Nonlinear Control of Systems with Non-smooth Nonlinearities

*S. Jiffri¹, J.E. Mottershead¹

¹ Centre for Engineering Dynamics, School of Engineering, The University of Liverpool, Brownlow Hill, Liverpool L69 3GH, United Kingdom.

*Corresponding author: s.jiffri@liv.ac.uk

Abstract

Application of feedback control in nonlinear systems is an area of active research. Control algorithms utilising Lyapunov methods, Describing Functions, backstepping etc. are some of the approaches being explored. Feedback Linearisation, which effectively renders the nonlinear system exactly linear through the application of nonlinear feedback, is another approach that has been investigated. Many publications presenting analytical, numerical and also experimental findings have emerged. Much of this work addresses systems with smooth nonlinearities, often described by a polynomial function. The underlying theory of feedback linearisation is well-defined for such systems and is readily available through classical texts and also other publications. For non-smooth systems, however, the applicability of the method is not quite as obvious. The present work aims to demonstrate that at least for some types of non-smooth nonlinearity, the theory of feedback linearisation holds soundly. Successful application of the method in closed-loop control is demonstrated through a numerical example.

Keywords: nonlinear, control, feedback linearisation, non-smooth

Nomenclature and abbreviations

$\omega_{\!\alpha}, \omega_{\!\beta}, \omega_{\!\xi}$	– uncoupled natural frequency in α , β , ξ DOFs
$\zeta_{\alpha}, \zeta_{\beta}, \zeta_{\xi}$	– viscous damping coefficients in α , β , ξ DOFs
a	– distance from aerofoil mid-chord to rotational axis, normalised by b
b	– aerofoil semi-chord
С	– distance from aerofoil mid-chord to aileron hinge line, normalised by <i>b</i>
$K_{\alpha}, K_{\beta}, K_{\xi}$	– structural stiffness in α , β , ξ DOFs
$\mathbf{D}, \mathbf{E}_1, \mathbf{E}_2, \mathbf{F}$	- matrices relating to the augmented states of the aeroelastic system
$\mathbf{M}_{t}, \mathbf{C}_{t}, \mathbf{K}_{t}$	- overall mass, damping and stiffness matrices of aeroelastic system
r_{α}, r_{β}	– radius of gyration in α , β normalised by b
U, U*	- air velocity, reduced air velocity $(U^* = U / b\omega_{\alpha})$
x_{α}	– COM distance of wing+aileron from rotational axis, normalised by b
x_{β}	– COM distance of aileron from hinge line, normalised by b
СОМ	– centre of mass
DOF(s)	– degree(s) of freedom
LFS	– linear flutter speed
UoL	– University of Liverpool
WTAR	- wind tunnel aerofoil rig

1 Introduction

Suppression of vibration is among the major considerations not only in the design and manufacture of new systems, but also in improving existing and well-established ones. A variety of active and passive control methods have been explored. Active control poses the advantage of being able to alter the control inputs based on observed response, thus allowing greater control of the plant. The modelling of nonlinearities in the system being controlled is becoming increasingly important, fuelled by the evergrowing desire to increase effectiveness of existing control methods or develop new ones altogether. In this work, the numerical illustration considered is that of flutter suppression in a 3-DOF pitch-plunge-flap aeroelastic system.

There have been many publications in the literature dealing with the control of systems with smooth nonlinearities, including aeroelastic systems. The application of feedback linearisation on nonlinear aeroelastic systems with smooth structural nonlinearities, mainly of the hardening type, was investigated in (Platanitis and Strganac 2004, Strganac, et al. 2000, Ko, et al. 1999, Jiffri, et al. in press, Jiffri, et al. 2013, Jiffri, et al. 2013, jiffri, et al. 2013); both theoretical and experimental aspects have been addressed. Papers related to non-smooth systems are also available, albeit in less abundance. A method for adaptive control with feedback linearisation of systems containing a freeplay input was presented in (Recker, et al. 1991), which was extended subsequently to include also a freeplay output (Tao and Kokotovic 1997). The cases of partial feedback linearisation with and without relative degree were addressed subsequently in (Ma and Tao 2000). Other papers related to control of non-smooth nonlinear systems include (Zheng, et al. 2013, Tao, et al. 2013).

The present work applies partial input-output feedback linearisation on a 3-DOF aeroservoelastic numerical model with a piece-wise linear stiffness in the pitch DOF, with the aim of stabilising the linearised response through pole-placement. The model employed is that developed by Edwards et al. (Edwards, et al. 1979), which includes actuator dynamics and approximates unsteady behaviour using two additional augmented aerodynamic states. Other work in which this model has been used include (Conner, et al. 1997, Li, et al. 2010). In the present work, the parameters of the model are tuned to match the dynamics of the wind tunnel aerofoil rig (WTAR) at the University of Liverpool.

This paper commences with a description of the nonlinear aeroelastic system. Equations of motion are given, and are followed by frequency and time-domain simulation results based on the WTAR parameters. Expressions for input-output linearisation of the plunge DOF are derived, including those for the zero-dynamics. Numerical simulation results from the closed-loop system are then presented, demonstrating successful control of the system with a piecewise linear non-smooth nonlinearity both when full knowledge of the nonlinearity is assumed, and when there is uncertainty associated with the nonlinearity.

2 Model description

In this section, a detailed description of the aeroelastic model employed in this work is given. Thereupon, numerical simulation results performed using aeroelastic parameters pertaining to the WTAR at the University of Liverpool will be presented.

2.1 Equations of motion

The aeroelastic model of Edwards et al. (Edwards, et al. 1979) featuring approximation of the unsteady aerodynamic loads through the use of augmented states is employed in the present work. This model consists of a total of 8 states in the first-order state-space representation. Six of these are structural states, namely plunge (normalised with respect to the semi-chord *b*), pitch, aileron flap (ξ, α, β respectively) and their time-derivatives ($\dot{\xi}, \dot{\alpha}, \dot{\beta}$ respectively). The remaining two are the augmented aerodynamic states mentioned above (x_{a_1}, x_{a_2}). Equations of motion for the model are given as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \text{ where } u = \boldsymbol{\beta}_{com}, \quad \mathbf{x} = \left\{ x_1 \quad x_2 \quad \cdots \quad x_8 \right\}^T = \left\{ \mathbf{q} \quad \mathbf{v} \quad \mathbf{q}_a \right\}^T, \\ \mathbf{q} = \left\{ \boldsymbol{\xi} \quad \boldsymbol{\alpha} \quad \boldsymbol{\beta} \right\}^T, \quad \mathbf{v} = \dot{\mathbf{q}} = \left\{ \dot{\boldsymbol{\xi}} \quad \dot{\boldsymbol{\alpha}} \quad \dot{\boldsymbol{\beta}} \right\}^T, \quad \mathbf{q}_a = \left\{ x_{a_1} \quad x_{a_2} \right\}^T, \quad (1) \\ \mathbf{A} = \begin{bmatrix} \mathbf{0}_{(3\times3)} & \mathbf{I}_{(3\times3)} & \mathbf{0}_{(3\times2)} \\ -\mathbf{M}_t^{-1}\mathbf{K}_t & -\mathbf{M}_t^{-1}\mathbf{C}_t & \mathbf{M}_t^{-1}\mathbf{D} \\ \mathbf{E}_1 & \mathbf{E}_2 & \mathbf{F} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{(3\times1)} \\ \mathbf{M}_t^{-1}\mathbf{G} \\ \mathbf{0}_{(2\times1)} \end{bmatrix}, \quad \mathbf{G} = \left\{ \begin{matrix} 0 \\ 0 \\ r_{\boldsymbol{\beta}}^2 \omega_{\boldsymbol{\beta}}^2 \\ \end{matrix} \right\}$$

and the definition of all quantities appearing within the above equation may be found in (Edwards, et al. 1979, Li, et al. 2010). The input in the above equation is the desired flap angle of the aileron.

This particular model is chosen as it models the dynamics of the actuator, the means through which the input will be applied. As will be seen later, the existence of a nonsmooth nonlinearity in the system will necessitate a non-smooth input during closedloop control. Since such an input cannot be achieved in practice, modelling of the actuator dynamics is necessary to produce a numerical model that is representative of reality. This becomes an even more important issue when implementing feedback linearisation in practice, as using a control law that is based on a model without actuator dynamics will give rise to a discrepancy between the required non-smooth input and the actual smooth input provided by the actuator. It is expected that such a discrepancy will degrade controller performance.

The nonlinear case of the above model may be readily expressed in the affine form

$$\dot{\mathbf{x}} = \mathbf{\underline{f}}(\mathbf{x}) + \mathbf{g}(\mathbf{x}) u , \qquad (2)$$

where, in the present case $\underline{\mathbf{g}}(\mathbf{x}) \equiv \mathbf{B}$ and the use of different symbols is aimed at maintaining conventionally accepted notation in the linear and affine nonlinear cases.

2.2 Aeroelastic Parameters of the WTAR

From previous experiments and related numerical simulations performed on the WTAR at UoL (Papatheou, et al. 24-26 June, 2013), aeroelastic parameters that describe well the aerofoil behaviour were extracted. These are given in Table 1, along

with estimates for the parameters describing flap dynamics (not found during the experiments).

Parameter	Value	Parameter	Value	Parameter	Value
ω_{α} (rad/s)	35.354	С	0.5428	ω_{β} (rad/s)	100
r_{α}	0.4	<i>b</i> (m)	0.175	r_{β}	0.079057
x _α	0.09	μ	69.0	x _β	0.0125
ω_{ξ} (rad/s)	22.948	ζ_{ξ}	0.002	ζ_{β}	0.002
а	-0.33333	ζα	0.015		

Table 1 – Parameters of the UoL wind tunnel aerofoil rig, used in the present numerical model

This format of parameters is widely used in the literature, and is defined in (Theodorsen 1935) in addition to the papers referenced earlier.

2.3 Frequency domain results for the linear system

For the linear case of the aeroelastic system, one may plot the variation of the eigenvalues with respect to reduced air speed. For a speed range of $U^* = 0.1 - 3.0$, the resulting plot is shown in Fig. 1.



Fig. 1 – Normalised eigenvalues of structural modes varying with airspeed

The linear flutter speed (LFS) is located at the point where the normalised real part of an eigenvalue becomes positive. It is evident from Fig. 1 that this occurs with the plunge mode. The reduced LFS in the present system is found to be $U^* = 2.793$ (this translates to an absolute airspeed of 17.28 m/s).

2.4 Nonlinear time-domain response with piece-wise linear stiffness in pitch

A symmetric piece-wise linear nonlinearity is now introduced into the pitch DOF. The parameters describing the nonlinearity are given in Table 2.

Parameter	Description	Value
g_{α}	initial (lower) stiffness region on either side of $\alpha = 0^{\circ}$	1°
λ	$K_{\alpha_{(\alpha \leq g_{\alpha})}} = (1 - \lambda) K_{\alpha}, \text{ where } K_{\alpha_{(\alpha \leq g_{\alpha})}}$ is the initial (lower) stiffness	0.6
K_{lpha}	stiffness in the outer regions $(\alpha > g_{\alpha})$, chosen to be equal to linear pitch stiffness K_{α}	$r_{\alpha}^2 \omega_{\alpha}^2$

Table 2 – Nonlinearity parameters for piece-wise linear pitch stiffness

The resulting pitch moment profile is depicted by the solid line in Fig. 2.



Fig. 2 – Pitch moment profile in the presence of piece-wise linear stiffness

For the purpose of applying feedback linearisation, it will be necessary to define a target linear system, i.e. the desired system once the nonlinearity has been eliminated. This is especially relevant if the feedback linearisation cancels out only the nonlinear terms and not the entire open loop dynamics. Naturally, the target linear system may be chosen as a system whose pitch stiffness is equal to the slope of the outer regions in the nonlinear case. The pitch moment profile in this case is shown by the dash-dot line in Fig. 2. It is now possible to define also the nonlinear moment, i.e. the moment which, when added to the linear moment produces the net, nonlinear moment profile. This non-smooth nonlinear moment profile is shown by the dashed line in Fig. 2.

The nonlinear system is now simulated at a reduced velocity $U^* = 2.0$ with plunge and pitch initial values of $\xi = 0.01$, $\alpha = 3^\circ$ respectively, and with all other states set to zero. The resulting structural responses are shown in Fig. 3.



Fig. 3 – Structural states of nonlinear system at U*=2.0

It is evident that the response settles into an LCO, which occurs at an airspeed which is less than the LFS $U^* = 2.793$, which is expected as the initial stiffness in the nonlinear case is lower than that of the linear system.

3 Feedback linearisation

This section presents the application of feedback linearisation on the nonlinear aeroelastic system described by eq. (2). Feedback linearisation (Isidori 1995, Khalil 2002) is a process whereby a nonlinear system is rendered linear through the application of nonlinear feedback and a co-ordinate transformation. The system in (2) is first expressed as

$$\mathbf{f}(\mathbf{x}) = \begin{cases} \mathbf{v} \\ \Psi \mathbf{q} + \Phi \mathbf{v} + \Lambda \mathbf{q}_a + \Omega \mathbf{f}_{nl} \\ \mathbf{E}_1 \mathbf{q} + \mathbf{E}_2 \mathbf{v} + \mathbf{F} \mathbf{q}_a \end{cases}, \qquad \mathbf{g}(\mathbf{x}) = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Xi} \\ \mathbf{0} \end{bmatrix}, \qquad (3)$$

where

$$\Psi := -\mathbf{M}_{t}^{-1}\mathbf{K}_{t}, \quad \Phi := -\mathbf{M}_{t}^{-1}\mathbf{C}_{t}, \quad \Omega := -\mathbf{M}_{t}^{-1}, \Xi := \mathbf{M}_{t}^{-1}\mathbf{G}, \quad \Lambda := \mathbf{M}_{t}^{-1}\mathbf{D},$$
(4)

The nonlinear force vector arising from the piece-wise linearity, as illustrated in Fig. 2 above, is expressed as

$$\mathbf{f}_{nl} = -\lambda \Delta \mathbf{K}_{s} \mathbf{q} \quad \text{for } |\boldsymbol{\alpha}| \leq g_{\alpha}$$
$$\mathbf{f}_{nl} = -\lambda \Delta \mathbf{K}_{s} \mathbf{g}_{\alpha} \quad \text{for } |\boldsymbol{\alpha}| > g_{\alpha}, \ \boldsymbol{\alpha} > 0$$
$$\mathbf{f}_{nl} = \lambda \Delta \mathbf{K}_{s} \mathbf{g}_{\alpha} \quad \text{for } |\boldsymbol{\alpha}| > g_{\alpha}, \ \boldsymbol{\alpha} < 0$$
(5)

where

$$\Delta \mathbf{K}_{s} = K_{\alpha} \mathbf{e}_{2} \mathbf{e}_{2}^{T}, \qquad \mathbf{g}_{\alpha} = \left\{ 0 \quad g_{\alpha} \quad 0 \right\}^{T}, \qquad K_{\alpha_{\left(|\alpha| \le s_{\alpha} \right)}} = \left(1 - \lambda \right) K_{\alpha} \tag{6}$$

and where \mathbf{e}_2 is the second column of a 3×3 identity matrix. The feedback linearisation method requires that the outputs are continuously differentiable, and therefore smooth. The non-smooth nature of the nonlinearity would result in non-smooth – but continuous – forces/accelerations. However, the resulting changes in the system states (both displacement and velocity) will be smooth, as they are obtained as time-integrals of the accelerations (which are continuous, albeit non-smooth). Thus, all the states of the system are continuously differentiable, satisfying the condition for feedback linearisability.

3.1 Plunge output linearisation

The classical input-output linearisation approach (Isidori 1995, Khalil 2002) is now followed to apply feedback linearisation by controlling the plunge displacement. The co-ordinates of the linear system are obtained as

$$z_1 = y = x_1, \qquad z_2 = \dot{y} = \dot{x}_1 = x_4$$
 (7)

using equation (1). Here, the output y is chosen as the plunge displacement $\xi = x_1$. The partially linearised system may then be obtained as

$$\begin{cases} \dot{z}_1 \\ \dot{z}_2 \end{cases} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{cases} z_1 \\ z_2 \end{cases} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v, \quad v = \mathbf{f} \left(\mathbf{x} \right)_{(4)} + \xi_1 u, \quad \mathbf{\Xi} = \left\{ \xi_1 \quad \xi_2 \quad \xi_3 \right\}^T,$$
(8)

with v being an artificial input associated with the linearised system. Since there remains an un-linearised set of 6 states, it is necessary to examine the zero-dynamics to ensure their stability when designing a controller. Expressions for the remaining linear co-ordinates are first required to complete the transformation. These are chosen as

$$z_{3} = x_{2}, \qquad z_{4} = x_{3} + \xi_{2}x_{4} - \xi_{1}x_{5}, \qquad z_{5} = \xi_{3}x_{4} - \xi_{1}x_{6}, z_{6} = \xi_{3}x_{5} - \xi_{2}x_{6}, \qquad z_{7} = x_{7}, \qquad z_{8} = x_{8},$$
(9)

completing the 8×8 transformation from nonlinear to linear co-ordinates as

$$\mathbf{z} = \mathbf{T}_{\mathbf{z}\mathbf{x}}\mathbf{x} \,. \tag{10}$$

The resulting zero-dynamics are found as

$$\dot{\mathbf{z}}_{(3:8)_{ZD}} = \begin{bmatrix} \mathbf{P}_{1} \\ \mathbf{T}_{\mathbf{zx} \ (4:6,4:6)} \begin{bmatrix} \mathbf{\Psi} & \mathbf{\Phi} & \mathbf{\Lambda} \end{bmatrix} (\mathbf{T}_{\mathbf{zx}}^{-1})_{(1:8,3:8)} + \mathbf{P}_{2} \\ \begin{bmatrix} \mathbf{E}_{1} & \mathbf{E}_{2} & \mathbf{F}_{p} \end{bmatrix} (\mathbf{T}_{\mathbf{zx}}^{-1})_{(1:8,3:8)} \end{bmatrix} \mathbf{z}_{(3:8)_{ZD}} + \begin{bmatrix} \mathbf{0} \\ \mathbf{T}_{\mathbf{zx} \ (4:6,4:6)} \mathbf{\Omega} \mathbf{f}_{nl} (z_{3}) \\ \mathbf{0} \end{bmatrix}$$
(11)

where

The zero-dynamics are checked to verify stability of the internal dynamics of the partially linearised system. A stability investigation of the zero-dynamics yields that there exist 3 equilibrium points – one zero-equilibrium and two non-zero equilibria. The eigenvalues pertaining to the trivial equilibrium point are found to have negative real parts, viz.,

$$\begin{array}{rrrr} -4.31592 + 98.20627i & -0.84754 + 16.22586i & -37.91684 + 0.00000i \\ -4.31592 - 98.20627i & -0.84754 - 16.22586i & -3.79316 + 0.00000i \end{array}$$

demonstrating stability of this equilibrium point.

3.2 Linearised response with pole-placement implemented

A desired natural frequency $\omega_{n_{\xi}}$ and damping ratio $\zeta_{n_{\xi}}$ may be set for the controlled DOF ξ by choosing the artificial input as

$$v = -\omega_{n_{\xi}}^{2} z_{1} - 2\zeta_{n_{\xi}} \omega_{n_{\xi}} z_{2}.$$
 (13)

For this simulation, target values are chosen as $\omega_{n_{\xi}} = 1$ Hz, $\zeta_{n_{\xi}} = 0.1$. The resulting closed-loop response, for the same initial conditions as the open-loop case, is shown in Fig. 4.



Fig. 4 – Closed-loop response of system at *U**=2.0

It is evident from the first subplot that the target natural frequency of 1 Hz is achieved in the plunge motion, as expected. The pitch motion, confined to the internal dynamics settles down to the stable zero equilibrium, as seen in the middle plot. The flap motion, given by the final subplot, is plotted alongside the commanded input in Fig. 5, where the difference between the two is highlighted.



Fig. 5 – Comparing commanded and actual flap angles U*=2.0

Closer inspection of the input reveals non-smooth changes corresponding to the switching points between the two stiffness regimes in the pitch DOF. This is expected, as the input is designed to cancel the system dynamics which include the non-smooth nonlinear forcing terms. Since the dynamics of the actuator are accounted for in the model and consequently in the computation of the non-smooth input, there will be no degradation on closed-loop response during feedback linearisation, and exact pole placement will be achieved in the absence of nonlinearity parameter errors.

4 Adaptive feedback linearisation

In real situations, complete cancellation of the nonlinearity will not be achievable. This could be due to a variety of reasons such as inaccurate measurement of the nonlinearity, incorrect assumption of the form of the nonlinearity etc. Adaptive Feedback Linearisation is a method that may be used to guarantee asymptotic closed-loop

stability in the presence of a discrepancy between the actual nonlinearity parameters and those assumed in the design of the controller. The assumed nonlinearity parameters are updated at every time step according to an adaptive law, which has the effect of driving the closed-loop controlled responses to zero.

The previous numerical simulation is continued. The inclusion of uncertainty/error in the description of the piece-wise linear stiffness would ideally require a few nonlinearity parameters (to describe the inner and outer stiffness and the range of the inner stiffness), but in this work we assume symmetry, knowledge of the inner stiffness; only the stiffness parameter K_{α} is considered uncertain. Since the zero-dynamics have an asymptotically stable equilibrium, and the nonlinearity is linearly parameterisable, the conditions for Adaptive Feedback Linearisation are satisfied. A 40% error in K_{α} is now assumed. Thus, $K'_{\alpha} = 1.4 K_{\alpha}$. Commencing with a scalar quadratic Lyapunov function

$$V = \mathbf{z}_{(1,2)}^{T} \mathbf{P} \mathbf{z}_{(1,2)} + \tilde{K}_{\alpha}^{2}, \qquad \tilde{K}_{\alpha} = K_{\alpha} - K_{\alpha}', \quad \mathbf{P} > 0, \qquad (14)$$

it can be shown that a parameter update law

$$\dot{K}'_{\alpha} = \mathbf{r}^{T} \left(\mathbf{\Omega}_{(1,:)} \right)^{T} \underline{\mathbf{B}}_{l}^{T} \mathbf{P} \mathbf{z}_{(1,2)}$$
(15)

can be derived, which asymptotically drives the closed-loop controlled response to zero by ensuring that V is a decreasing function. Inclusion of this update law translates to an increase in the dimension of the state vector. In (15),

$$\mathbf{r}(\alpha) = -\mathbf{e}_{2}\mathbf{e}_{2}^{T}\mathbf{q} \quad \text{for } |\alpha| \le g_{\alpha}$$

$$\mathbf{r}(\alpha) = -\mathbf{e}_{2}\mathbf{e}_{2}^{T}\mathbf{g}_{\alpha} \quad \text{for } |\alpha| > g_{\alpha}, \alpha > 0 \qquad (16)$$

$$\mathbf{r}(\alpha) = \mathbf{e}_{2}\mathbf{e}_{2}^{T}\mathbf{g}_{\alpha} \quad \text{for } |\alpha| > g_{\alpha}, \alpha < 0$$

and $\mathbf{B}_{l} = \begin{bmatrix} 0 & 1 \end{bmatrix}^{T}$ is the input matrix of the partially linearised system (equation (8)). The entries in the arbitrary matrix **P** are chosen judiciously so as to ensure rapid convergence of K'_{α} . For the same initial conditions as before and the same pole-placement requirement from the exact linearisation case above, the close-loop responses for the structural DOFs are given in Fig. 6.



Fig. 6 - Closed-loop response of system with Adaptive Feedback Linearisation at U*=2.0

It can be seen that the closed-loop response is characterized by higher frequency harmonics as compared with the exact linearisation case. Furthermore, the response takes longer to settle, although it eventually decays to zero. The pitch response is again driven to the zero equilibrium. A noticeable difference between the controlled response in this case and in the case of exact feedback linearisation is that the poleplacement objective is not achieved here. This is expected, as the adaptive law does not take into account this objective, and merely guarantees the convergence of the response to the origin.

5 Conclusions

This work has presented the application of partial feedback linearisation on a dynamical system having a piece-wise linear structural stiffness nonlinearity. Although the nonlinear forces and the required inputs are non-smooth, the structural states themselves are smooth and continuously differentiable, thereby satisfying the requirements for feedback linearisability. The non-smooth nature of the inputs necessitates modelling of the actuator dynamics, so as to replicate the situation one would encounter in practice, namely that a real actuator is only capable of applying smooth inputs. Numerical simulation results from the 3-degree of freedom aeroelastic model demonstrate successful linearisation of the plunge response, whilst driving the uncontrolled pitch response to zero, as expected from the zero dynamics. The final section presents a simple case of nonlinearity parameter uncertainty and application of the associated adaptive algorithm during feedback linearisation; it is shown from numerical results that the system responses are successfully driven to zero.

Acknowledgements

This research has been funded by EPSRC grant EP/J004987/1 under the project entitled "Nonlinear Active Vibration Suppression in Aeroelasticity".

References

- Platanitis, G. and Strganac, T. W. (2004) Control of a nonlinear wing section using leading- and trailing-edge surfaces, *Journal of Guidance Control and Dynamics* 27(1): 52-58.
- Strganac, T., Ko, J. and Thompson, D. (2000) Identification and control of limit cycle oscillations in aeroelastic systems, *Journal of Guidance Control and Dynamics* **23**(6): 1127-1133.
- Ko, J., Strganac, T. W. and Kurdila, A. J. (1999) Adaptive Feedback Linearization for the Control of a Typical Wing Section with Structural Nonlinearity, *Nonlinear Dynamics* 18(3): 289-301.
- Jiffri, S., Paoletti, P., Cooper, J. E. and Mottershead, J. E. (in press) A tutorial introduction to feedback linearisation for nonlinear vibration problems, *Shock and Vibration*.
- Jiffri, S., Mottershead, J. E. and Cooper, J. E. (2013) Adaptive Feedback Linearisation and Control of a Flexible Aircraft Wing, *Topics in Modal Analysis, Volume 7, Proceedings of the 31st IMAC, A Conference on Structural Dynamics Orange County CA, USA*, Springer.
- Jiffri, S., Mottershead, J. E. and Cooper, J. E. (2013) Active Control of a Nonlinear Flexible Aircraft Wing, International Conference on Structural Engineering Dynamics, Sesimbra, Portugal.
- Jiffri, S., Mottershead, J. E. and Cooper, J. E. (2013) Nonlinear Control of a Flexible Aeroelastic System, International Forum on Aeroelasticity & Structural Dynamics, Bristol Marriott Royal Hotel, Bristol, UK.
- Recker, D. A., Kokotovic, P. V., Rhode, D. and Winkelman, J. (1991) Adaptive Nonlinear Control of Systems containing a Deadzone, Proceedings of the 30th IEEE Conference on Decision and Control, Vols 1-3, New York, IEEE: 2111-2115.
- Tao, G. and Kokotovic, P. V. (1997) Adaptive control of systems with unknown non-smooth nonlinearities, *International Journal of Adaptive Control and Signal Processing* **11**(1): 81-100.
- Ma, X. L. and Tao, G. (2000) Adaptive actuator compensation control with feedback linearization, *Ieee Transactions on Automatic Control* **45**(9): 1705-1710.

- Zheng, Y. F., Wen, C. Y. and Li, Z. G. (2013) Robust adaptive asymptotic tracking control of uncertain nonlinear systems subject to nonsmooth actuator nonlinearities, *International Journal of Adaptive Control and Signal Processing* 27(1-2): 108-121.
- Tao, G., Burkholder, J. O., Guo, J. X. and Ieee (2013) Adaptive Output Feedback Actuator Nonlinearity Compensation for Multivariable Systems, 2013 American Control Conference, New York, Ieee: 3356-3361.
- Edwards, J. W., Ashley, H. and Breakwell, J. V. (1979) Unsteady aerodynamic modeling for arbitrary motions, *AIAA Journal* **17**(4): 365-374.
- Conner, M. D., Tang, D. M., Dowell, E. H. and Virgin, L. N. (1997) Nonlinear behavior of a typical airfoil section with control surface freeplay: A numerical and experimental study, *Journal of Fluids and Structures* **11**(1): 89-109.
- Li, D., Guo, S. and Xiang, J. (2010) Aeroelastic dynamic response and control of an airfoil section with control surface nonlinearities, *Journal of Sound and Vibration* **329**(22): 4756-4771.
- Papatheou, E., Tantaroudas, N. D., Da Ronch, A., Badcock, K., Mottershead, J. E. and Cooper, J. E. (24-26 June, 2013) Active Control for Flutter Suppression: An Experimental Investigation, *International Forum on Aeroelasticity and Structural Dynamics (IFASD), Bristol, UK.*
- Theodorsen, T. (1935) General theory of aerodynamic instability and the mechanism of flutter, *NACA Report no.* 496: 413-433.
- Isidori, A. (1995) Nonlinear Control Systems, Berlin Heidelberg New York, Springer.

Khalil, H. K. (2002) Nonlinear Systems, Prentice Hall.