

**A Differential Quadrature Hierarchical Finite Element Method and Its Application to  
Thickness-shear Vibration Analysis of Rectangular Quartz Plates**

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This paper presented a differential quadrature hierarchical finite element method (DQHFEM) and applied it to thickness-shear vibration analysis of rectangular quartz plates. The DQHFEM introduced the key ideas and skills of the differential quadrature finite element method (DQFEM) into the hierarchical finite element method (HFEM) to overcome the numerical stability problems and the difficulty in computing high-order hierarchical elements in the HFEM. The incorporation of the DQFEM into the HFEM also improved its computational accuracy and efficiency due to the simplicity and high accuracy of the DQFEM. The high frequency thickness-shear vibration of a rectangular quartz plate was modeled by using several DQHFEM elements. The degrees of freedom of the DQHFEM elements were adjusted according to a balance between accuracy and computational cost. The DQHFEM, as an improved version of the HFEM, was shown to have the potential of becoming the mainstream in developing finite element software in future.

**Keywords:** Thickness-shear vibration, Quartz plate, Frequency, Differential quadrature method, Hierarchical finite element method