A New Method for Hybrid of Probability and Interval Uncertainty Analysis

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Abstract

This paper will propose a new uncertain analysis method for dynamics problems involving hybrid uncertain parameters. The Polynomial Chaos (PC) theory is systematically integrated with the Chebyshev inclusion function theory to deliver a new hybrid uncertain analysis approach termed as PCCI method, in which the former is applied to solve the random uncertainty and the latter is used to account for the interval uncertainty. The PCCI method is non-intrusive, which does not require the amendment of the original solver for different and complicated dynamics problems. As a result, the PCCI method can be implemented easily. The interval mean (IM) and the other is interval variance (IV) are proposed as the evaluation indexes. The proposed hybrid uncertain analysis method may produce the similar accuracy of the combination of Monte Carlo method and scanning method, and it saves the computational cost much.

Keywords: Hybrid uncertainty, Polynomial Chaos, Chebyshev inclusion function

1. Introduction

In the field of uncertain research, the probabilistic method has been widely studied with the development of a series of techniques. The probabilistic methods can be classified into two types: statistic methods and non-statistic methods. Statistic methods have been derived to include a variety of approaches. The Monte Carlo method is the widely used statistic method for uncertain analysis due to the easiness of its implementation. However, the large number of sampling points of the Monte Carlo method limits its scope of applications, especially for complicated or time-consuming problems. Thus, the Monte Carlo method is often used as the reference of other probability methods. The non-statistic methods mainly include differential analysis approaches and the spectral-based stochastic finite element methods.The spectral-based stochastic finite element method employs a series expansion, like Karhunen-Loeve expansion, to represent stochastic processes, in which the Galerkin method is used to transform the original control equation with uncertain parameters to several equations without uncertain parameters. This is the main process of Polynomial Chaos (PC) expansion, and also the basis for Stochastic Response Surface Methodology (SRSM) (Isukapalli 1999). The (Xiu and Karniadakis 2003) presented an algorithm to model the input uncertainty and its propagation in incompressible flows, where the stochastic input was represented spectrally via an orthogonal polynomial functional from the Askey scheme. Compared with the solutions obtained by the Monte Carlo simulation, the generalized PC method shows better efficiency.

In the non-probabilistic uncertain methods, the interval model is experiencing popularity, because it makes it possible to measure the uncertainties for uncertain-but-bounded parameters, without requiring complete information of the system and only with knowing lower and upper bounds of an uncertain parameter. After the appearance of Moore's work (Moore 1966), several interval methods have been proposed to solve the static problems (Moore 1966; Ishibuchi and Tanaka 1990). Based on the interval arithmetic, the interval method can directly calculate the upper and lower bounds of the response, but one of the shortcomings is the overestimation caused by the wrapping effect, which is inherent in interval computation.

In interval analysis, how to reduce the overestimation becomes one of the key issues in interval analysis (Wu, Luo et al. 2013; Wu, Zhang et al. 2013). Some special techniques should be contained in interval computation to control the overestimation, such as the interval Taylor series method (Alefeld and Mayer 2000; Jackson and Nedialkov 2002) and Taylor model method (Berz and Makino 1999; Wu, Zhao et al. 2005), or a combination of these two methods (Lin and Stadtherr 2007). However, the Taylor series and model based interval methods generally require the explicit expression of the govern equations. Recently, Wu et al (Wu, Zhang et al. 2013) proposed a Chebyshev inclusion function for ODEs with interval parameters, without requiring the explicit expression of governing equations. For nonlinear ODEs, the Chebyshev inclusion function-based method can control overestimation better than the Taylor model method. This method has also been applied to solve the DAEs with interval parameters (Wu, Luo et al. 2013) for multi-body dynamics. The Chebyshev inclusion-based method (Wu, Luo et al. 2013; Wu, Zhang et al. 2013) has shown several merits in solving dynamic problems with uncertainty, including effective control of interval overestimation, and non-intrusive characteristic which can also be applied as a general method to solve black-box type problems.

As aforementioned, it can be seen that most works in this field were mainly focused on either the random parameters or the interval variables. The studies including both types of uncertainties are relatively small, although many engineering problems in nature involve both types of uncertainties simultaneously. The research about the mixed uncertainties is mainly focused on the reliability-based design (RBD). In (Qiu and Wang 2003; Du, Sudjianto et al. 2005), the authors attempted to deal with variables characterized by a mixture of probability distributions and interval uncertainty, where the optimization method was used to find the values of random and interval variables while the reliability index was the worst scenario. It can be found that these RBD methods are mainly focused on static problems, while vehicle analysis problems often involve dynamics, which requires the solution of the differential equations with longer computational time. In dynamics problem, the simulation period is often divided to many discrete time steps, so the computational cost will be prohibitive, if the optimization is directly incorporated in each discrete time step.

This paper will mainly focus on the dynamics problems with hybrid uncertainties of random parameters and interval parameters. Due to the complexity and high computational cost of vehicle dynamics, this study will propose a more effective and efficient PCCI method to solve the dynamic problems with hybrid uncertainties. The PC theory is applied to solve the random uncertainty and the Chebyshev inclusion function theory is used to handle the interval uncertainty.

2. Polynomial Chaos theory for random parameters

The fundamental idea of polynomial chaos is that the random process of interest can be approximated by sums of orthogonal polynomials of random independent variables (Xiu and Karniadakis 2003). For a deterministic model with random inputs, if the inputs are represented in terms of the set $\left\{\xi_i\right\}_{i=1}^{n}$, the output metrics can also be represented with the same set, as the uncertainty of the outputs is solely because of the uncertainty of the inputs (Isukapalli 1999) . A random process $Y(\kappa)$, viewed as a function of the random event κ, can be expanded in terms of the orthogonal polynomial chaos as:

$$Y(\kappa)=\sum_{j=0}^{\infty} y_j\phi_j\left(\boldsymbol{\xi}\left(\kappa\right)\right) \tag{1}$$

Here y_j represents the deterministic coefficients to be estimated, $\phi_j(\boldsymbol{\xi})$ are the generalized Askey-Wiener polynomial chaos of order *j*, according to the multi-dimensional random variable $\boldsymbol{\xi}=(\xi_1,...,\xi_n)$ (Xiu and Karniadakis 2003). For uniformly distributed random variables the basis are Legendre polynomials, for Gaussian random variables the basis are Hermite polynomials, and more basis for other random variables can be find in (Xiu and Karniadakis 2003). In this paper, only the uniformly distribution random variables are considered, and other random variables can be dealt with in the same way. In the numerical implementation, we have to employ finite terms to approximate the accuracy value. If we remain *s* terms, $Y(\kappa)$ can be expressed by

$$Y(\kappa)=\sum_{j=0}^{s\text{-}1} y_j\phi_j\left(\boldsymbol{\xi}\left(\kappa\right)\right) \tag{2}$$

The Legendre polynomial forms a complete orthogonal basis in the L_2 space consisting of the uniformly random variables, i.e.

$$\left\langle \phi_i,\phi_j\right\rangle=\left\langle \phi_i^2\right\rangle\delta_{ij} \tag{3}$$

where δ_{ij} is the Kronecker delta, and $\langle .,. \rangle$ denotes the ensemble average inner product.

$$\langle f(\xi), g(\xi) \rangle = \int f(\xi) g(\xi) w(\xi) d\xi \tag{4}$$

Here $w(\xi) = (1/2)^n$ is the weighting function of Legendre polynomials. With the orthogonality, the coefficient y_i in Eq. (2) can be obtained via the following expression

$$y_i = \frac{\langle Y, \phi_i \rangle}{\langle \phi_i^2 \rangle} = \frac{1}{\langle \phi_i^2 \rangle} \int Y \phi_i(\xi) \omega(\xi) d\xi \tag{5}$$

Once getting the coefficients, the statistics characteristics can be obtained. The mean of Y is given by the 0^{th} order term in the stochastic expansion, and the variance of Y can be expressed by the sum of square of other terms multiplying with $\langle \phi_i^2 \rangle$

$$\mu = y_0, \ \sigma^2 = \left\langle \left(Y - \bar{Y} \right)^2 \right\rangle = \sum_{i=1}^{s\text{-}1} y_i^2 \langle \phi_i^2 \rangle \tag{6}$$

The coefficients of the PC expansion can also be obtained through the collocation method, by using the model outputs at some selected collocation points to regress the coefficients (Isukapalli 1999). The collocation points are selected from the roots of the polynomial, which is one order higher than the polynomial chaos expansion. Once the collocation points selected, the least square method can be used to produce the coefficients, i.e.

$$\mathbf{y} = \left(\mathbf{X}(\xi)^T \mathbf{X}(\xi) \right)^{-1} \mathbf{X}(\xi)^{\mathrm{T}} \mathbf{Y}, \ \mathbf{X}(\xi) = \begin{bmatrix} \phi_0(\xi_1) & \cdots & \phi_{s\text{-}1}(\xi_1) \\ \vdots & \ddots & \vdots \\ \phi_0(\xi_N) & \cdots & \phi_{s\text{-}1}(\xi_N) \end{bmatrix} \tag{7}$$

where $\mathbf{Y} = \left[Y(\xi_1) ... Y(\xi_N) \right]^{\mathrm{T}}$ denotes the model output vector at the collocation points, N denotes the number of collocation points, $\mathbf{y} = \left[y_0, y_1 ... y_{s-1} \right]^{\mathrm{T}}$ is the coefficients vector of polynomial chaos expansions, $\mathbf{X}(\xi)$ is the transform matrix, and $\xi_1,...,\xi_N$ denote the collocation points in a n-dimensional space.

3. Chebyshev method for interval parameters

The PC method expanses the function with random variables by the sum of some orthogonal polynomials, and the corresponding orthogonal polynomials are determined by the distribution of random variables. This section will consider the interval variables.

Define a real interval $[x]$ as a connected nonempty subset of real set R, expressed as

$$[x] = [\underline{x}, \overline{x}] = \{ x \in R : \underline{x} \le x \le \overline{x} \}, \tag{8}$$

where $\underline{x}$ and $\overline{x}$ denotes the lower and upper bounds of $[x]$, respectively. Any interval $[x]$=[a, b] can be transformed to the expression of $[\eta]=[-1, 1]$, so we only consider the interval $[\eta]=[-1, 1]$ in this paper. (Wu, Zhang et al. 2013) proposed the Chebyshev inclusion function to estimate the bounds for an interval function, and to control the overestimation in interval arithmetic. The Chebyshev inclusion function will be introduced here briefly, more information can be found in (Wu, Luo et al. 2013; Wu, Zhang et al. 2013).

Firstly, for one dimensional interval function $f([\eta])$, the pth order Chebyshev inclusion function is

$$[f]([\eta]) = \frac{1}{2} f_0 + \sum_{i=1}^{p} f_i C_i([\eta]) = \frac{1}{2} f_0 + \sum_{i=1}^{p} f_i \cos i[\theta] \tag{9}$$

where $\theta = \arccos(\eta) \in [0, \pi]$, and $C_i(\eta) = \cos i\theta$ denotes the Chebyshev polynomial with order i. The coefficients f_i can be calculated by

$$f_i = \frac{2}{\pi} \int_{-1}^{1} \frac{f(\eta) C_i(\eta)}{\sqrt{1-\eta^2}} dx \approx \frac{2}{\pi} \frac{\pi}{p+1} \sum_{j=1}^{p+1} f(\eta_j) C_i(\eta_j) = \frac{2}{p+1} \sum_{j=1}^{p+1} f(\cos\theta_j) \cos i\theta_j \tag{10}$$

where the interpolation points η_j are defined as the zeros of Chebyshev polynomial with order p+1:

$$\eta_j = \cos\theta_j,\ \theta_j = \frac{2j-1}{p+1}\frac{\pi}{2},\ j = 1, \ldots, p+1 \tag{11}$$

Equation (10) is given with the Gaussian-Chebyshev interpolation integral formula.

Denote the Chebyshev polynomial $C_i(\eta)$ as $\psi_i(\boldsymbol{\eta})$, the Chebyshev inclusion function can be expressed as

$$[f]([\boldsymbol{\eta}]) \approx \frac{1}{2} f_0 + \sum_{i=1}^{p} f_i C_i([\eta]) = \sum_{i=0}^{p} \gamma_i \psi_i([\boldsymbol{\eta}]) \tag{12}$$

Similar to the SRSM, the interpolation points are chosen to build the Chebyshev inclusion function, and the least square method is used. The Legendre polynomials in the transform matrix are required to change to the Chebyshev polynomials as

$$\boldsymbol{\gamma} = \left(\mathbf{X}(\boldsymbol{\eta})^T \mathbf{X}(\boldsymbol{\eta})\right)^{-1} \mathbf{X}(\boldsymbol{\eta})^{\mathrm{T}} \mathbf{f},\ \text{where } \mathbf{X}(\boldsymbol{\eta}) = \begin{bmatrix} \psi_0(\boldsymbol{\eta}_1) & \cdots & \psi_{k\text{-}1}(\boldsymbol{\eta}_1) \\ \vdots & \ddots & \vdots \\ \psi_0(\boldsymbol{\eta}_M) & \cdots & \psi_{k\text{-}1}(\boldsymbol{\eta}_M) \end{bmatrix} \tag{13}$$

where M denotes the number of interpolation points, $\mathbf{f} = [f(\boldsymbol{\eta}_1)\ldots f(\boldsymbol{\eta}_M)]^{\mathrm{T}}$ denotes the model output vector at the interpolation points, $\boldsymbol{\gamma} = [\gamma_0, \gamma_1 \ldots \gamma_{k-1}]^{\mathrm{T}}$ denotes the coefficients vector of Chebyshev polynomials. It is noted that $C_0([\eta])$=1, $C_i([\eta])$=$\cos i[\theta]$=[-1,1], $i \geq 1$, so $\psi_0([\boldsymbol{\eta}])$=1, and $\psi_i([\boldsymbol{\eta}])$=[-1,1], $i \geq 1$. Based on the interval arithmetic, we can calculate the bounds of the interval function as follows:

$$[f]([\boldsymbol{\eta}]) = \gamma_0 + \left(\sum_{i=1}^{k\text{-}1} |\gamma_i|\right)[\text{-}1,1] \tag{14}$$

4. The hybrid uncertain analysis method

In this section, both the random and interval variables are considered in $F(\boldsymbol{\xi},[\boldsymbol{\eta}])$. The function contains an n-dimensional random variable $\boldsymbol{\xi} \in U(-1,\ 1)^n$ and an m-dimensional interval variable $[\boldsymbol{\eta}]$=[-1, 1]^m. Hence, the output of the function will have the characteristics of both random and interval variables, and the PCCI method will integrated the PC method with the Chebyshev based interval method.

Consider the random variable $\boldsymbol{\xi}$ only, and use Eq. (2) to expand the function $F(\boldsymbol{\xi},[\boldsymbol{\eta}])$

$$F(\boldsymbol{\xi},[\boldsymbol{\eta}]) = \sum_{j=0}^{s-1} \beta_j \phi_j(\boldsymbol{\xi}) \tag{15}$$

Here we use β_j denotes the PC coefficients. Since the left side of Eq. (15) contains both the interval variable $[\boldsymbol{\eta}]$ and the random variable $\boldsymbol{\xi}$, while $\phi_j(\boldsymbol{\xi})$ at the right side is the Legendre polynomials which is only the function of $\boldsymbol{\xi}$, the coefficients β_j will be a function with respect to $[\boldsymbol{\eta}]$, namely $\beta_j([\boldsymbol{\eta}])$. Use the Chebyshev expansion Eq. (12) to the coefficients $\beta_j([\boldsymbol{\eta}])$, obtaining its Chebyshev inclusion function

$$[\beta_j]([\boldsymbol{\eta}]) = \sum_{i=0}^{k-1} \beta_{j,i} \psi_i([\boldsymbol{\eta}]) \tag{16}$$

Here $\beta_{j,i}$ denotes the element in the coefficient matrix $\boldsymbol{\beta}$ with k rows and s columns. Substitute Eq. (16) into Eq. (6), the mean and variance will be obtained as follows:

$$\mu([\boldsymbol{\eta}]) = \beta_0([\boldsymbol{\eta}]) = \sum_{i=0}^{k-1} \beta_{0,i} \psi_i([\boldsymbol{\eta}]),\ \sigma^2([\boldsymbol{\eta}]) = \sum_{j=1}^{s-1} \beta_j^2([\boldsymbol{\eta}]) \left\langle \phi_j^2 \right\rangle = \sum_{j=1}^{s-1} \left(\sum_{i=0}^{k-1} \beta_{\mathrm{j},i} \psi_i([\boldsymbol{\eta}]) \right)^2 \left\langle \phi_j^2 \right\rangle \tag{17}$$

Since the expression of the mean and variance contain interval variables, the two statistics will also be interval numbers: interval mean (IM) $[\mu]$ and interval variance (IV) $[\sigma^2]$, respectively. Based on the Chebyshev polynomials, the IM $[\mu]$ can be expressed as

$$[\mu]([\boldsymbol{\eta}]) = [\beta_0]([\boldsymbol{\eta}]) = \sum_{i=0}^{k-1} \beta_{0,i} \psi_i([\boldsymbol{\eta}]) = \beta_{0,0} + \left(\sum_{i=1}^{k-1} |\beta_{0,i}| \right)[\text{-}1,1] \tag{18}$$

Similarly, the IV $[\sigma^2]$ may be expressed by

$$[\sigma^2]([\boldsymbol{\eta}]) = \sum_{j=1}^{s-1}\left(\left(\sum_{i=0}^{k-1}\beta_{j,i}\psi_i([\boldsymbol{\eta}])\right)^2\left\langle\phi_j^2\right\rangle\right) = \sum_{j=1}^{s-1}\left(\left(\beta_{j,0}+\left(\sum_{i=1}^{k-1}\left|\beta_{j,i}\right|\right)[\text{-}1,1]\right)^2\left\langle\phi_j^2\right\rangle\right) \tag{19}$$

Since $[\mu]$ and $[\sigma^2]$ are the functions with respect to the interval numbers [-1,1], the above equations (18) and (19) still involve the overestimation (Moore 1966) according to the interval arithmetic, particularly, when the evaluated functions are multimodal. Here, the bounds of IM and IV can be calculated respectively as

$$[\mu] = \left[\underline{\mu}, \overline{\mu}\right] = \left[\min_{-1\le\boldsymbol{\eta}\le 1}\sum_{i=0}^{k-1}\beta_{0,i}\psi_i(\boldsymbol{\eta}),\ \max_{-1\le\boldsymbol{\eta}\le 1}\sum_{i=0}^{k-1}\beta_{0,i}\psi_i(\boldsymbol{\eta})\right] \tag{20}$$

$$\left[\sigma^2\right] = \left[\underline{\sigma}^2, \overline{\sigma}^2\right] = \left[\min_{-1\le\boldsymbol{\eta}\le 1}\sum_{j=1}^{s-1}\left(\left(\sum_{i=0}^{k-1}\beta_{j,i}\psi_i(\boldsymbol{\eta})\right)^2\left\langle\phi_j^2\right\rangle\right),\ \max_{-1\le\boldsymbol{\eta}\le 1}\sum_{j=1}^{s-1}\left(\left(\sum_{i=0}^{k-1}\beta_{j,i}\psi_i(\boldsymbol{\eta})\right)^2\left\langle\phi_j^2\right\rangle\right)\right] \tag{21}$$

In interval analysis, the scanning method or global optimization algorithms are often applied to the above equations, in order to solve the "min" and "max" problems to obtain the bounds. In this case, the overestimation of the interval computation can be well controlled.

Since the evaluated functions may be multimodal, the global optimization algorithms have to be used in order to find the minimum and maximum values for the lower bound and upper bound of IM and IV, respectively. Based on the explicit expressions of IM and IV, both the scanning method and the global optimization algorithm can efficiently find the bounds for IM and IV. If the dimension of the interval variables is less than 3 ($m<3$), the scanning method (Buras, Jamin et al. 1996)can directly produce accurate bounds. However, for the high dimensional problems, some global optimization algorithms, such as the genetic algorithm, particle swarm algorithm, and simulated annealing algorithm, may be more effective.

5. The uncertain analysis of vehicle dynamics

To demonstrate the effectiveness of the proposed PCCI method in engineering, the 4-DOF roll plan model of vehicles (Blanchard, Sandu et al. 2009) is studied in this section. The roll plan model is shown in Fig. 1.

There is an added mass *M* on the roll bar, which denotes the driver, the passenger, and other object in the vehicle. The *d* denotes the distance from the added mass position to the left end of the roll bar. The vehicle body is presented by a roll bar with mass *m*, length *l*, and inertia *I*. The mass of left tyre and right tyre is m_{t1} and m_{t2}, respectively, and the tyre stiffness is k_{t1} for the left side and k_{t2} for the right side. Considering the nonlinear stiffness of suspension, the linear stiffness is denoted by k_i and the nonlinear stiffness is represented by k_i', where i=1for the left suspension and i=2 for the right suspension. The damping ratio for the left suspension and right suspension is noted as c_1 and c_2, respectively.

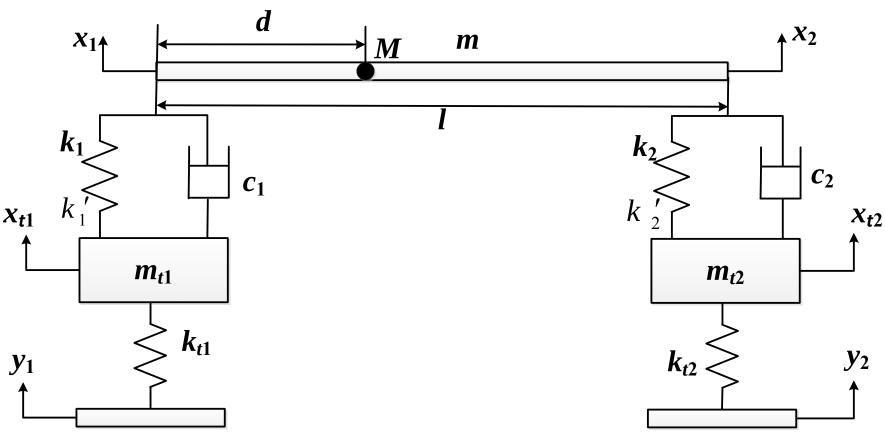

Figure. 1 The roll plan model of a vehicle

Assume that there are some uncertain parameters in this system, including the stiffness of the suspension k_1, k_2, k_1', k_2', the added mass M, and the position of added mass d. The stiffness parameters are considered as random variables, assuming that they satisfy the uniform distribution. For the added mass and its position, it is practically hard to obtain their probability distribution, but their variation ranges are limited inside some intervals. Therefore, the added mass M and its position d are described as interval parameters. The uncertain and other parameters are shown in Table 1.

Table 1 Parameters of the roll plan model

Parameters	m (kg)	m_{t1},m_{t2} (kg)	c_1,c_2 (N/(m/s))	k_1, k_2 (N/m)	k_1', k_2' (N/m^3)
values	580	36.26	710.7	U(19000, 20000)	U(95000,105000)
Parameters	l (m)	I (kg.m^2)	k_{t1},k_{t2} (N/m)	M (kg)	d (m)
values	1.524	63.3316	96319.76	[150, 250]	[0.5, 1]

The road input is given in Fig. 2, and the vehicle velocity is 16 km/h. The left tyre moves upgrade from 0m, and reaches the highest position 0.1m where the horizontal displacement is 1m. Keeping the height 0.1m unchanged until the left tyre goes downgrade, which is asymmetrical to the upward slope. The right tyre moves along a similar track to the left one, but its upgrade starts from 0.6m of the horizontal displacement, and its maximum height is 0.08m. The output of the roll plan model are defined as the deformation of the suspension, i.e. z_1=x_1-x_{t1} and z_2=x_2-x_{t2}.

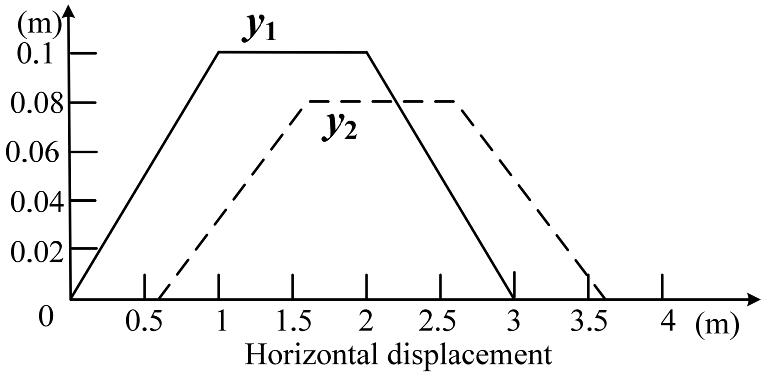

Figure 2 The road input

Due to the uncertainty of suspension stiffness, the added mass, and its position, the output should also be uncertain. The hybrid uncertain analysis method is used to solve this problem. Replacing the function $F(\boldsymbol{\xi},[\boldsymbol{\eta}])$ in Fig. 1 by the output of this roll plan model, we can obtain the IM and IV of the output. In this paper, we choose the order of PC and Chebyshev inclusion function as 4 (p=4), and the scanning method with 20 symmetrical scanning points in each dimension of interval parameters is used to compute Eqs. (20) and (21), which provides accurate bounds information. To validate the proposed method, the Monte-Carlo-Scanning test is also performed, in which the number of Monte Carlo sampling points is 1000, and 20 scanning points are used in each dimension of interval parameters, so the total number of the system is 1000×20^2=400,000. The proposed method takes about 180.4s to obtain the results, while the Monte-Carlo-Scanning test takes 9915.3s, which is more than 50 times than that of the proposed method.

The IM and IV of the output are shown in Fig. 3-6. The results show that the IM of the PCCI method are close to the IM of the Monte-Carlo-Scanning test, and the test results of IM are contained in that of PCCI tightly. So the PCCI method can provide sufficient accuracy to the interval mean. For the IV, the intervals of the PCCI method do not contain all the intervals obtained by the reference test, but there is only small difference between them. Thus, the PCCI method can also provide good estimation for interval variance.

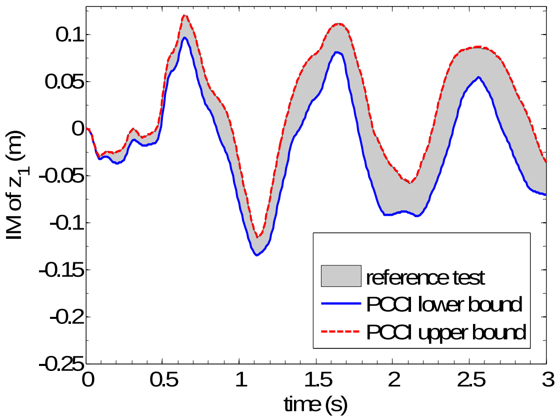

Figure 3 IM of z_1

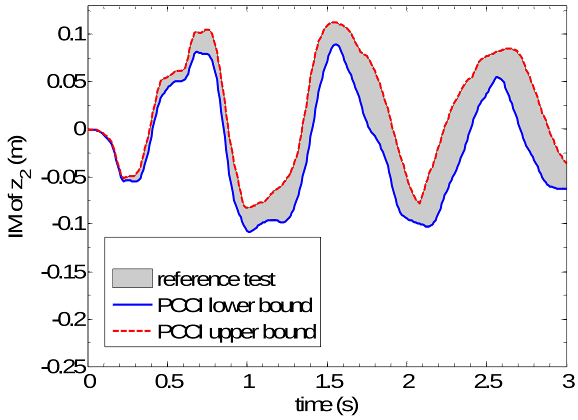

Figure 4 IM of z_2

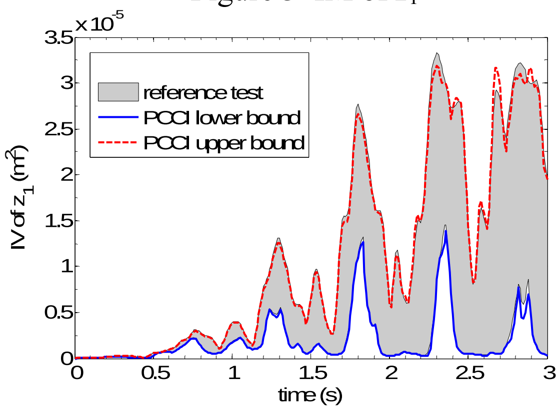

Figure 5 IV of z_1

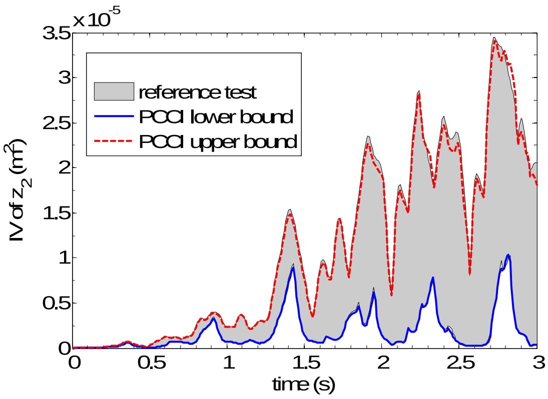

Figure 6 IV of z_2

6. Conclusions

This paper has proposed an uncertain analysis method, termed as PCCI, for systems involving hybrid uncertain parameters, namely the random parameters and interval variables. In this method, the PC method is applied to deal with the random uncertainty and the Chebyshev-based interval method is proposed to handle the interval uncertainty. The evaluation indexes are proposed, which include the interval mean (IM) and interval variance (IV). To validate the PCCI method, a Monte-Carlo-Scanning test scheme is proposed, by combining the Monte Carlo method and the scanning method to calculate the two types of evaluation indexes. A 4-DOF vehicle roll plan model is used to demonstrate the effectiveness of the proposed PCCI method, in which the stiffness of the suspension are regarded as random parameters, while the added mass and its position are considered as interval parameters. The numerical results show that the PCCI method can provide accurate numerical results for both types of the evaluation indexes. Furthermore, the PCCI method only takes 180.4s, but the Monte-Carlo-Scanning test takes 9915.3s. In addition, the PCCI method is a kind of non-intrusive method, so it can be used to solve black-box type problems.

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