# The Stiffness of Tensegrity Structures

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## Extended Abstract

Tensegrity frameworks form remarkable structures. A classic example is the 'tensegrity icosahedron' shown in Figure 1. From an aesthetic perspective, the remarkable characteristic of this structure is that the compression members 'float', supported at their ends by only cables. The support provided implies that the structure is prestressed, so that the cables carry tension even when the structure is unloaded, and are able to react as conventional structural members if additional load is applied to the structure. The level of prestress can be altered (by, for instance, turnbuckles), showing that tensegrities are *statically indeterminate*, i.e., the internal forces are not uniquely given by ensuring equilibrium with external loads.

From a structural perspective, it is also remarkable that tensegrity structures frequently do not have enough members to satisfy Maxwell's 1864 rule for the rigidity of frameworks [Maxwell, 1864], and yet form stable structures. Such structures are *kinematically indeterminate* and must have some *infinitesimal mechanism*, a set of nodal displacements which, to a linear approximation, do not cause the members to change in length.

For the tensegrity icosahedron, with $j = 12$ nodes, and $b = 24$ members (6 bars, 24 cables), the Maxwell rule, in the form given by Calladine (1978) gives

$$m - s = 3j - b - 6 = 0,$$

and as there is $s = 1$ state of self-stress (easily shown by, for instance, physical experiment), there must also be $m = 1$ infinitesimal mechanism. In fact, symmetry methods [Guest and Fowler, 2000] show that both the state of self-stress, and the infinitesimal mechanism, are totally symmetric. The

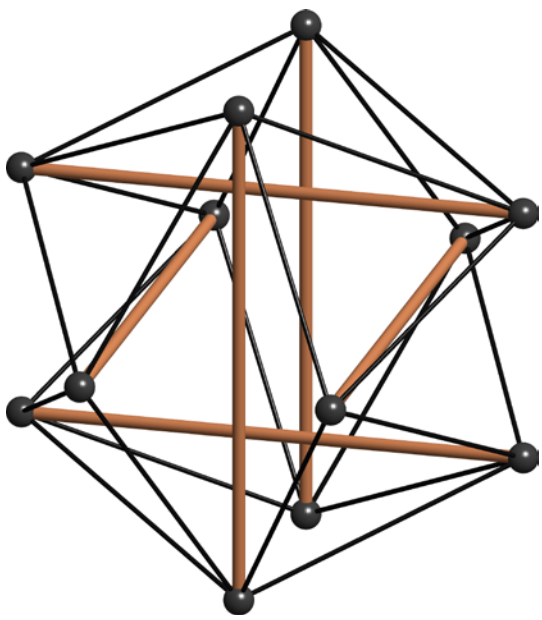
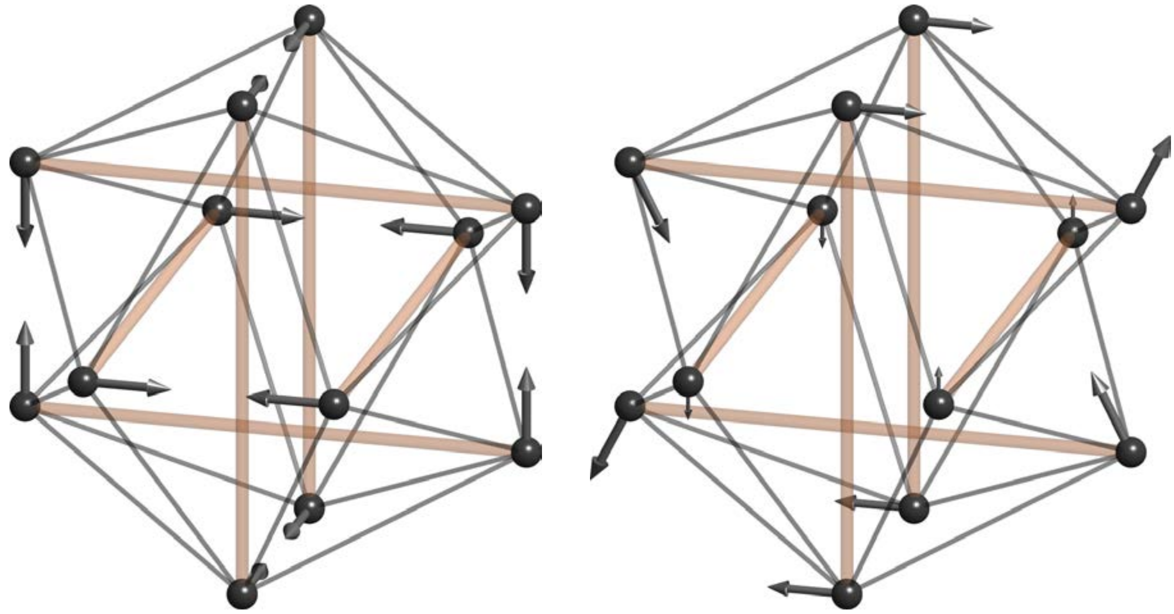


**Figure 1. A 'tensegrity icosahedron', formed of 6 bars and 24 cables.**



(a) (b)

**Figure 2. Two modes of deformation for a tensegrity icosahedron: (a) shown the infinitesimal mechanism, which is the softest mode for structures where the elastic strain is small; (b) shows a shear mode, which is the softest mode for structures where the elastic strain is large.**

state of self-stress consists of equal tension in every cable, and equal compression in every bar, while the infinitesimal mechanism is a ‘breathing mode’, where every node moves with a component of displacement toward the centre of the structure. This mode is shown in Figure 2(a).

This talk will discuss the stability and stiffness of tensegrity structures, and will use the tensegrity icosahedron as an example [Guest (2011)]. For the tensegrity icosahedron to remain stable, the geometric stiffness of the mode that is an infinitesimal mechanism must be positive – that is, the reorientation of already stressed members must be able to equilibrate the applied load. For members made of conventional structural materials, which are restricted to a strain of the order of 0.01 (1%), the talk will show that this mode will be by far the softest mode for the structure, and hence the infinitesimal mechanism will dominate the response of the structure to loading. However, tensegrity models are often not made of conventional structural materials. For instance, tension members are often made from ‘elastic’ or ‘rubber’ materials which may be stretched to a strain of the order of 1 (100%). The talk will show that this changes the basic structural response: the softest mode for these structures is now the shear mode shown in Figure 2(b).

**References**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