aerodynamics in the near space need to be investigated in detail.

At low altitude, traditional CFD method is generally used in continuum flow regime. As increasing of altitude, flow changes gradually from continuum flow regime to near free molecule flow regime. In that processing， it need to be answered that when continuum breakdown is starting, and how much error that will result in. The continuum breakdown criteria was investigated by many researchers[2-6]. The rarefied gas effect on aerodynamics of hypersonic vehicles near space have been investigated little[7-9].

The applicability of different slip models will be investigated around cylinder, and then The comparison of the aerodynamics of hypersonic vehicle near space using slip method and noslip method in transitional flow regime will be made.

## 1.Computational approach

A. N-S equation

3-D laminar N-S equation can be written as $\frac{\partial \vec{Q}}{\partial t}+\frac{\partial \vec{F}}{\partial x}+\frac{\partial \vec{G}}{\partial y}+\frac{\partial \vec{H}}{\partial z}=\frac{\partial \vec{F}_v}{\partial x}+\frac{\partial \vec{G}_v}{\partial y}+\frac{\partial \vec{H}_v}{\partial z}$

Here, $\vec{Q}$ is a conservative vector. $\vec{F}$ , $\vec{G}$ , $\vec{H}$ are convection vector flux in three direction respectively, and $\vec{F}_v$ , $\vec{G}_v$ , $\vec{H}_v$ are viscous vector flux in corresponding direction.

Roe scheme is used for space discretization under the finite volume frame, minmod limiter is adopted. The center-difference method is used for viscous numerical flux.LU-SGS implicit time integration method is introduced.

B. Slip boundary models

Four type models are investigated in this paper. Type1 is defined by Maxwell model. Type2 is Gokcen[10] slip model. Lockerby[11] model is defined as Type3. HS model is Maxwell slip model modified by hard sphere model.

Type1 (Corresponding to CFD(1) in reference [9]) is given by

$$U_S=\frac{2-\sigma}{\sigma}\lambda\frac{\partial U}{\partial n}|_0 \quad , \quad T_S-T_w=\frac{2-\alpha}{\alpha}\frac{2\gamma}{(\gamma+1)P_r}\lambda\frac{\partial T}{\partial n}|_0 \ , \ \lambda=\frac{\mu}{\rho}\sqrt{\frac{\pi}{2RT}}$$

Type2 (Corresponding to CFD(2) in reference [9]) is given by

$$\sigma U_\lambda=2\lambda\frac{\partial U}{\partial n}|_0 \quad , \quad \alpha(T_\lambda-T_w)=2\frac{2\gamma}{(\gamma+1)P_r}\lambda\frac{\partial T}{\partial n}|_0 \ , \ \lambda=\frac{\mu}{\rho}\sqrt{\frac{\pi}{2RT}}$$

Type3 (Corresponding to CFD(3) in reference [9]) is given by

$$U_S=A\frac{2-\sigma}{\sigma}\lambda\frac{\partial U}{\partial n}|_0, \ T_S-T_w=\frac{2-\alpha}{\alpha}\frac{2\gamma}{(\gamma+1)P_r}\lambda\frac{\partial T}{\partial n}|_0, \ \lambda=\frac{\mu}{\rho}\sqrt{\frac{\pi}{2RT}} \ , \quad \mu=\mu\psi^{-1}$$

Here, $\psi(\frac{n}{\lambda})=1+\frac{7}{10}(1+\frac{n}{\lambda})^{-3}$ , $A=\sqrt{2/\pi}$

HS is given by

$$U_S=\frac{2-\sigma}{\sigma}\lambda\frac{\partial U}{\partial n}|_0, \ T_S-T_w=\frac{2-\alpha}{\alpha}\frac{2\gamma}{(\gamma+1)P_r}\lambda\frac{\partial T}{\partial n}|_0 \ , \text{Here, } \lambda=\frac{m}{\sqrt{2}\pi d^2\rho}, \text{ m is molecule mass.}$$

C. Main dimensionless parameters

Wall pressure coefficient $C_P=\frac{(p_w-p_\infty)}{0.5\rho_\infty U_\infty{}^2}$

Wall heat flux coefficient $C_H=\frac{q}{0.5\rho_\infty U_\infty^3}$

Wall friction coefficient $C_F=\frac{\tau}{0.5\rho_\infty U_\infty^2}$

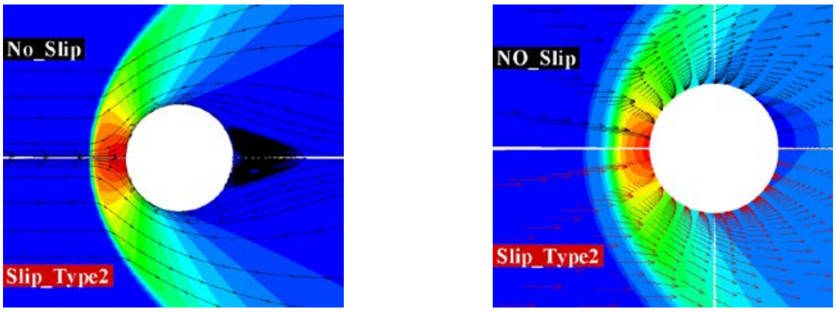
Error can be written as $Error = \dfrac{q_{noslip} - q_{slip2}}{q_{slip2}} \times 100\%$

Here， $q_{noslip}$ is no_slip results， $q_{slip2}$ is slip results with Type2 model.

## 2.Code validation and applicability analysis of different slip models

The flow over cylinder is simulated to analyse applicability of above several slip models at Mach number of 10, 25. Cylinder diameter is 304.8mm. The argon flow is considered in this work[9]. Knudsen number based on diameter is 0.002、0.05、0.25,respectively. Free steam temperature is 300K.

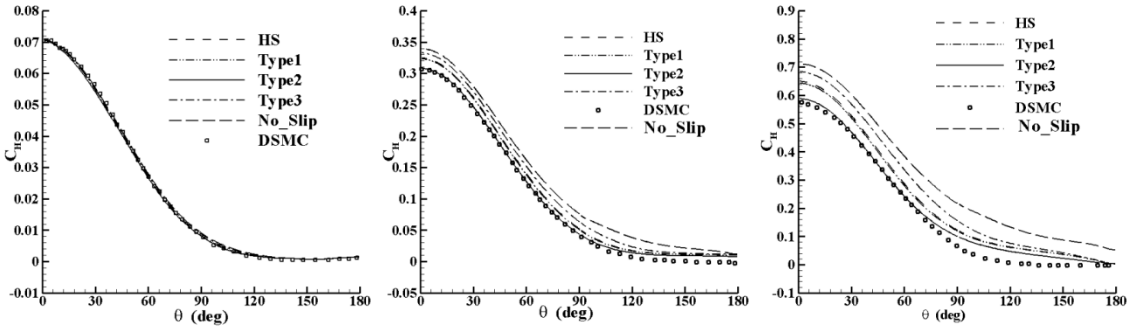
### A. M=10，U=2624m/s，$T_W$=500K



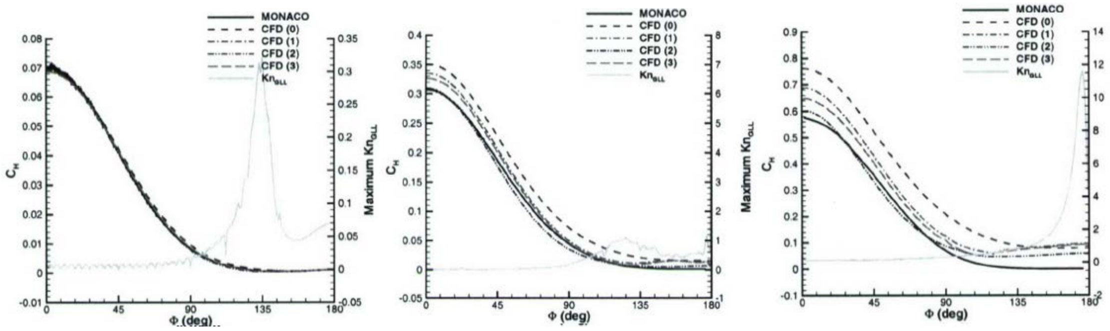

**(a)Pressure contours and streamline (Kn=0.002)** **(b)Pressure contours and vector (Kn=0.05)**

**Fig.1Flow field with different methods(Ma=10)**

Fig1(a) show pressure contours at Knudsen number 0.002, and Fig.1(b) show pressure contours and vector. It can be concluded from Fig.1(a) that at the continuum regime, the agreement of flow between slip boundary conditions and no slip boundary conditions is very good. Present slip method can capture the shock structure and wake eddy. As Knudsen number increases to 0.05, the flow is well within the slip regime. The flow demonstrates breakdown in a larger area of the flow in each of the three regions(shock, boundary layer and wake). The slip CFD shock is much thicker than the no_slip CFD shock. Velocity-slip at the wall can be seen clearly with slip boundary conditions. The difference of pressure contours at the wake can be found evidently. It can be predicted from previous several flow difference that wall pressure, heat flux may be very different.




**(a)Present results**

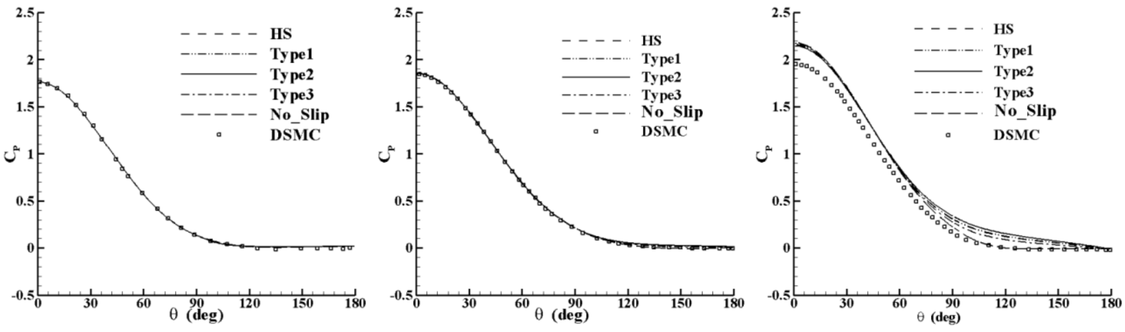



**(b)Computational results in reference [9]**

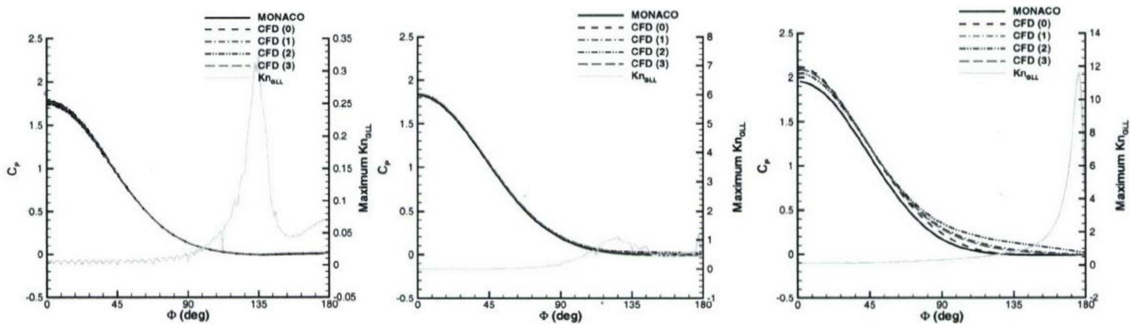
**Kn=0.002 Kn=0.05 Kn=0.25**

**Fig.2 Heat transfer rate on the surface at different Knudsen numbers (Ma=10)**

Figure 2 quantify the differences in the surface properties by comparing CFD and DSMC predictions for heat flux coefficient at Mach number 10. Where Fig.2(a) shows present results, and Fig.2(b) shows computational results in reference [9]. At a Knudsen number of 0.002, the flow is within the continuum regime, and the results predicted by CFD and DSMC are in excellent agreement. As Knudsen increasing to 0.05, 0.25, the results predicted by different slip models keeps no longer in agreement. It can be found the heat flux coefficient shows surprisingly good agreement between the type-2 case and DSMC even at the higher Knudsen number of 0.05,0.25. The surface properties predicted by DSMC tended to be lower than those predicted by CFD, even for the CFD cases implementing slip conditions. From comparison among several different types of slip boundary conditions, the best agreement appeared to be obtained using Type-2 slip model. It can be concluded that present results are consistent with computational results in reference [9].




**(a)Present results**



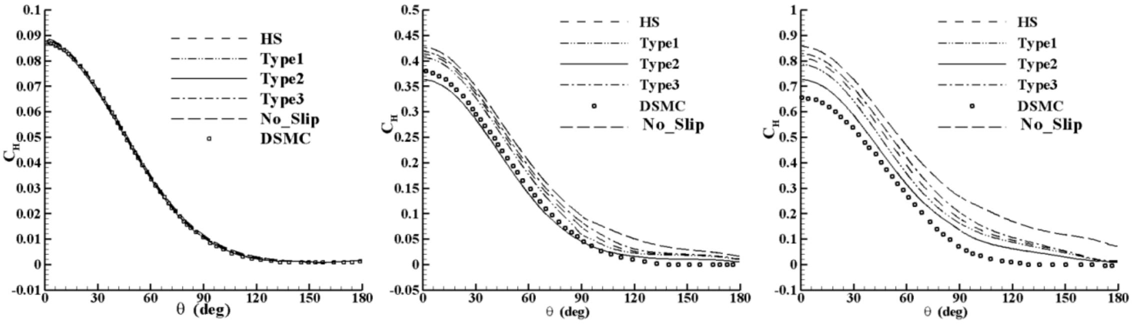

**(b)Computational results in reference [9]**

**Kn=0.002 Kn=0.05 Kn=0.25**

**Fig.3 Pressure coefficient on the surface at different Kn numbers( Ma=10)**

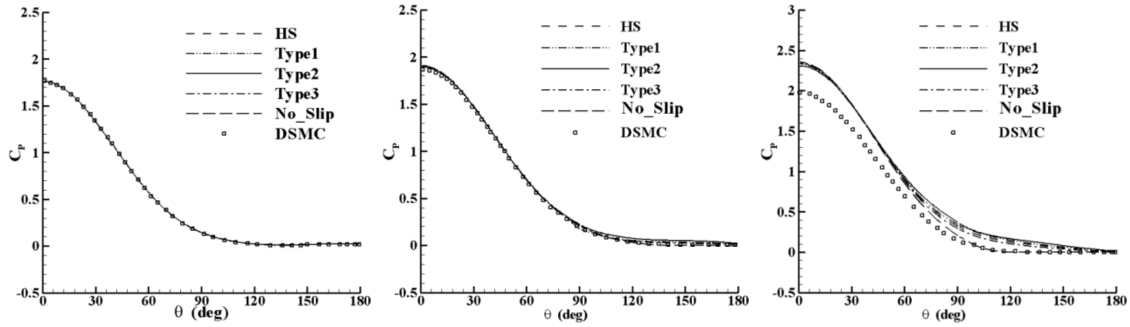
Pressure coefficient on the surface at different Kn numbers are shown in Fig.3, and Fig.3(a) is present results, Fig.3(b) is reference's results. At Knudsen number of 0.002,0.05, the results predicted by slip NS keep a good agreement with those by DSMC. Pressure coefficient by DSMC tend to be lower than those by CFD at Kn=0.25. The main reason can be explained by that shock thickness predicted by DSMC is bigger than that by CFD, and compressibility predicted by DSMC is more weakly than that predicted by CFD. Above all will result in a bigger pressure after shock wave. It can be concluded from Figs.(2-3) that pressure coefficient variation is more sensitive to rarefied gas effect than heat flux coefficient, and present results keep an excellent agreement with computational results in reference [9].

B. **M=25，U=6585m/s，$T_W$=1500K**




**Kn=0.002 Kn=0.05 Kn=0.25**

**Fig.4 Heat transfer rate on the surface at different Kn numbers( Ma=25)**




**Kn=0.002 Kn=0.05 Kn=0.25**

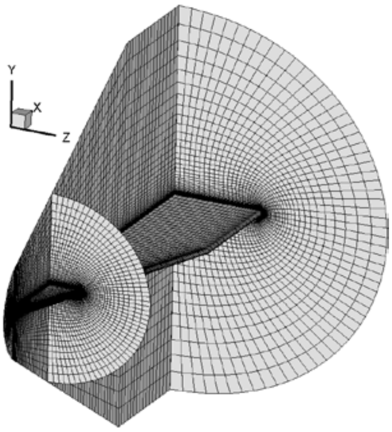
**Fig.5 Pressure coefficient on the surface at different Kn numbers( Ma=25)**

Fig.4 shows heat flux coefficient on the surface at different Kn numbers with mach number of 25.Fig.5 shows corresponding pressure coefficient distribution. They are computed by different slip models, and compared with DSMC results in reference [9]. The surface properties at mach number of 25 is similar to that at mach number of 10. Even at Mach-25 case, type-2 CFD solution also keeps a less error than others.

It can found form above all that present code is valid for predicting slip regime flow by comparing with results in reference[9]. Type-2 slip model can be adopted in more extensive rarefied limit. At a bigger Knudsen number, surface properties predicted by DSMC tend to be lower than those by slip CFD. Comparing with heat flux rate, pressure is more sensitive to rarefied gas effect. Like conclusion in reference[9], all of the slip boundary conditions increases the computational expense of the CFD simulations, with the Type-2 being the most expensive. If possible for designing, other slip boundary conditions may be a good choice.

## 3. Aerodynamics of trapezoid wing in near space

Aerodynamics of trapezoid wing in near space is investigated with Type-2 slip model at mach number of 15. The altitude range is from 50km to 80km, and the angle of attack is 10°. Wall temperature is 500K. Wing head diameter is 30mm.Wing is with 1.96m root chord length, 0.63m tip chord length, 0.5m span length. Swept back angle is 20°. Fig.6 shows grid in the computation region.Fig.7 shows slice station of trapezoid wing.



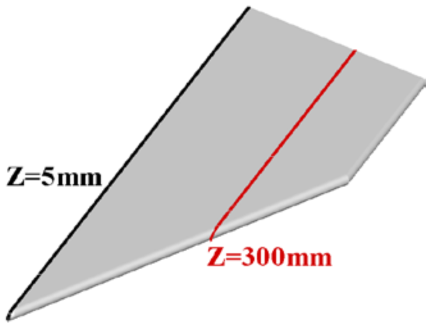
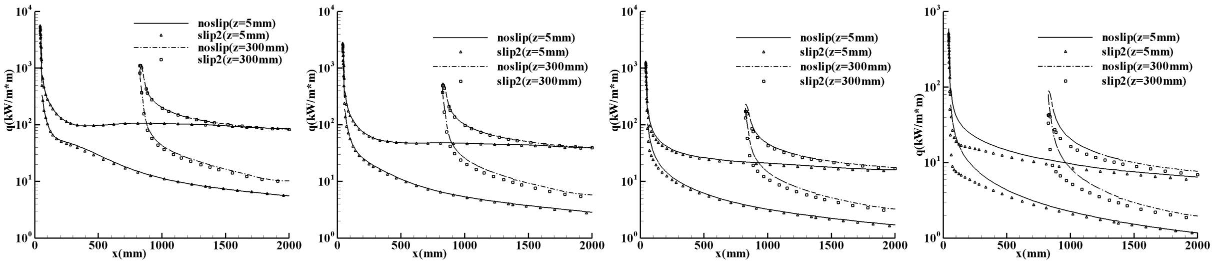
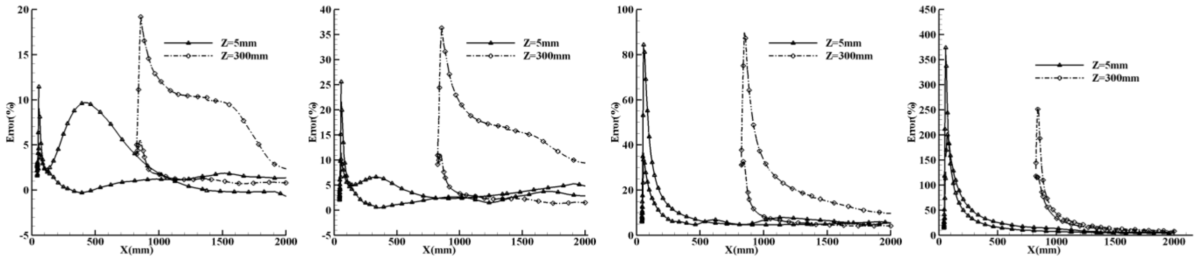




**Fig.6 Grid in the computation region** **Fig.7 Slice at different stations**




**50km 60km 70km 80km**

**Fig.8 heat flux at different stations**




**50km 60km 70km 80km**

**Fig.9 Error at different stations**

Figs.(8-9) show respectively heat flux and its error along both the slice stations. At a lower altitude, the heat transfer rate along slice station shows surprising agreement between slip case and no_slip case. As altitude increasing, the difference of results predicted by different methods tends to be evident. With windward compressed and leeward expanded, rarefied gas effect firstly appear on the windward, and so the difference of results predicted by different methods is smaller on the windward than on the leeward. It can seen from Fig.9 that the maximum error is at the expanding region of wing leading edge, and error of heat transfer on the surface of the wing range from 5% for 50km altitude to almost 15% for 80km altitude.

**Table1 Aerodynamics with different methods**

| H (km) | | $C_{Af}$ (×100) | $C_A$ (×100) | $C_N$ (×100) | L/D |
|---|---|---|---|---|---|
| 50 | No_slip | 0.4496 | 0.7410 | 5.339 | 3.096 |
| | Slip | 0.4493 | 0.7410 | 5.334 | 3.094 |
| | Error | 0.07% | 0.00% | 0.09% | 0.06% |
| 60 | No_slip | 0.8509 | 1.150 | 5.473 | 2.492 |
| | Slip | 0.8454 | 1.145 | 5.423 | 2.485 |
| | Error | 0.65% | 0.44% | 0.92% | 0.28% |
| 70 | No_slip | 1.770 | 2.083 | 5.757 | 1.740 |
| | Slip | 1.738 | 2.051 | 5.448 | 1.689 |
| | Error | 1.84% | 1.56% | 5.67% | 3.02% |
| 80 | No_slip | 4.245 | 4.593 | 6.342 | 0.676 |
| | Slip | 4.083 | 4.422 | 5.382 | 0.591 |
| | Error | 3.97% | 3.87% | 17.84% | 14.38% |

Table1 shows comparison of aerodynamics by different methods. It can be concluded that as flight altitude increases, friction coefficient, axial force coefficient and normal force coefficient increase, and the ratio of Lift-to-Drag decrease. It can be seen from the error results that rarefied gas effect enhance much as flight altitude increasing, traditional CFD method is invalid in continuum breakdown region and cannot give an accurate velocity jump boundary condition. So error tends to be increasing as rarefied gas effect enhancing.

**Table2 Peak heat flux with different methods**

| H (km) / Q (kW/m*m) | 50 | 60 | 70 | 80 |
|---|---|---|---|---|
| No_slip | 5701.9 | 2816.6 | 1348.4 | 594.1 |
| Slip | 5614.9 | 2755.7 | 1272.1 | 518.8 |
| Error | 1.55 % | 2.21% | 5.99% | 14.52% |

Table2 shows variation of peak heat transfer rate as flight altitude. The density increases , and peak heat transfer rate decreases sharply as flight altitude increasing. Like aerodynamics, the error of peak heat transfer rate also increases as flight altitude increases with continuum breakdown.

## 4.Conclusions

Different slip model validation was investigated by comparing cylinder flow. Hypersonic aerodynamics of trapezoid wing in near space flying near continuum flow regime was analyzed with slip CFD method in this paper. The difference between slip CFD and no_slip CFD simulations was quantified lastly.The main contents and achievements are concluded as follows:

（1）Present code was valid for predicting slip regime flow by comparing with results in reference.

（2）Type-2 slip model proposed by Gokcen had more extensive rarefied limit, and could give a best agreement with DSMC results in higher Knudsen numbers comparing with other slip models.

（3）The surface properties predicted by DSMC tended to be lower than those by CFD, so CFD solutions could give much redundancy for thermal protection designing.

（4）The surface pressure was less affected by continuum breakdown than heat transfer rate.

（5）For present trapezoid wing, as flight altitude ranged from 50km to 80km, error of heat transfer on the surface of the wing is from 5% to 15%, with peak heat transfer rate ranging from 1.6% to 14.5% at stagnation, and the error of the ratio of Lift-to-Drag is from 0.06% to 14.38%.

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