Investigating the influence of bending in the structural behavior of tensegrity

modules using dynamic relaxation

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Abstract

Tensegrity modules are spatial structures composed of tensile and compression components. Tensile and compression components are assembled together in a self-equilibrated state that provides stability and stiffness to the structure. Modules can be combined to create lightweight structures with good structural efficiency. Furthermore, tensegrity structures are good candidates for adaptive and deployable systems having thus applications in various scientific and engineering fields. Research into tensegrity structures has resulted in reliable techniques for their form-finding and analysis. Although bending is not considered in these techniques, tensegrity structures often sustain bending in their elements due to dead load and imperfections. Therefore, this paper investigates the effect of bending in a tensegrity "simplex" module. Dynamic relaxation is used to analyze the module with strut and strut-beam elements. The study reveals that bending increases stresses in elements and therefore should not be neglected.

Keywords: Tensegrity, Structural Behavior, Bending Elements, Dynamic Relaxation

Introduction

Tensegrity structures are spatial reticulated structures composed of cables and struts in a selfequilibrated pre-stressed state that provides stability and stiffness to the structure. The concept exists for almost 60 years now and has received significant interest in disciplines such as structural engineering [Motro (2005)], aerospace engineering [Skelton and de Oliveira (2009)] and biology [Ingber et al. (2014)]. In biology, tensegrity is used to explain cell mechanics [Ingber (2003)] while in aerospace engineering and structural engineering it is used to design strong yet lightweight modular structures [Skelton and de Oliveira (2009); Adriaenssens and Barnes (2001); Rhode-Barbarigos et al. (2010)]. Tensegrity is also attractive for adaptive applications as actuators and structural elements can be combined [Skelton and de Oliveira (2009); Rhode-Barbarigos et al. (2012a)]. Therefore, tensegrity systems have also been proposed for deployable structures [Sultan and Skelton (2003); Rhode-Barbarigos et al. (2012b)] and robots [Paul et al. (2006); SunSpiral et al. (2013)].

Research into tensegrity systems has resulted in reliable techniques for their form-finding and analysis [Tibert and Pellegrino (2003); Masic et al. (2005)]. In these techniques, compressive elements are modeled as struts with no bending as a pure compression state is desired. However, in reality elements in tensegrity structures are strut-beam elements sustaining bending due to dead load and imperfections such as initial curvature or eccentricity in their joints. Therefore, this paper focuses on the effect of bending in a tensegrity "simplex" module. Dynamic relaxation is used to analyze the module numerically and study the effect of considering strut-beam elements with initial curvature in its structural behavior.

"Simplex" module topology and geometry

The tensegrity structure studied in this paper is the "simplex" module. The "simplex" module is the basic spatial tensegrity system [Motro (2006)]. It is composed of 3 struts and 9 cables (Figure 1). The module topology is given in the appendix. The module has a single state of self-stress and a single infinitesimal mechanism which involves the translation and rotation of the upper triangle [Motro (2005)].



Figure 1. The tensegrity "simplex" module

In this study, strut elements and strut-beam elements are steel hollow tubes with a modulus of elasticity of 210GPa, while tensile elements are stainless steel with a modulus of elasticity of 120GPa. Strut and strut-beam elements have a length of approximately 1.4m, a diameter of 76.1mm and a thickness of 4mm. Cables have a cross-sectional area of 0.2826mm² and a tensile strength of 31.8kN. Cable lengths depend on their topology with horizontal cables having a length of 0.866m and vertical cables having a length of 1.032m. Finally, vertical displacements on all nodes on the basis of the module are restrained.

Dynamic Relaxation

In this study, dynamic relaxation is employed for the static analysis of the "simplex" module. Dynamic relaxation is an explicit numerical form-finding and analysis method of tensile structures [Barnes (1999); Adriaenssens and Barnes (2001); Bel Hadh Ali et al. (2011)] that avoids stiffness-matrix calculations [Brew and Brotton (1971)]. Therefore, it is suitable for the analysis of nonlinear structures such as tensegrity modules.

In dynamic relaxation, a structure is modeled as a mesh of elements connected with nodes. A mass is assigned to every node. Loading is also applied to the nodes, while pre-stress is applied through the definition of an initial element length. The method explores the fact that the static solution for a structure subject to loading can be seen as the equilibrium state of a series of damped vibrations. Consequently, the governing equation is:

$$F_{ext} - F_{int} = M\dot{v} + Dv \tag{1}$$

where F_{ext} and F_{int} are the external and internal forces at each node respectively, *M* corresponds to the nodal mass and *D* corresponds to damping. However, mass *M* and damping *D* are fictitious parameters optimized for the stability and convergence of the method [Belytschko and Hugues (1983)]. \dot{v} and *v* are the acceleration and the velocity at each node respectively. In this study, kinetic damping is employed [Cundall (1976)]. Therefore, kinetic energy is monitored and when a peak in kinetic energy is detected, the velocity is reset to zero, the geometry is updated and convergence is

checked. Expressing the acceleration in a finite difference form gives the velocity and the updated geometry for each node:

$$v^{t+\Delta t/2} = v^{t-\Delta t/2} + \frac{F_{ext} - F_{int}}{M} \Delta t$$
⁽²⁾

$$x^{t+\Delta t} = x^t + v^{t+\Delta t/2} \Delta t \tag{3}$$

where $v^{t+\Delta t/2}$ and $v^{t-\Delta t/2}$ are the nodal velocities at times $t+\Delta t/2$ and $t-\Delta t/2$ respectively. $x^{t+\Delta t}$ is the nodal position at time $t+\Delta t$ and Δt is the time step applied. The new geometry obtained allows updating the internal forces F_{int} and thus starting over. Convergence is obtained when the term F_{ext} - F_{int} is sufficiently small (equilibrium).

To encounter the effect of bending in the "simplex" module, the bending-element formulation by Adriaenssens and Barnes (2001) is employed for strut-beam elements. Strut-beam elements are thus decomposed on a series of links and bending moments are estimated based on a finite difference modeling of a continuous beam. Bending moments are decomposed into shear forces that are added to the existing nodal forces and convergence is checked according to the general calculation scheme (Eq. 1). The formulation allows thus the method to maintain its computational advantages.

Structural analysis

Dynamic relaxation is used to analyze the structural response of the "simplex" module (stresses in the elements) under self-stress as well as under the combination of self-stress with vertical loading. In order to investigate the effect of bending in the "simplex" module, compressive elements are first modeled using struts (purely axially loaded elements) and then with strut-beam elements. Moreover, since tensegrity structures have pinned connections an initial curvature is also given to the strutbeam elements ($1/(10*l_{strut})$) to initiate bending action in them.

Figures 2 shows the stresses in the cables and struts of the "simplex" module for different self-stress levels (5%, 10% and 15% of the tensile strength of the cables) with strut elements (left) and strutbeam elements (right). The analysis shows that in both configurations cables are the most load bearing elements of the system. Furthermore, when strut-beam elements are employed stresses in cables reduce (up to 40%) while stresses in strut-beam elements increase (up to 44%).



Figure 2. Stresses in the elements of the "simplex" module for different self-stress levels (5%, 10% and 15% of the tensile strength of the cables) with strut elements (left) and strut-beam elements (right).

Figures 3 shows the stresses in the elements of the "simplex" module for different vertical loads applied on the top nodes (10kN, 20kN and 30kN per top node) with strut elements (left) and strutbeam elements (right). A self-stress level of 5% is also applied in the cables to guarantee the stability and stiffness of the module. Similar to the self-stress study, the analysis reveals that cables are the load bearing elements and that when strut-beams are employed, stresses in these elements increase significantly (close to 100%) while stresses in cables remain in approximately the same level (decrease of 5%).



Figure 3. Stresses in the elements of the "simplex" module for different loads applied on the top nodes (0kN, 10kN and 20kN per top node) with struts modelled as axial elements (left) and bending elements (right).

Bending increases stresses in strut-beam elements. Consequently, it can lead to failure at lower loading levels than originally predicted with form-finding and analysis techniques that model compressive elements with struts. Therefore, bending should be taken into account when designing tensegrity systems especially for load-bearing applications.

Discussion

Current design theory holds that bending is undesirable in tensegrity elements. However, by integrating bending in the form-finding process, novel tensegrity structures constructed from flexible yet strong engineering materials that have low Young's modulus and high strength such as Fibre Reinforced Plastics (FRP) could be explored. Applying such materials reduces significantly bending stresses in the strut-beam elements avoiding failure and thus opening the door to the development of a whole new realm of novel tensegrity systems that can sustain large elastic deformations without failure similar to natural systems [Ingber et al. (2014)].

Conclusions

This paper investigates the effect of bending in a tensegrity "simplex" module. Dynamic relaxation is used to analyze a "simplex" module with strut and strut-beam elements. It is found that considering bending increases stresses in the elements which can lead to failure at lower loading levels than predicted with traditional form-finding and analysis techniques. Therefore, it is important to consider bending when designing tensegrity structures. Moreover, integrating bending in the form-finding process could lead to bending-active tensegrity systems and thus novel applications of tensegrity systems.

References

Motro, R. (2005). Tenségrité, Hermes Science.

Skelton, R. E. and M. C. de Oliveira (2009). Tensegrity systems, Springer.

Ingber, D.E., Wang, N., Stamenović, D. (2014). Tensegrity, cellular biophysics, and the mechanics of living systems. Reports on Progress in Physics, 77(4), 046603.

Ingber, D.E. (2003) Tensegrity: I. Cell structure and hierarchical systems biology. J Cell Sci 116: 1157-1173

Adriaenssens S., Barnes M.R. (2001). Tensegrity spline beam and grid shell structures, Eng Struct 23(1), 29-36.

Rhode-Barbarigos L., Bel Hadj Ali N., Motro R., Smith I.F.C. (2010). Designing tensegrity modules for pedestrian bridges. Eng Struct, 32(4): 1158-1167.

Rhode-Barbarigos L., Schulin C., Bel Hadj Ali N., Motro R., Smith I.F.C. (2012a). Mechanism-based approach for the deployment of a tensegrity-ring module. J of Struct Eng-ASCE, 138: 539-548.

Sultan, C. and Skelton R. (2003). Deployment of tensegrity structures. Int J of Solids and Struct 40(18): 4637-4657.

Rhode-Barbarigos L., Bel Hadj Ali N., Motro R., Smith I.F.C. (2012b). Design aspects of a deployable tensegrity-hollow-rope footbridge. Int J of Space Struct, 27(2): 81-96.

Paul C., Valero-Cuevas F.J., Lipson H. (2006). Design and Control of Tensegrity Robots for Locomotion, IEEE Trans. on Robotics, 22 (5), 944–957.

SunSpiral V., Gorospe G., Bruce J., Iscen A., Korbel G., Milam S., Agogino A., Atkinson D. (2013). Tensegrity Based Probes for Planetary Exploration: Entry, Descent and Landing (EDL) and Surface Mobility Analysis. Int J of Planetary Probes.

Tibert, A.G. and Pellegrino S. (2003). "Review of Form-Finding Methods for Tensegrity Structures." Int J of Space Struct 18: 209-223.

Masic, M., Skelton R., et al. (2005). "Algebraic tensegrity form-finding." Int J of Solids and Struct 42(16-17): 4833-4858.

Motro R. (2006) Tensegrity: Structural Systems for the Future. Butterworth-Heinemann, Boston.

Barnes, M.R. (1999). "Form Finding and Analysis of Tension Structures by Dynamic Relaxation." International Journal of Space Structures 14: 89-104.

Bel Hadj Ali N., Rhode-Barbarigos L., Smith I.F.C. (2011). Analysis of clustered tensegrity structures using a modified dynamic relaxation Algorithm. Int. J. of Solids and Structures, 48(5): 637-647.

Brew JS, Brotton DM. (1971). Non-linear structural analysis by dynamic relaxation. Int J Numer Methods Eng. 3(4), 463-83.

T.B. Belytschko, T.J.R. Hugues, (1983) Computational Methods for Transient Analysis: Computational Methods in Mechanics; Mechanics and Mathematical Methods, a Series of Handbooks. North-Holland, Amsterdam-New York.

P.A. Cundall, Numerical Methods in Engineering (Proc. of the EF Conf. on Numerical Methods in Geomechanics), 1, 132-150, (1976).

Appendix

Table 1. Nodal coordinates for the "simplex" module

Node	X [mm]	Y [mm]	Z [mm]
1	500	0	0
2	-250	433	0
3	-250	-433	0
4	-433	250	1000
5	0	-500	1000
6	433	250	1000

 Table 2. Nodal connectivity for the "simplex" module

Element	Node	Node
Strut	1	4
Strut	3	6
Strut	2	5
Cable	1	2
Cable	2	3
Cable	3	1
Cable	4	5
Cable	5	6
Cable	6	4
Cable	1	6
Cable	3	5
Cable	2	4