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# Long-term analysis of crown-pinned concrete-filled steel tubular arches

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## Abstract

Concrete-filled steel tubular (CFST) arches with crown-pins are widely used in engineering practice. Investigation on the long-term features of CFST arches is essential to the stability and serviceability of CFST arches. This paper presents a long-term analysis of crown-pinned CFST arches subjected to sustained central concentrated load, accounting for the coupling effects of creep and shrinkage of its concrete core and temperature change. The algebraically tractable age-adjusted effective modulus method is used to model the creep behaviour of the concrete core and the analytical solutions of time-dependent structural behaviour are obtained. It is found that the coupling effects cause significant increase of deformation and internal force, which may affect the local strength reserve and the routine service of the CFST arches.

**Keywords:** Crown-pinned arch, CFST, Creep, Shrinkage, Temperature

## Introduction

Among composite structures, concrete-filled steel tubular (CFST) cross-section is widely employed in engineering practice. As can be seen from the cross section in Fig. 1, the composite section is composed of two components including the steel tube and the concrete core, which facilitates the resistance of the structure to the external force.

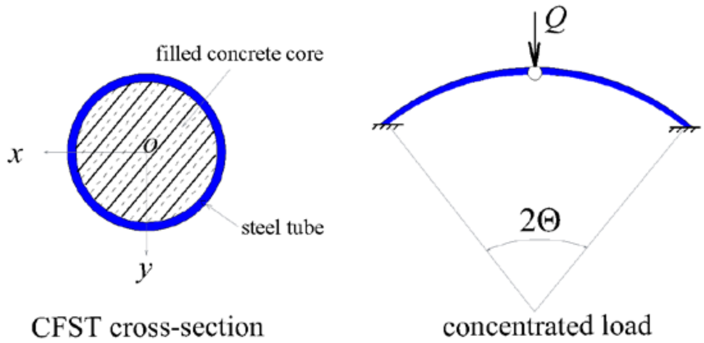


**Figure 1. CFST crown-pinned arch subjected to a central concentrated load**

As time goes on, the creep deformation increases when a CFST arch undergoes a sustained load, while the shrinkage strain emerges all the time even when there is no applied load. The visco-elastic effects of the concrete core exert significant effects on the structural behavior of CFST arches. Studies of the buckling behaviour have been focused on pin-ended and fixed arches [Pi et al. (2002); Bradford et al. (2002)]. In these studies, the arch is assumed to be continuous without any pins between its ends. It is known that in many cases, arches are built by joining two separate curvilinear segments together at the crown, thereby reducing the arch size to meet transport requirements and to create a pin at the arch crown. Because of the crown-pin, the structural responses and buckling behaviour of the arch are different from those of arches without the crown-pin. However, investigations on crown-pinned shallow circular arches do not appear to be reported in the open literature. This paper, therefore, presents an analytical analysis to investigate the non-linear long-term behaviour of crown-pinned CFST shallow circular arches subjected to a central concentrated load.

## Long-term effects of concrete

Due to the different Young's moduli of materials and the effects of creep and shrinkage in the concrete core, the stress $\sigma_s$ in the steel tube and the stress $\sigma_c$ in the concrete core are quite different with each other.

The stress in the steel tube can be expressed as

$$\sigma_s = E_s \varepsilon \tag{1}$$

in which $E_s$ is Young's modulus of steel, while the stress in the concrete core can be obtained based on the model proposed by ACI Committee-209 and the Australian design code for concrete structures AS3600 as

$$\sigma_c = E_{ec}\left(\varepsilon + \varepsilon_{sh} + \varepsilon_T\right) \tag{2}$$

in which the age-adjusted effective modulus of concrete $E_{ec}$ is given by [Gilbert (2010), Bazant (1972)]

$$E_{ec} = \frac{E_c}{1+\chi(t,t_0)\phi(t,t_0)} \tag{3}$$

in which $\phi(t,t_0)$ is the creep coefficient and $\chi(t,t_0)$ is the aging coefficient and they are given by

$$\phi(t,t_0) = \left[\frac{(t-t_0)^{0.6}}{10+(t-t_0)^{0.6}}\right]\phi_u \quad \text{and} \quad \chi(t,t_0) = 1 - \frac{(1-\chi^*)(t-t_0)}{20+(t-t_0)} \tag{4}$$

respectively, where $\phi_u$ is the final creep coefficient (the value when $t \to \infty$) and given by

$$\phi_u = 1.25 t_0^{-0.118}\phi_{\infty,7} \quad \text{and} \quad \chi^* = \frac{k_1 t_0}{k_2 + t_0} \tag{5}$$

With $k_1 = 0.78 + 0.4e^{-1.33\phi_{\infty,7}}$ and $k_2 = 0.16 + 0.8e^{-1.33\phi_{\infty,7}}$.

$\varepsilon_{sh}$ is the shrinkage strain of the concrete and can be expressed by ACI Committee-209 and Australian design code for concrete structures AS3600 as

$$\varepsilon_{sh} = \left(\frac{t}{t+d}\right)\varepsilon_{sh}^* \tag{6}$$

in which $t$ is the time in days, because the egress of the moisture in the concrete core is prevented by the steel tube, $d = 35$ days for moist curing can be used for the concrete core of CFST members, and $\varepsilon_{sh}^*$ is the final shrinkage strain (the value when $t \to \infty$). Although experimental studies of the shrinkage strain $\varepsilon_{sh}$ and creep coefficient $\phi(t,t_0)$ of CFST columns have been reported by several researchers [Zhong (1994); Terrey (1994); Uy (2001); Han (2004)] and the empirical values for the final shrinkage strain $\varepsilon_{sh}^*$ and the final creep coefficient $\phi_u$ of CFST columns were proposed, these values for the CFST member cannot be used directly for the time-dependent analysis of CFST members. Instead, they can be derived from the creep and shrinkage test results of CFST columns. From the creep and shrinkage test results [Uy (2001)], the shrinkage strain and creep coefficient at time $t$ = 140 days can be derived. Based on the derived data, the empirical value of the final

shrinkage strain $\varepsilon_{sh}^{*} = 340 \times 10^{-6}$ (the value when $t \to \infty$) and the creep coefficient $\phi_{\infty,7} = 2.5$ are used in the following investigation.
$\varepsilon_T$ is the thermal strain resulted by temperature change in concrete core and can be expressed as

$$\varepsilon_T = \alpha_c \left(T - T_0\right) \tag{7}$$

where $\alpha_c$ is the coefficient of thermal expansion, $T_0$ is the reference temperature and $\Delta T = T - T_0$ indicates the temperature change in the ambient environment.

**Non-linear in-plane equilibrium**

Assumptions adopted in this investigation are: 1. The Euler-Bernoulli hypothesis is applied so that the plain remains plane when it rotates about the neutral axis. 2. The dimension of the cross-section is much smaller than the length and radius of the arch to ensure sufficient slenderness. Because the arch and load system is symmetric, equilibrium of a half arch ($0 \le \theta \le \Theta$ and $\Theta$ is half of the included angle of the arch) is considered (Fig. 2). Based on the assumptions, differential equations of equilibrium for a crown-pinned circular arch can be derived from the principle of virtual work as

$$N' = 0 \quad \text{and} \quad \frac{\tilde{v}^{iv}}{\mu^2} + \tilde{v}'' = -1 \tag{8}$$

for the arches that are subjected to a central concentrated load $Q$ [Pi et al. (2008)], where $(\ )' \equiv d(\ )/d\theta$, $\theta$ denotes the angular coordinates, $\tilde{v} = v/R$ and $\tilde{w} = w/R$, $v$ and $w$ are the radial and axial displacements with $R$ being the radius of the arch, $\mu$ is the dimensionless axial force parameter defined by $\mu = NR^2/EI$ with $E$ being Young's modulus and $I$ being the second moment of area of the cross-section, and the axial compressive force $N$ is defined by

$$N = -\left(A_s E_s + A_c E_{ec}\right)\left[\tilde{w}' - \tilde{v} + \frac{1}{2}\left(\tilde{v}'\right)^2\right] - A_c E_{ec}\left(\varepsilon_{sh} + \varepsilon_T\right) \tag{9}$$

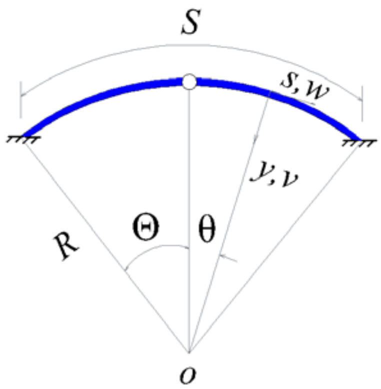


**Figure 2. Arch geometry**

The boundary conditions can also be derived as

$$\tilde{v}'' = 0 \quad \text{at} \quad \theta = 0 \text{ and } \tilde{w} = 0 \quad \tilde{v} = 0 \quad \text{at} \quad \theta = \Theta \tag{10}$$

Solving the equations given by Eqs. (8) and (9) simultaneously will lead to the solution of the radial displacement $\tilde{v}$ for crown-pinned arches subjected to a central concentrated load, and leads to the non-linear equilibrium equation between the internal force parameter $\mu$ and external force $Q$ as

$$A_1\tilde{Q}^2 + A_2\tilde{Q} + A_3 = 0 \tag{11}$$

where $\tilde{Q}$ is the dimensionless load defined by $\tilde{Q} = R^2\Theta Q / 2EI$ , and the expressions for coefficients for $A_1$, $A_2$, $A_3$ can be obtained accordingly .

## Long-term analysis

The long-term deformation and internal force of a crown-pinned CFST shallow arch are illustrated in the following figures including three scenarios accounting for different long-term effects. In the example shown below, the crown-pinned arch is fixed at two ends. The geometry is given as follows: the span is $L = 15$ m, the rise-to-span ratio is $f / L = 1/6$ and the thermal expansion coefficient of concrete is assumed as $\alpha_c = 340\times10^{-6}$ with the temperature change being $\Delta T = -20^{\circ} C$ . The dimension of the cross-section is assumed that the outer radius of the steel tube is $r_0 = 250$ mm and the inner radius of the concrete core is $r_i = 240$ mm.

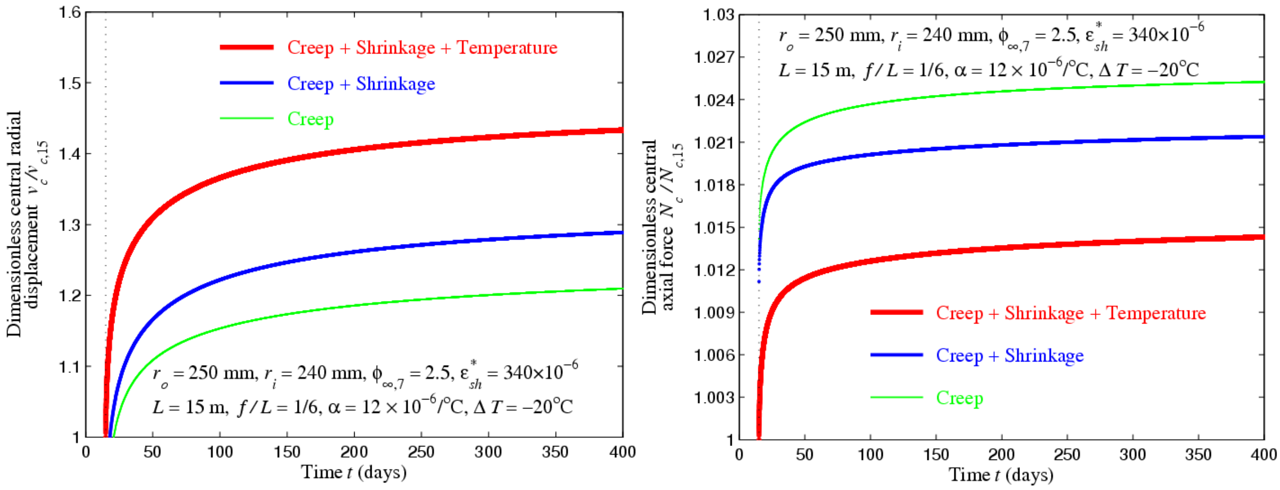


**Figure 3. Long-term deformation and internal force**

In Fig. 3, the central radial displacement of the arch is shown with respect to the increased time. $v_{c,15}$ and $N_{c,15}$ represents the central radial displacement and central axial force at time $t = 15$ days, due to the coupling effects of creep, shrinkage and temperature change. Among these three curves, one situation that all of these non-mechanical factors are combined together will lead to a drastic increase of displacement in the long-term. So the coupling effects of creep, shrinkage and temperature change influence the structural behaviour most significantly. In this case, the large deformation may exceed the maximum limit of amount referred in design practice. The routine serviceability cannot be assured in the long-term either. For a crown-pinned CFST shallow arch, the long-term internal axial force will also increase but only slightly in the life time. It can be seen that the change scope is the greatest when the coupling effects of creep, shrinkage and temperature change is accounted for, although the axial forces are initially different at the first loading time $t = 15$ days. The temperature change is assumed to be decreased in this example so the displacement is increased in radial direction, while the axial force is less than that of the other situations in which the temperature change is not considered.

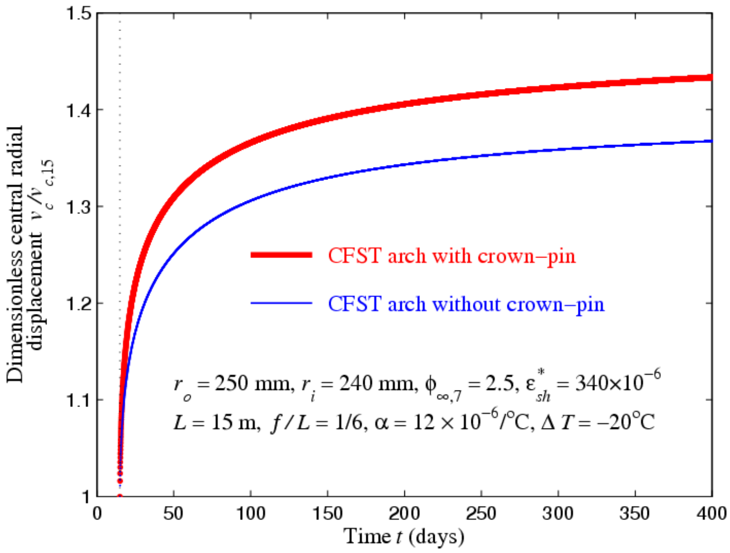


**Figure 5. Long-term displacement**

Following the same geometry and the other parameters of the arch above, the effects of the crown-pin can be also investigated by comparing the structural behaviour of two CFST arches: one of them is crown-pinned and the other one is continuous along the arch body. The boundary conditions are different when solving the differential equations of equilibrium. As can be found from Fig. 5, the comparison reveals an obvious gap of increasing rates between the crown-pinned CFST arch and the continuous CFST arch. The long-term central displacement of the CFST arch with the crown-pin is greater than that of the CFST arch without the crown-pin. The crown-pinned CFST arch will deflect within a broader range in the long-term and may become unsafe and cannot remain functional when it cannot withstand large deformation. The crown-pin acts as a degrader to the resisting ability of CFST arches to the external load in the long-term.

## Conclusions

This paper studied the non-linear long-term behaviour of crown-pinned shallow CFST arches subjected to central concentrated load. It was found that the coupling effects of the creep, shrinkage and temperature change in the concrete core influence the long-term response significantly and may affect the local strength reserve and the routine service of the CFST arches. Hence, these effects need to be accounted for in design practice of CFST arches. It was also shown that the crown-pin also plays an important role in the long-term response. As time increases, the change scopes of deformations and internal forces were found to be different from that of the continuous CFST arches. In design practice, all these factors should be considered to ensure a safe structure that can satisfy the strength requirement and also the normal serviceability.

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