

# Deep learning for reliability analysis with epistemic uncertainty

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## Abstract

Modern engineering products are becoming more and more complex, sophisticated and intelligent in physical mechanisms and functional structures, which may result in more failure modes and higher failure probability of the product. Therefore, reliability analysis is critical for modern engineering products, and it often encounters epistemic uncertainty due to the lack of reliability information. Evidence theory is an important model for dealing with reliability analysis with epistemic uncertainty. At present, reliability analysis based on evidence theory has become an important research direction in the field of reliability.

This paper proposes a deep learning method to solve the reliability analysis problem based on evidence theory. The stacked autoencoder is constructed by extracting the spatial location features of the sampled focal elements to achieve high-precision classification of the remaining focal elements, and the confidence interval of reliability are calculated efficiently according to the classification results of the focal elements. The core innovation of this method is to use the deep learning model to classify focal elements to solve the reliability analysis problem based on evidence theory, and a spatial location features extraction method of focal elements which takes into account the features correlation and integrity is proposed to ensure the classification accuracy and efficiency of the deep learning model. The efficiency and accuracy of the proposed method were demonstrated using numerical examples.

**Keywords:** Reliability analysis; Epistemic uncertainty; Deep learning; Stacked autoencoder; Evidence theory

Consider the following performance function [1]:

$$g(X_1, \dots, X_n) = (n + 0.6\sqrt{n}) - \sum_{i=1}^n X_i, n = 10 \quad (1)$$

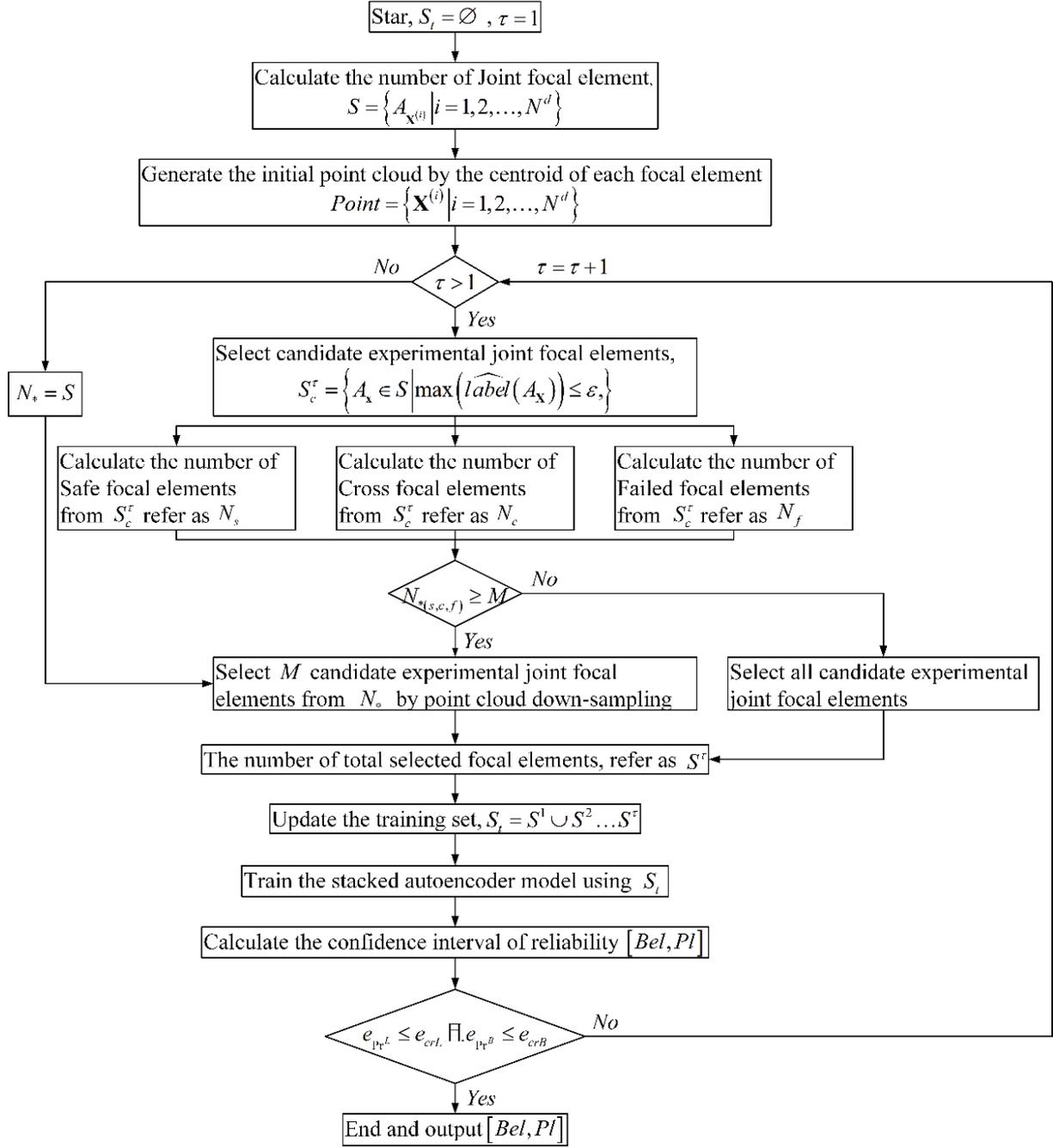
where  $X_i, i = 1, \dots, 10$  are independent evidence variables in Eq. (1), and the BPA structures of the variables are shown in [Table 1](#).

The Bel and Pl result obtain using the conventional method of SQP optimization algorithm are  $[bel, pl] = [0.7221, 1.0000]$ . In the proposed method, Firstly, the point cloud down-sampling method is used to sample 100 initial training focal elements in the uncertainty domain. The above focal elements are evenly distributed in the uncertainty domain to ensure that three different types of focal elements are obtained for subsequent focal elements classification learning. Secondly, the spatial location features of focal elements are extracted and arranged according to the way of image pixels. The spatial location feature of the focal element is the coordinates of the specific vertex of the focal element. In this problem, each focal element is actually a hypercube and each focal element selects 11 specific vertices, which are a vertex of a focal element and all other vertices connected with this vertex through edges. Since the coordinates of each vertex are a 10-dimensional vector, so each focal element extracts a total of 110 features, and then the coordinates of the 11 specific vertex are arranged in a column to form a  $11 \times 10$  matrix, each value in the matrix can be regarded as image pixel. The stacked autoencoder is constructed by using the features of the focal elements and their real category labels. It is worth noting that real category labels of focal elements are calculated by the SQP optimization algorithm and the original features are used to train the stacked autoencoder directly rather than the normalized characteristics, because of the numerical difference between the evidence variables is not significant in this problem. Finally, the stacked autoencoder is used to predict the categories of the remaining focal elements, and the stacked autoencoder is updated according to the prediction results. The confidence interval of reliability is also calculated according to the classification results of the focal elements. The computational flowchart of the proposed method is shown in [Figure 1](#).

The iterative updating process stops after 10 iterations to obtain the final reliability confidence interval as  $[bel, pl] = [0.7223, 1.0000]$ . The relative error curves of belief function and plausibility function during the iterative process are shown in [Figure 2](#). The accuracy and efficiency between the proposed method and conventional method are compared and the result is shown in [Table 2](#). It can be found that the proposed method has high computational efficiency and accuracy comparing with the conventional method. The proposed method only takes 190 function evaluations to obtain high-precision reliability results, compared with 1,048,576 function evaluations for the conventional method, which is 4 orders of magnitude more than the proposed method.

**Table 1. The BPA structure for evidence variables  $X_i (i = 1, \dots, 10)$**

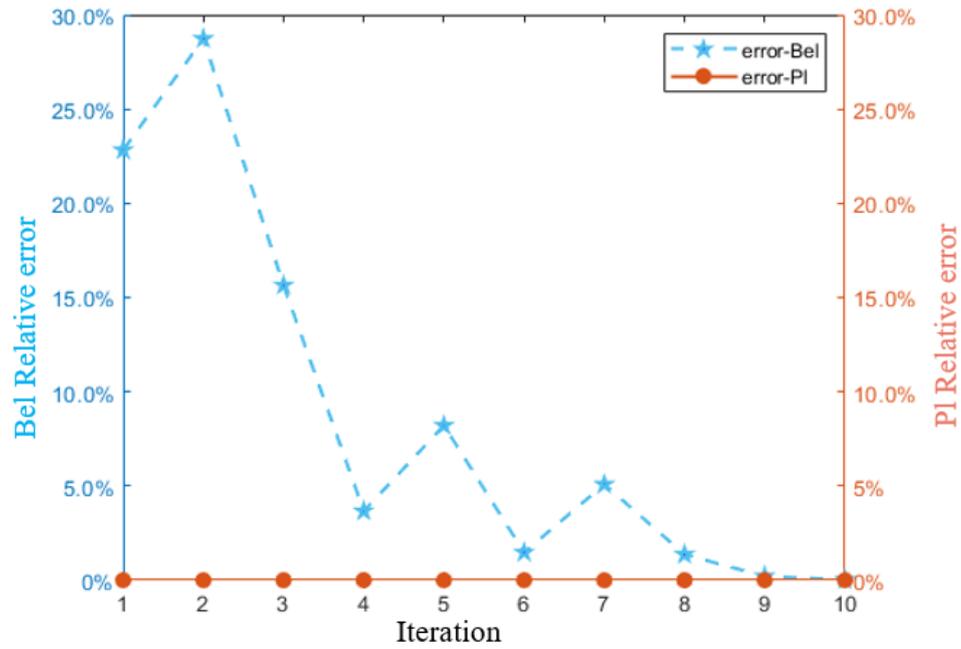
$X_i (i = 1, \dots, 10)$	
Focal element	BPA
[0.4, 0.7]	0.1
[0.7, 1.0]	0.4
[1.0, 1.3]	0.4
[1.3, 1.6]	0.1



**Figure 1. Flowchart of the proposed deep learning method**

**Table 2. Comparison of the computational cost and accuracy between the proposed method and conventional method**

method	$N_{call}$	[Bel, Pl]	Deviations
Conventional method	1048576	[0.7221, 1.0000]	-
Proposed method	190	[0.7223, 1.0000]	[0.028%, 0.000%]



**Figure 2. The history for the relative error of belief function and plausibility function**

**References**

- [1] Xiang, Z., Chen, J., Bao, Y., and Li, H., 2020, "An active learning method combining deep neural network and weighted sampling for structural reliability analysis," *Mech Syst Signal Pr*, 140.