

# Optimization of stiffened shell structures with stability objective/constraint based on kriging surrogate model and the explicit FEM

\*Bin Wang<sup>1</sup>, Jiandong Tian<sup>1</sup>, Cheng Huang<sup>1</sup>

<sup>1</sup> Beijing Institute of Astronautical Systems Engineering, Beijing, 100076, P. R. China,

\*Corresponding author: [b\\_wang@189.cn](mailto:b_wang@189.cn)

## Abstract

In this paper, a parametric model of a stiffened shell is built with Python language in Abaqus. The explicit FEM is used as an analysis tool in the optimal design of stiffened shell structures. The skin thickness and stiffener size are designed and optimized. The optimization contains two strategies: one is to obtain the minimum mass subjected to the structural performance, and the other is to obtain the high structural performance subject to the mass. In spite of the advantages of computer capacity and speed, the enormous computational cost of complex simulations makes it impractical to rely exclusively on simulation codes for the purpose of design optimization. To solve this problem, a surrogate model is built employing the experimental design and Kriging model, constructing the relationship between variables and standard deviation of the objective, reduced the computing time of uncertainty analysis in optimization to improve computing efficiency.

**Keywords:** Stiffened shell structures, Post-buckling, Explicit FEM, Optimization, Kriging surrogate model.

## Introduction

Stiffened shell structures are by far the most consumed structural components in the aerospace industry due to good stability, design ability and low cost. Buckling of such structures is often of mayor concern to designers. However, it is still a difficult task to perform a stability analysis of stiffened shells when the post-buckling behavior is considered. In fact the post-buckling analysis is quite necessary for completely describing the stability characteristics of stiffened shells. In this situation, the optimization problem with stability objective/constraints of these structures becomes difficult to evaluate.

Literature [1,2] pointed out that the explicit finite element method is an effective way to analysis the post-buckling behavior. However, because of explicit finite element analysis aims to simulate structural response under impact loads, an appropriate loading rate is important to quasi-static loading process simulation. Before optimization performing, the appropriate loading rate is determined firstly by comparing the effect of loading rate on numerical results. We set skin thickness, rib height, width and quantity as design parameters, using the Python language to build a parametric model for optimization. This procedure uses the commercial finite element software Abaqus as working platform. Because of discrete design parameters existing, Kriging surrogate model method and multi-island genetic algorithm are employed firstly to get a preliminary design, and then applying sequential linear programming algorithm to obtain the optimized design of other continuous variables' optimization problem based on the preliminary design.

In this paper, the optimization contains two strategies: one is to obtain the minimum mass subjected to the structural performance, and the other is to obtain the high structural performance subject to

the mass. The numerical results show that the mass decreases 14.2% or the loading capacity increases 19.3% through the above optimal processes, respectively. The proposed optimization procedure provides an effective tool for the safe exploitation of stiffened shell structures.

## 1 Model introduction

The parametric model (skin thickness, the number of ribs, height, width) built by Python language is shown in Figure 1. The stiffened shell's height is 1200.0mm, diameter is 2000.0mm, and the skin thickness is 2.0mm. There are nine hoop ribs and fifty longitudinal ribs in the stiffened shell. The rib's height is 10.0mm, width is 4.0mm. The material of the structure is aluminum alloy, which elastic modulus is 70GPa, Poisson's ratio is 0.3, yield strength is 350MPa, ultimate strength is 450MPa, elongation is 10%, and the material density is  $2.7 \times 10^9 \text{ton/mm}^3$ . Full integration 4-node shell element is employed to mesh the structure, and one skin cell between ribs has 5\*5 elements. To ensure accuracy, the height direction along the rib is meshed by two elements.

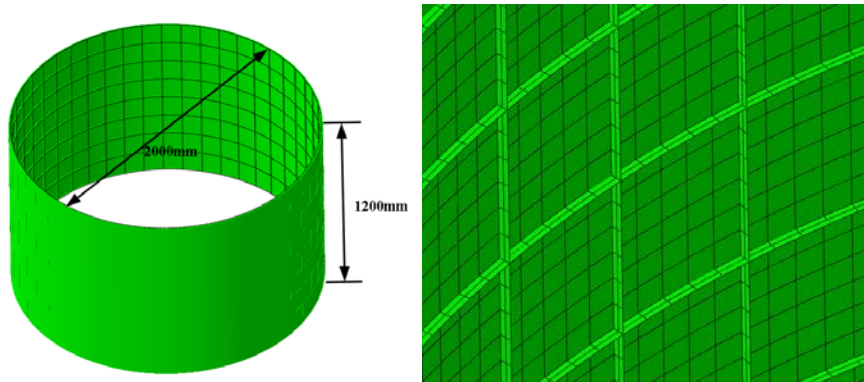
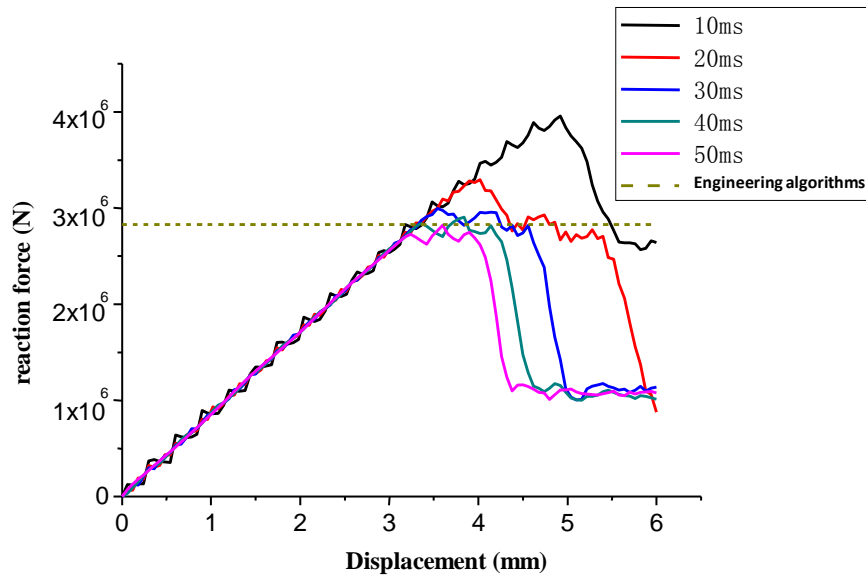


Figure 1. Finite element model

## 2 Loading rate selection

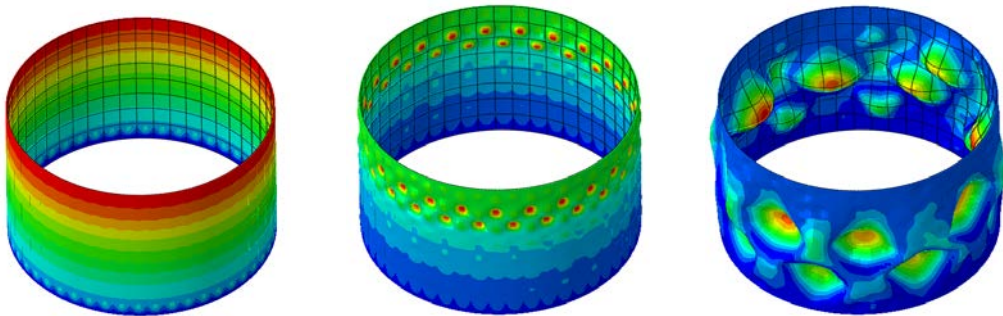
For the structure shown in Figure 1, post-buckling load of the structure is 282.81 ton though the engineering algorithms [3]. To apply explicit finite element algorithm to obtain the post-buckling load, the boundary and loading condition are clamping the lower end and applying a displacement on vertical downward face of the structure, the load size is 6mm.

Because of explicit finite element analysis aims to analysis structural response under impact loads, we need to select the appropriate loading speed in order to simulate quasi-static loading process firstly. Solving 5 load time conditions (10ms, 20ms, 30ms, 40ms, 50ms) respectively, and the support reaction force - displacement curves are shown in Figure 2. One can observe from the results, when the loading speed relatively faster, the support reaction force - displacement curve becomes unstable, and the post-buckling load of the structure is also higher. When load time is 10ms, the post-buckling load obtained from explicit finite element analysis is 395.87 ton higher than the engineering algorithm's result. And when the loading time increases, the calculated post-buckling load gradually decreases. If load time is chosen as 50ms, the analysis result is 281.93ton agreeing well with the engineering algorithm.

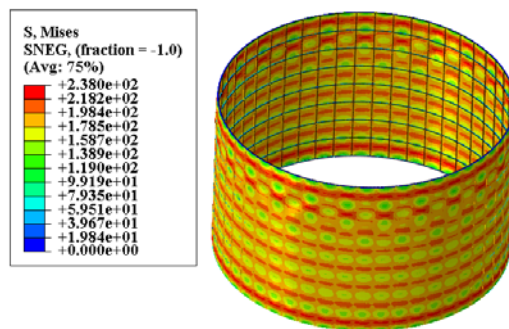


**Figure 2. Support reaction force - displacement curves**

From the comparison of the results of different loading rates, we can also find that the post-buckling load getting smaller when loading time over 30ms and the result of loading time is 50ms can be treated as a convergence solution. Because the computing time is proportional to the loading time, we choose 50ms as the loading time in the following calculations but not a longer loading time.



**Figure 3. Instability process**



**Figure 4. Stress distribution when post-buckling load reaching**

Figure 3 and Figure 4 shows steps of instability process of the structure when loading time is 50ms and the stress distribution when post-buckling load reaching. Though observation of the figures, one can find that the skin local buckled firstly, and then whole structure buckled. During this process,

the maximum stress in the structure is 230MPa lower than the yield stress. So the structure not makes use of the material because of instability.

### 3 Parameter Optimization

#### 3.1 Optimal column type and solution strategy

The two optimization formulations are showed following, the (a) formulation is the minimize structure weight optimization, and the (b) formulation is for maximize the structure performance optimization:

$$\begin{array}{ll}
 \text{find} & mpt, zjn, hjn, jg, jk \\
 \text{min} & mass \\
 \text{s.t.} & 40 \leq zjn \leq 60 \\
 & 6 \leq hjn \leq 10 \\
 & 1.5 \leq mpt \leq 2.5 \\
 & 7.5 \leq jg \leq 12.5 \\
 & 3.0 \leq jk \leq 5.0 \\
 & RF \geq 282ton \\
 \end{array} \quad \dots(a)
 \qquad
 \begin{array}{ll}
 \text{find} & mpt, zjn, hjn, jg, jk \\
 \text{max} & RF \\
 \text{s.t.} & 40 \leq zjn \leq 60 \\
 & 6 \leq hjn \leq 10 \\
 & 1.5 \leq mpt \leq 2.5 \\
 & 7.5 \leq jg \leq 12.5 \\
 & 3.0 \leq jk \leq 5.0 \\
 & mass \leq 53.2kg \\
 \end{array} \quad \dots(b) \quad (1)$$

The  $mpt$  is the thickness of skin, and the  $zjn$  and  $hjn$  are respective the number of longitudinal ribs and ring ribs, which are integer variables. It should be noted that the actual number of ring ribs is  $hjn+1$ .  $jg$  and  $jk$  are respective the height and width of the ribs.  $mass$  is the total mass of the structure.  $Rf$  is the maximum post-buckling load of the structure. As to the structure in the figure, the parameters are,  $mpt=2.0$ ,  $zjn=50$ ,  $hjn=8$ ,  $jg=10.0$ ,  $jk=4.0$ , and the total mass and the post-buckling load are 53.22Kg and 281.93ton, respective.

Because of the integer variables, the optimization problem could not use the sequential linear programming algorithm directly. So the Kriging surrogate model method is first use to get an approximate model, and then the multi-island genetic algorithm is used to obtain a preliminary design based on the approximate model. At last, applying sequential linear programming algorithm to get the optimized design of continuous variables' optimization problem based on the preliminary design.

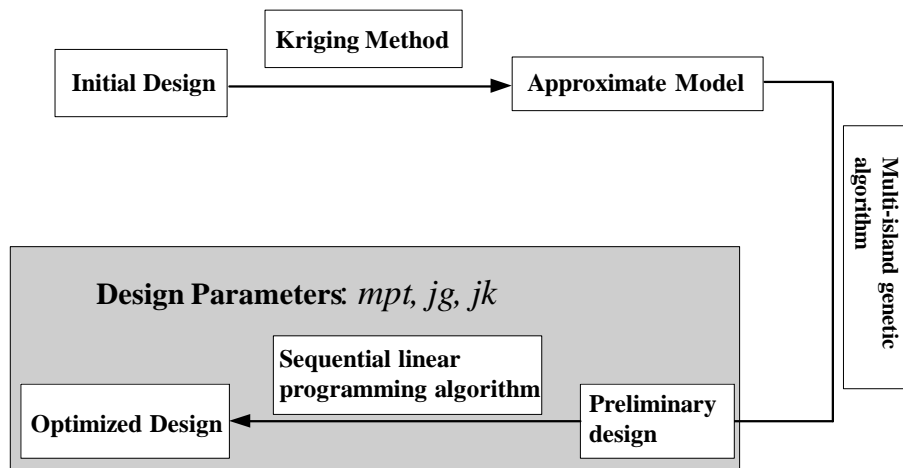


Figure 5. Optimization Process

### 3.2 Numerical examples

As mentioned in the previous content, the initial parameters are  $mpt=2.0$ ,  $zjn=50$ ,  $hjn=8$ ,  $kg=10.0$ ,  $jk=4.0$ . Firstly, establish an approximate model using Kriging surrogate model (in this paper, we randomly select 100 design points to build an approximate mode). The two optimization formulations have same design parameters and the needed structure response, so one approximation model is sufficient. The parameters and objective values of final optimized designs under these two optimization formulations are both showed in table 1. The results show that the optimized designs reduce the 14.2% weight of the structure or increase 19.3% of the post-buckling load, respectively.

**Table 1. Optimization Results Summary**

	Initial Design	minimize structure weight		maximize the structure post-buckling load	
$mpt(mm)$	2.00	1.51	1.52	1.77	1.79
$zjn$	50	46	46	41	41
$hjn$	8	10	10	9	9
$kg(mm)$	10.00	11.94	12.50	11.14	11.25
$jk(mm)$	4.00	3.32	3.53	4.93	4.93
$mass$ ( $kg$ )	Approximate model		43.94		52.58
	Finite element model	53.22	43.97	45.67	52.55
$RF$ ( $ton$ )	Approximate model		283.26		368.55
	Finite element model	281.93	266.55	282.50	332.16

Figure 6 shows support reaction force - displacement curves of two optimized designs. Figure 7 and Figure 8 show the buckling modes and stress distributions when total structure buckled (the left figure is optimized design got by minimizing the total structure weight). Compare with the initial design, the stress when total structure buckled of the two optimized designs both reaching the yield stress, which means that the material are fully utilized. And because of it, the support reaction force - displacement curves are stable without any flutter.

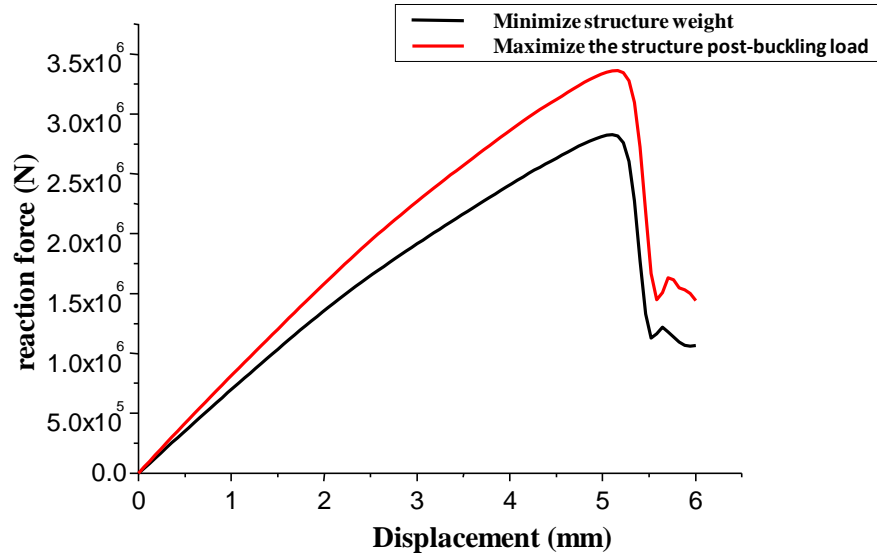


Figure 6. Support reaction force - displacement curves of two optimized designs

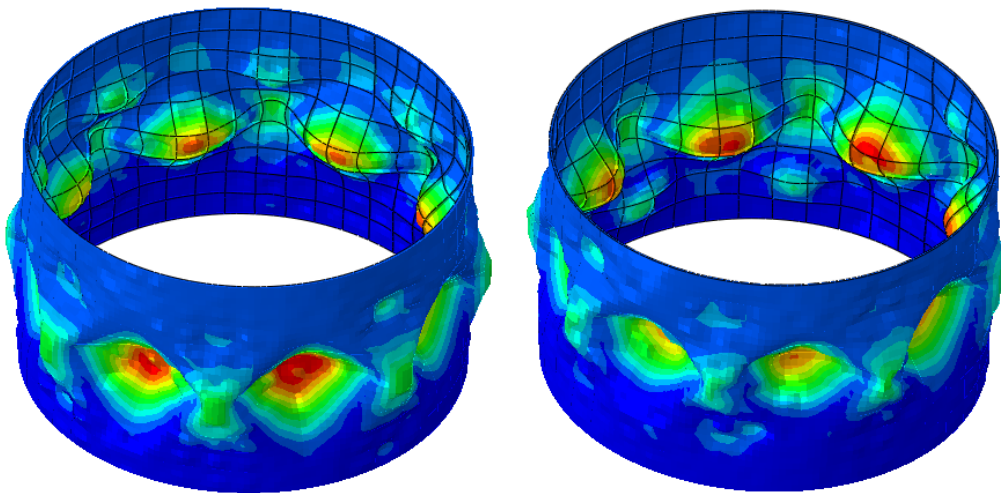


Figure 7. Buckling modes of optimized designs

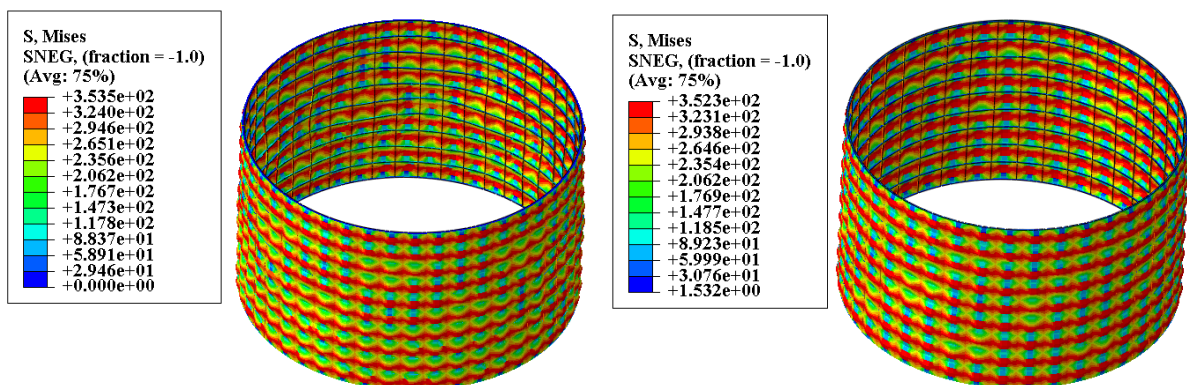


Figure 8. Stress distributions when total structure buckled

## 4 Conclusions

In this paper, explicit finite element algorithm is used to solve the grid stiffened cylindrical shell post-buckling load. The skin thickness, rib height, width and quantity are set as the design parameters, and two optimized designs under two optimization formulations are obtained. Because of there are integer variables, the Kriging surrogate model method and the multi-island genetic algorithm are firstly used to get a preliminary design based on the approximate model, and then using sequential linear programming algorithm to get the optimized design of other continuous variables' optimization problem based on the preliminary design. The results show that the method used in this paper can get a reasonable and effective design for an optimization problem which contains both integer variables and continuums variables.

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