Scaled boundary finite element analysis for elastic problems with interval parameters

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Abstract

This paper presents an approach to tackle with the elastic problem with interval uncertainty. The scaled boundary finite element method and Taylor expansion technique are combined to estimate the uncertain intervals of displacements and stresses when elastic constitutive parameters or length of crack are interval variables. A numerical comparison with the combinatorial method is provided to verify the proposed approach.

Keywords: Uncertainty, Scaled boundary finite element method, Interval estimation

Introduction

Non-deterministic approaches are gaining momentum in the field of numerical modeling techniques (Moens and Hanss, 2011). For the FE based numerical analysis of interval uncertainty, one of the major challenges is how to obtain reliable interval results for secondary finite element output quantities, such as stress distributions (Moens and Hanss, 2011). Due to its good behavior of computing accuracy for the displacement/stress solutions (Wolf and Song, 1996), Scaled Boundary Finite Element Method (SBFEM) is combined with a Taylor expansion technique to estimate the uncertain intervals of displacements/stresses when elastic constitutive parameters/ geometric parameters are interval variables.

Scaled boundary finite element method

Only some key equations of 2-D SBFEM, instead of entire formulae, are given here (Wolf and Song, 1996).

$$\mathbf{u}(\xi,s)=\mathbf{N}(s)\sum_{i=1}^{n}c_i\xi^{-\lambda_i}\boldsymbol{\varphi}_i\,; \qquad \boldsymbol{\sigma}(\xi,s)=\mathbf{D}\sum_{i=1}^{n}c_i\xi^{-\lambda_i-1}[-\lambda_i\mathbf{B}^1(s)+\mathbf{B}^2(s)]\boldsymbol{\varphi}_i \tag{1}$$

where $\mathbf{N}(s)$ are the circumferential finite element (FE) shape function. c_i are the modal participation factors, $\boldsymbol{\varphi}_i$ are the modal boundary displacements and λ_i are the modal scaling factors for the 'radial' direction.

Taylor series expansion based interval estimation

Assume the material or geometric parameter vector $\mathbf{b}$ is an interval vector and is described by $\mathbf{b}^I$

$$\mathbf{b}^I=\left[\underline{\mathbf{b}},\overline{\mathbf{b}}\right]=\mathbf{b}^c+\Delta\mathbf{b}\cdot e_\Delta \text{ where } \mathbf{b}^c=(\overline{\mathbf{b}}+\underline{\mathbf{b}})/2 \quad \Delta\mathbf{b}=(\overline{\mathbf{b}}-\underline{\mathbf{b}})/2 \quad e_\Delta=[-1,1] \tag{2}$$

Near the neighbor of $\mathbf{b}^c$, $\mathbf{b}$ can be described by

$$\mathbf{b}=\mathbf{b}^c+\delta\mathbf{b} \quad \delta\mathbf{b}\in\Delta\mathbf{b}^I=[-\Delta\mathbf{b},\Delta\mathbf{b}] \tag{3}$$

Utilizing the Taylor series expansion, the first-order approximation of the displacement solution can be written as (Xue and Yang, 2013)

$$\mathbf{u}(\mathbf{b}) = \mathbf{u}(\mathbf{b}^c + \delta\mathbf{b}) = \mathbf{u}(\mathbf{b}^c) + \sum_{j=1}^{n} \frac{\partial \mathbf{u}\left(\mathbf{b}^c\right)}{\partial b_j}\delta b_j \quad \text{where} \quad \mathbf{u}^c = \mathbf{u}(\mathbf{b}^c) \tag{4}$$

Therefore, the lower and upper bounds of $\mathbf{u}^I$ are estimated by

$$\overline{\mathbf{u}} = \mathbf{u}^c + \Delta\mathbf{u} = \mathbf{u}(\mathbf{b}^c) + \sum_{j=1}^{n} \frac{\partial \mathbf{u}\left(\mathbf{b}^c\right)}{\partial b_j}\delta b_j \qquad \underline{\mathbf{u}} = \mathbf{u}^c - \Delta\mathbf{u} = \mathbf{u}(\mathbf{b}^c) - \sum_{j=1}^{n} \frac{\partial \mathbf{u}\left(\mathbf{b}^c\right)}{\partial b_j}\delta b_j \tag{5}$$

Regarding to the difficulty to acquire $\frac{\partial \mathbf{u}\left(\mathbf{b}^c\right)}{\partial b_j}$ analytically, a finite difference approximation (Song et al., 2014) is employed to calculate the derivative for each variable, i.e.

$$\frac{\partial \mathbf{u}\left(\mathbf{b}^c\right)}{\partial b_j} = \frac{\mathbf{u}\left(b_j^c + \Delta b_j\right) - y\left(b_j^c - \Delta b_j\right)}{2\Delta b_j} \tag{6}$$

Numerical example

Consider a single edge cracked plate subjected to a uniaxial tension as illustrated in Fig.1. Due to the symmetry only half of the problem is modeled. The length of crack $c^c = 0.25$, $\Delta c = \alpha \cdot c^c$, $\alpha = 5\%$, α is defined as the degree of uncertainty. Tab. 1 shows that the Taylor series expansion-based estimations for the stress σ_x are very close to those given by the combinatorial method.

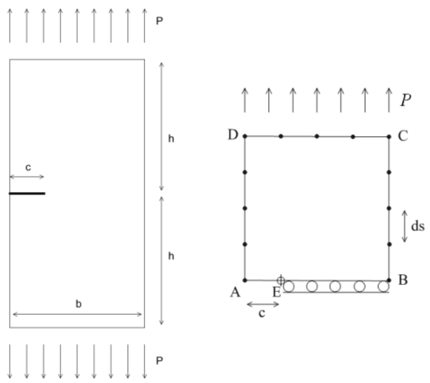

Fig.1 A single edge cracked plate

Tab.1 The stress solutions

Position	First-order Taylor	Combinatorial Method
（0.325，0）	[1.2713，1.4671]	[1.2738，1.4697]
（0.4，0）	[0.7279，0.8503]	[0.7295，0.8519]
（0.475，0）	[0.4820，0.5680]	[0.4831，0.5691]
（0.55，0）	[0.3326，0.3944]	[0.3334，0.3953]
（0.625，0）	[0.2287， 0.2724]	[0.2293， 0.273]
（0.7，0）	[0.1508，0.1801]	[0.1512，0.1805]
（0.775，0）	[0.0903，0.1080]	[0.0905，0.1082]

Conclusions

The paper presented a SBFEM based approach for the displacement/stress interval analysis when material/geometric parameters are interval variables. The lower and upper bounds of the uncertain intervals can be estimated by solving two sets of deterministic equations in terms of central values and radius.

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