Stress Concentration Factors of Matrix in a Composite Subjected to Transverse Loads

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EXTENDED ABSTRACT

In order to predict failure behavior and ultimate strength of a fibrous composite only based on its original constituent material properties, three challenging problems must be resolved with high success. First, the internal stresses in the constituent fiber and matrix materials must be accurately evaluated. This can be accomplished by using well established micromechanics theories such as the Bridging Model theory^[11] or FEM (finite element method) simulation. Second, efficient failure criteria must be established to detect failure status of the composite upon the obtained internal stresses in the constituent fiber and matrix. Last but not the least, input data for the in-situ properties of the constituents specifically the matrix in-situ strengths must be correctly defined. This is because the matrix in-situ strengths are fifferent from its original counterparts measured using monolithic material specimens. Experiments have shown that a transverse tensile strength of a UD (unidirectional) composite is much lower than the tensile strength of the pure matrix material. No boby, however, is able to directly measure an in-situ strength of the matrix in a composite.

Many factors such as stress magnification, strain magnification, residual stresses, etc. have been attributed to the variation of matrix strengths once reinforced with fibers^[2], but no attempt has been made in the past literature to quantitatively correlate the matrix in-situ strengths with its original counterparts. We have found it is a stress concentration due to introduction of fibers that has caused the difference between the matrix in-situ and original properties. As long as the stress concentration factors are determined, the matrix in-situ strengths are obtained from its original ones divided by these factors.

Based on a CCA (concentric cylinder assemblage) model, the stress fields in the fiber and matrix under a transverse load can be determined in terms of rigorous elasticity theory. However, a stress concentration factor cannot be defined as the maximum stress devided by the overall applied stress, as done in a classical definition for the stress concentration factor of an infinite plate with a hole. Otherwise, the thus obtained factor would be independent of a fiber volume fraction and would be unlimited large (and thus the matrix in-situ strength would be zero) when there exist a crack on the fiber/matrix interface. In view of the fact that the stress concentration factor of a plate containing a hole is defined as the maximum stress (point-stress) on the hole divided by the far-filed applied stress, which, in fact, is a surface-averaged quantity, it is very reasonable that in composite a stress concentration factor is defined as the maximum line-averaged stress divided by a volume-averaged one.

Where should be the straight line for the stress averaging? Many experiments have revealed that the failure planes of a UD composite under a transverse tension and transverse compression are apparently different. A transverse tensile failure plane is perpendicular whereas a transverse compression failure plane is in an inclined angle to the transverse tensile and compressive loads, respectively^[3]. Therefore, the straight line used must be related to a transverse failure plane. We have chosen the straight line to be outward normal to a failure plane, as schematically shown in the following figures.



A stress concentration is then defined as the averaged stress component in the external load direction along the outward normal in the matrix domain divided by the stress component, which is an volume averaged quantity, of the matrix in the same direction determined by the Bridging Model. In the above figures, 2a is the fiber diameter whereas 2b is the matrix ouside diameter with $b = a / \sqrt{V_f}$. It is noted that a CCA model correspondes to a case of $b \rightarrow \infty$. In this way, the stress concentration factor of the matrix in the composite subjected to a transverse tension is given by

$$K_{22}^{t} = \left[1 + \frac{\sqrt{V_{f}}}{2}A + \frac{\sqrt{V_{f}}}{2}(3 - V_{f} - \sqrt{V_{f}})B\right] \left[\frac{(V_{f} + V_{m}\beta)E_{22}^{f} + V_{m}(1 - \beta)E^{m}}{\beta E_{22}^{f} + (1 - \beta)E^{m}}\right], \quad (1)$$

$$A = \frac{\left[1 - v^m - 2(v^m)^2\right]E_{22}^f - \left[1 - v_{23}^f - 2(v_{23}^f)^2\right]E^m}{E_{22}^f(1 + v^m) + E^m\left[1 - v_{23}^f - 2(v_{23}^f)^2\right]},$$
(2.1)

$$B = \frac{E^{m}(1+v_{23}^{f}) - (1+v^{m})E_{22}^{f}}{E_{22}^{f}[v^{m}+4(v^{m})^{2}-3] - E^{m}(1+v_{23}^{f})},$$
(2.2)

whereas that under a transverse compression is derived as

$$K_{22}^{c} = \left\{1 - \frac{\sqrt{V_{f}}}{2}A\frac{\sigma_{u,c}^{m} - \sigma_{u,t}^{m}}{\sigma_{u,c}^{m} + \sigma_{u,t}^{m}} + \frac{B}{2(1 - \sqrt{V_{f}})}\left[-V_{f}^{2}\left(1 - 2\left(\frac{\sigma_{u,c}^{m} - \sigma_{u,t}^{m}}{\sigma_{u,c}^{m} + \sigma_{u,t}^{m}}\right)^{2}\right) + \frac{B}{2(1 - \sqrt{V_{f}})}\left[-V_{f}^{2}\left(1 - 2\left(\frac{\sigma_{u,c}^{m} - \sigma_{u,t}^{m}}{\sigma_{u,c}^{m} + \sigma_{u,t}^{m}}\right)^{2}\right)\right]\right]$$

$$\frac{4\sigma_{u,t}^{m}V_{f}}{\sigma_{u,c}^{m}+\sigma_{u,t}^{m}}\left(1+2\frac{\sigma_{u,c}^{m}-\sigma_{u,t}^{m}}{\sigma_{u,c}^{m}+\sigma_{u,t}^{m}}\right)-\sqrt{V_{f}}\left(2\frac{\sigma_{u,c}^{m}-\sigma_{u,t}^{m}}{\sigma_{u,c}^{m}+\sigma_{u,t}^{m}}+1-2\left(\frac{\sigma_{u,c}^{m}-\sigma_{u,t}^{m}}{\sigma_{u,c}^{m}+\sigma_{u,t}^{m}}\right)^{2}\right)\right]\right\}\times\left[\frac{(V_{f}+V_{m}\beta)E_{22}^{f}+V_{m}(1-\beta)E^{m}}{\beta E_{22}^{f}+(1-\beta)E^{m}}\right]$$
(3)

Having determined the stress concentration factors, the matrix in-situ transverse tensile and compressive strengths are defined as the tensile and compressive strengths of the pure matrix devidided by the corresponding stress concentration factors, K_{22}^t and K_{22}^c , respectively. These in-situ strengths are used together with other independent constituent property parameters for strength predictions of 8 typical UD composites^[4-5]. The predicted results are summarized in the following table. Considering the complexity of the problem under study and large descrepancy generally involved in measurement of composite strengths, the present predictions are satisfactorily accurate. It is noted that all of the predictions are made with only elastic deformation assuption.

Composite	Fiber		Matrix						Measured UD		Predicted UD	
							β	V_f	strengths		strengths	
	E_{22}	G_{23}	E	v	$\sigma_{\!u,t}$	$\sigma_{u,c}$		_	Trans	Trans.	Trans	Trans.
					-	· ·			tension	Compr.	tension	Compr.
E-Glass LY556	80	33.33	3.35	0.35	80	120	0.4	0.62	35	114	54.2	123.4
E-Glass MY750	74	30.8	3.35	0.35	80	120	0.4	0.6	40	145	54.3	124.1
AS4 3501-6	15	7	4.2	0.34	69	250	0.4	0.6	48	200	52.9	306.8
T300 BSL914C	230	7	4	0.35	75	150	0.4	0.6	27	200	57.1	162.4
IM7 8511-7	276	7	4.08	0.38	99	130	0.4	0.6	73	185	72.8	129.1
T300 PR319	230	7	0.95	0.35	70	130	0.4	0.6	40	125	48.0	143.5
AS carbon Epoxy	231	7	3.2	0.35	85	120	0.4	0.6	38	150	63.1	120.8
S2-Glass Epoxy	87	36	3.2	0.35	73	120	0.4	0.6	63	180	49.3	127.9

References

- 1. Huang ZM. Simulation of the mechanical properties of fibrous composites by the bridging micromechanics model. Compos Part A. 2001; 32(2): 143-172.
- 2. Fiedler B., Hojo M., Ochiai S., Schulte K., Ando M., Failure behavior of an epoxy matrix under different kinds of static loading, Comp. Sci. Tech., 2001, 61: 1615-1624.
- 3. Vaughan T.J., McCarthy C.T., Micromechanical modelling of the transverse damage behavior in fiber reinfroced composites, Comp. Sci. Tech., 2011, 71: 388-396.
- Soden P.D., Hinton M.J., Kaddour A.S., Lamina properties, lay-up configurations and loading conditions for a range of fibre-reinforced composite laminates, Comp. Sci. Tech., 1998, 58(7): 1011-1022.
- 5. Kaddour A.S., Hinton M., Input data for test cases used in benchmarking triaxial strength of laminates, J. Comp. Mater., 2012, 46(19-20): 2295-2312.