General Ray Method and Rotating Projection Algorithm for Fast Recognition of Discreet Micro Scale Compound Structures

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Abstract

A new fast method of high resolvability for identification of compound structures is proposed. Its mathematical model is constructed on the basis of General Ray Principle, proposed by the author for distribution of different, in particular electrostatic and thermostatic, fields. Proposed model leads to the classic Radon transformation that appears as specific element in new General Ray Method, constructed using explicit formulas and realized as a fast numerical algorithm. Proposed General Ray and Rotating Projection algorithms open possibility of high resolvability and fast computer recognition of compound structures with micro components. Computer simulation of developed scheme is realized as MATLAB software and justified by numerical experiments.

Key-Words: Micro Scale Compound Structures, General Ray Method, Rotating Projection algorithm.

Introduction

In creation and construction of modern artificial materials it appears the necessity of recognition of compound structures with micro and nano-particles. Recognition of compound structures with elements that have different thermo-conductivities o electro-conductivities (permittivity) is possible by Computer Tomography (CT). In a plane case considering problem can be mathematically described (Williams, R.A. and Beck, M.S., 1995) as a coefficient inverse problem for the Laplace type equation, written in the divergent form

$$\frac{\partial}{\partial x}\left(\varepsilon(x,y)u_x^{'}(x,y)\right)+\frac{\partial}{\partial y}\left(\varepsilon(x,y)u_y^{'}(x,y)\right)=0\,, \qquad (1)$$

where $(x, y) \in \Omega$ some limited open region on a plane, $u(x, y)$ is a temperature o electrical potential, the function $\varepsilon = \varepsilon(x, y)$ characterize thermo-conductivity o electro-conductivity (permittivity) of a media.

In traditional statement (Williams, R.A. and Beck, M.S., 1995) it is supposed also that functions $J_n(x, y)$, $u^0(x, y)$ are known on the boundary curve Γ and the next boundary conditions are satisfied:

$$\varepsilon(x, y)\frac{\partial u(x, y)}{\partial n} = J_n(x, y), \quad (x, y) \in \Gamma, \tag{2}$$

$$u(x, y) = u^0(x, y), \quad (x, y) \in \Gamma, \tag{3}$$

where $\frac{\partial}{\partial n}$ is the normal derivative in the points of the curve Γ.

Eq. (1) – (3) serve as the mathematical model of CT, if there is a family of boundary conditions that corresponds to different angles of scanning scheme. Traditional approach for resolving CT leads to nonlinear ill-posed problem.

General Ray Method

To construct a new mathematical model for CT we use the General Ray Principle (Grebennikov, 2005), i.e., consider the physic field as the stream flow of "general rays". Each one of these rays corresponds to some straight line l.The main idea of the General Ray Principle consists in reduction a Partial Differential Equation to a family of Ordinary Differential Equations. Let the line l has the parametric presentation: $x = p\cos\varphi - t\sin\varphi$, $y = p\sin\varphi + t\cos\varphi$, where $|p|$ is a length of the perpendicular, passed from the centre of coordinates to the line l, φ is the angle between the axis X and this perpendicular (Radon, J.,1917). Hence, using this parameterization for the line l, we shall consider the functions $u(x, y)$ and $\varepsilon(x, y)$ for $(x, y) \in l$ as functions (traces) $\bar{u}(t)$ and $\bar{\varepsilon}(t)$ of variable t. Let suppose that domain Ω is the circle of radius r. Considering Eq. (1) on the line l, we obtain for every fixed p and φ the ordinary differential equation for traces

$$\left(\bar{\varepsilon}(t)\bar{u}_t'(t)\right)_t' = 0, \quad |t| < \bar{t}, \; \bar{t} = \sqrt{r^2 - p^2}. \tag{4}$$

We suppose that functions $v(p, \varphi)$ and $J(p, \varphi)$ are given and we can write boundary conditions

$$\bar{\varepsilon}(-\bar{t})\bar{u}_t'\left(-\bar{t}\right)=J\left(p,\varphi\right) \ , \tag{5}$$

$$\bar{u}\left(\bar{t}\right)-\bar{u}\left(-\bar{t}\right)=v\left(p,\varphi\right). \tag{6}$$

Eq. (4) – (6) are considered as the new basic mathematical model for CT (Grebennikov, 2005).

If different components in the considered structure have the smooth distribution, therefore functions $\bar{u}_t'\left(t\right)$ and $\left(\bar{\varepsilon}\left(t\right)\bar{u}_t'\left(t\right)\right)'$ are continuous. Out of the domain Ω we define extensions of functions $u(x,y)$, $\varepsilon(x,y)$ and the function $v(p,\varphi)/J(p,\varphi)$ as zero. Integrating twice the Eq. (4) on t and using boundary conditions (5) - (6), we obtain for $\varepsilon(x,y)$ the next formula for scanning General Ray (*GR*) method

$$\varepsilon\left(x,y\right)=1/R^{-1}\left[\frac{v(p,\varphi)}{J(p,\varphi)}\right], \tag{7}$$

where R^{-1} is the inverse Radon transform operator (Radon, J.,1917). *GR*-method gives the explicit solution of the inverse coefficient problem for considering case. It is generalized and applied also for structures with piecewise constant characteristics (Reyes Mora, S. and Grebennikov, Alexandre I., 2009). We have constructed the numerical realization of Eq. (7) that we call "*GR*–algorithm". This algorithm is fast, because it does not require solving any equation and the Radon transform can be inversed by fast manner using discrete Fast Fourier Transform algorithm.

Numerical Experiments with *GR*–algorithm

We tested scanning *GR*-algorithm on mathematically simulated model examples for structure with piecewise-constant permittivity. The scheme of solution of the corresponding example consists of the steps presented in details at papers (Reyes Mora, S. and Grebennikov, Alexandre I., 2009), (Grebennikov, 2011).

Here we considered a plane circle of radius 1mm with basic permittivity $\varepsilon_0\left(x,y\right)=1$ and two different internal elements as circles of radiuses $r_{i,}$ i=1,2, with permittivity $\varepsilon_1\left(x,y\right)=2$, $\varepsilon_2\left(x,y\right)=3$.

At part (a) of Fig. 1, 2 the exact structures are shown; at part (b) – reconstructed structures, for r_1=0.05 mm, r_2 =0.005 mm, r_1=0.0015 mm, r_2 =0.001 mm correspondently. We have used discretization on the bi-dimensional red with *N*x*N* nodes and made calculation at the PC with processor INTEL CORE 2 DUO T5250. For the first example *N*=1001, time of reconstruction – 207.1 sec; in the second example *N*=3001, time of reconstruction – 5950 sec.

So, we see that constructed algorithms needs for its realization a lot of time for the case of micro particles in simulated compound structure, because it requires sufficiently large *N*. In paper (Grebennikov and others, 2007), it was proposed the simplified variant of scanning algorithm without application of the inverse Radon transform. We call it as Rotating Projection Algorithm (*RPA*). In (Grebennikov and others, 2007) this algorithm was applied for electric tomography. Here we use this idea and develop the approach for abstract Discrete Computer Tomography.

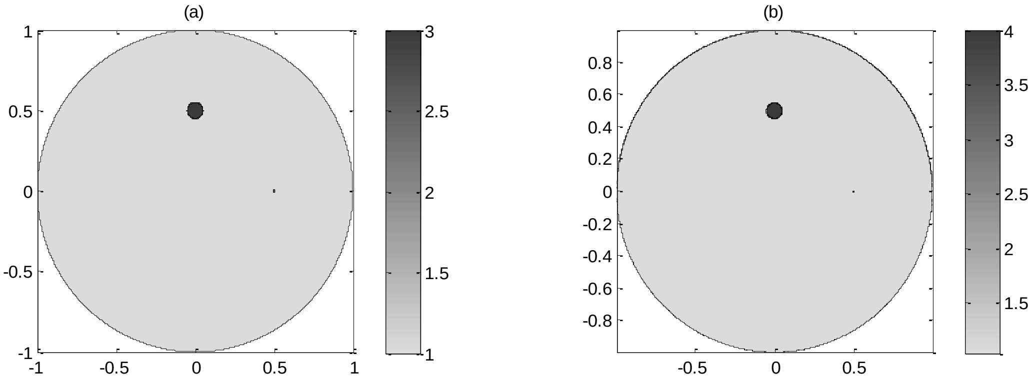

Figure 1. The first example, r_1=0.05, r_2 =0.005;
(a)– exact distribution; (b) – reconstructed by GR-algorithm distribution.
N=1001, time of reconstruction – 207.1 sec

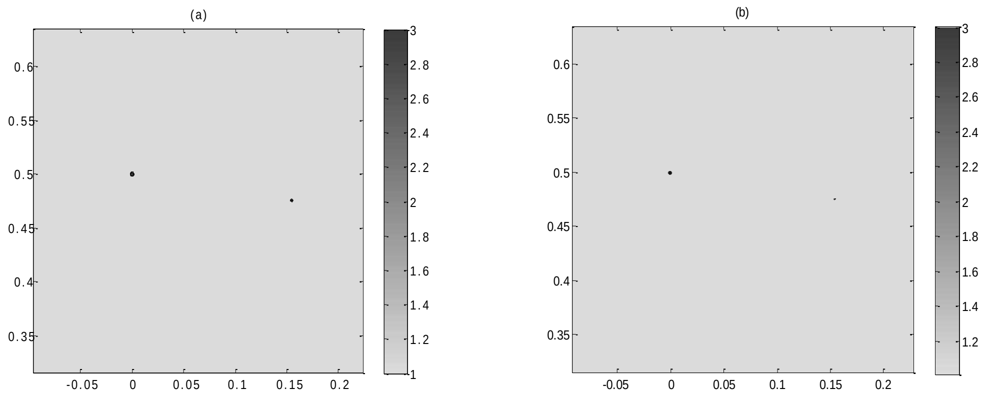

Figure 2. Scaled image of the second synthetic example; r_1=0.0015, r_2 =0.001;
(a)– exact distribution; (b) – reconstructed by GR-algorithm distribution.
N=3001, time of reconstruction – 5950 sec.

Rotating Projection Algorithm and Numerical Experiments

Let us explain the idea of this algorithm in the plane case. Suppose that we know values of function $\bar{v}(p,\varphi)$ (projections), which characterizes the intensiveness of rays, which have passed through the object for some fixed angles φ and all linear coordinates p of scanning. By another words, we know for some fixed angles

$\varphi_i, i = 1,...,n$ corresponding number of one-dimensional images (projections), which present intensiveness as functions of one variable *p*.

We need the next steps.

1. Prolongation on a plane of calculated projections $\bar{v}(p,\varphi_i)$ for every p along the direction, corresponding to angle φ_i, to obtain the extended two dimensional image.

2. Rotation, i.e., changing number *i* (scanning for different φ) of prolonged projections and localization of the areas of intersection of projections with the same values of intensiveness.

In considering simulated numerical example we have homogeneous substance in the region and two elements with other characteristics. If we do not interesting in the exact geometrical form of the element and want to localize them as a rectangles, it is sufficient (Grebennikov and others, 2007) to use *n=2.* We suppose also that $\varphi_1 = 0, \varphi_2 = 90\, grad$, that means we have two orthogonal projections. So, our variable *p* is *x* for *i=1,* or *y* for *i=2,* and we can localize one dimensional images at exes X and Y, construct it's prolongations and realize the rotation with localization of intersections with the same intensiveness. To obtain the adequate entrance data we can use as $\bar{v}(p,\varphi)$ recalculated function $v(p,\varphi)/J(p,\varphi)$ in accordance to straight lines, at which corresponding projections appear. Graphical illustrations of calculations by MATLAB program realization of RPA are presented at Fig. 3 and 4. The time of reconstruction by *RPA* for the first example is 0.7628 sec., for the second example - 1.7972 sec.

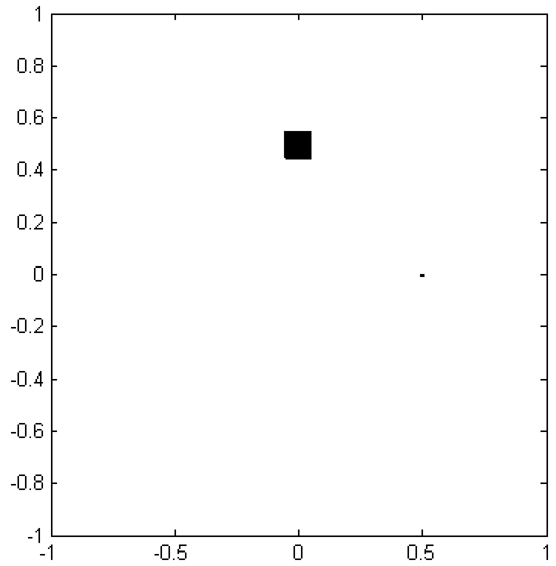

Figure 3. The structure of the first example, reconstructed by *RPA* for 0.7628 sec.

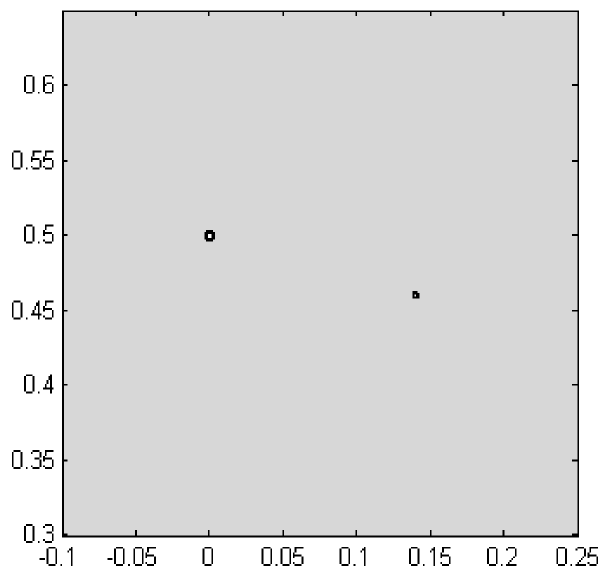

Figure 4. Scaled image of the second synthetic example, reconstructed by *RPA* for 1.7972 sec.

Conclusions

A new mathematical model on the base of the *GRP* for recognition of compound structures by CT is considered. To resolve this model new *GR*- method, fast algorithms and MATLAB software are constructed. The properties of the constructed algorithms and computer programs are illustrated by numerical experiments that demonstrate the possibility to reconstruct structures with micro-scale elements.

“Rotating Projection” algorithm is developed for the case of recognition of images on the base of data, obtained in computer tomography for objects that have the discreet structure. Good properties of the developed algorithm at recognition for very short time are demonstrated on numerical examples with simulated data.

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