A temporally-piecewise adaptive algorithm with SBFEM to solve viscoelastic problems

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Abstract

This paper combines the scaled boundary finite element method (SBFEM) with a temporally-piecewise adaptive algorithm to solve viscoelastic problems. By expanding variables at a discretized time interval, a coupled spatial-temporal problem is decoupled into a series of recursive spatial problems which are solved by SBFEM, and the local truncation order with respect to time is controlled adaptively via the increase of expansion powers. Numerical comparison is given to verify the proposed approach.

Keywords: Adaptive algorithm, Scaled boundary finite element method, Viscoelasticity.

Introduction

The scaled boundary finite element method (SBFEM) is a semi-analytical method developed by Wolf and Song (Wolf and Song, 1996). The method has been shown to be significantly efficient for problems involving stress singularities and unbounded domains.

This paper combines SBFEM and a temporally-piecewise adaptive algorithm to develop a new numerical model for viscoelastic problems. The temporally-piecewise adaptive algorithm was presented by Yang (Yang and Guo, 2000), all variables are expanded at a discretized time interval, and the local truncation order with respect to time is controlled adaptively via the number of expansion powers.

Recurrent governing equations

A differential form of constitutive relationship for the three-parameter solid viscoelasticity model is written as

$$q_0\{\varepsilon(t)\}+q_1\frac{d\{\varepsilon(t)\}}{dt}=[K]\left(\{\sigma(t)\}+p_1\frac{d\{\sigma(t)\}}{dt}\right)\quad(t>0)\;;\quad\{\varepsilon(t)\}=\frac{1}{E_2}[K]\{\sigma(t)\}\quad(t=0)\qquad(1\text{-}2)$$

In order to solve the Eq. (1) in time domain, a self-adaptive precise algorithm (Yang and Guo, 2000) is employed in this paper. At a discretized time interval, $\{\sigma(t)\}$ and $\{\varepsilon(t)\}$ are expanded by

$$s=\frac{t-t_{k-1}}{T_k},\; s\in[0,1],\; k=1,2,3,\ldots;\quad \{\sigma(t)\}=\sum_{m=0}\{\sigma_m\}s^m\;;\quad \{\varepsilon(t)\}=\sum_{m=0}\{\varepsilon_m\}s^m\qquad(4\text{-}6)$$

Substituting Eqs (4-6) into Eq. (1) and equating the power of the two sides of the equation give

$$\{\sigma_m\}=[D]\{\varepsilon_m\}+\{C_m\},\quad m=1,2,3,\ldots\ldots\quad\text{where }\{C_m\}=\frac{Tq_0}{mp_1}[K]^{-1}\{\varepsilon_{m-1}\}-\frac{T}{mp_1}\{\sigma_{m-1}\}.\qquad(7)$$

Implementation of SBFEM

The SBFEM is used to solved the recurrent equation in Eq. (7), the equation is obtained as

$$[K]\{u_h\}=\{P_m\}=\{PS_m\}+\{PC_m\}=\int_\eta\{N(\eta)\}\{p_m(\eta)\}\,\mathrm{d}\eta+\int_V\{\delta\varepsilon(\xi,s)\}^T\{C_m\}\,\mathrm{d}V\qquad(9)$$

The vector $\{PC_m\}$ depends on the result obtained from last iterative step

$$\{PC_m\} = \left([\Phi_1]^{-1}\right)^T \left([G_m^0] + [G_m^1] + [G_m^1]^T + [G_m^2]\right)[\Phi_1]^{-1} - \frac{T}{mp_1}\{PC_{m-1}\} \tag{10}$$

Each element of $[G_m^0]$, $[G_m^1]$ and $[G_m^2]$ can be evaluated analytically, for example

$$G_{m,ij}^0 = \int_0^1 \frac{1}{\xi}\left(-\lambda_i \xi^{-\lambda_i}\right) E_{m,ij}^0 \left(-\lambda_j \xi^{-\lambda_j}\right) d\xi = \left(\lambda_i \cdot \lambda_j\right) \cdot \frac{E_{m,ij}^0}{-\lambda_i - \lambda_j} \tag{11}$$

Numerical example

A square viscoelastic plate subjected to a uniform tension is examined as shown in Fig. 1. The material parameters of three-parameter solid viscoelasticity are $E_1 = 1000$, $E_2 = 2000$ and $\eta_1 = 1000$. Fig.2 shows the SBFEM solution for the displacement time-curve on point A, a good agreement with analytical solution is obtained.

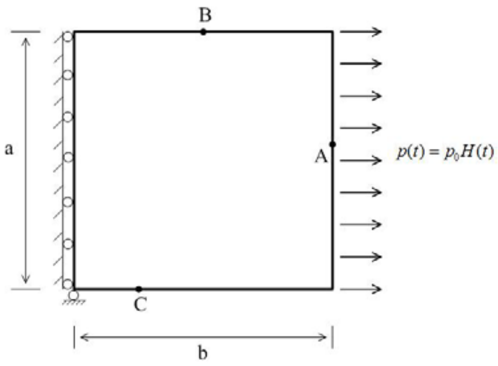

Fig. 1 A square plate

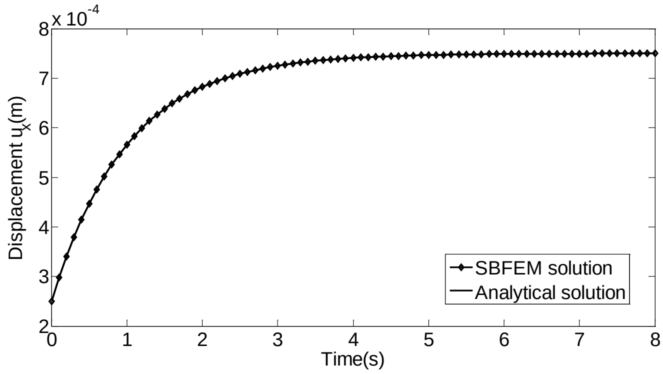

Fig. 2 The variation of displacement with time

Conclusions

A new numerical method for viscoelastic problems is presented in this paper. The SBFEM is used to solve the recurrent governing equations, which is obtained by a time-domain expansion method. The advantage of the presented method is that, firstly the accuracy in time-domain can be adaptively controlled by the orders of expansion; secondly SBFEM provides a suitable approach for the viscoelastic problems with stress singularity.

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Reference

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