Size and Scale Dependence of the Behavior of Truss Models: A Gradient Elasticity Approach

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The effect of size and scale dependence on the behavior of truss models is investigated, by introducing higher order strain gradient terms and an internal length parameter (the gradient elasticity coefficient) into the constitutive stress-strain relation of the bar element of the truss model. Instead of an algebraic equation (1D classical elasticity stress-strain relation), a differential equation governs the response of the elastic bar element and extra boundary conditions are required at the nodes. The displacement fields are obtained by deriving the associated stiffness matrix directly from the governing differential equation of the gradient elastic bar element.
Moreover by establishing a ratio between the micro-scale internal length and the macro-scale length of the bar elements the effect of changes in the truss bar element size and the microstructure internal length of the bar element material is revealed. A numerical example is presented as illustration.

Keywords: Size and Scale dependence, Gradient elastic coefficient, Gradient elastic truss bar, Gradient elasticity stiffness matrix

1. Introduction

In developing new engineering structures the analysis of their mechanical properties and behavior is often based on qualitative analysis carried out to enable a reasonable evaluation of the feasibility before embarking on any elaborate expensive research. In this process, computational analysis and simulation play a major role in developing these new structures.

Computationally, in order to study the nonlinear mechanical behavior of materials, a first choice is to use the standard Finite Element Method (FEM) with complex elements. While this method allows the computational stress analysis of a continuum with any boundary conditions and any loading, several problems and complications arise mainly due to the inclusion of a nonlinear constitutive law when updating the stiffness matrix of the finite element Argyris (1978, 1981, and 1984). However, from a geometrical standpoint the simplest finite elements are one-dimensional or a line element which is the two-node bar element. One-dimensional models can be very accurate and very cost effective in the proper applications. Hence, in search of a less complicated and efficient computational tool for testing constitutive equations, a truss model can be used for the linear and nonlinear analysis of a continuum, since the bars of the truss are the simplest possible finite elements.

In the nonlinear analysis of concrete, truss models have been used in Bazant et al. (1990) and Bazant (1997) and in Goel et al (1997) it was used for the analysis of steel structures. Akintayo et al. (1998, 2000), and Papadopoulos & Xenidis (1998, 1999) studied the response of concrete computationally using the plane truss model by considering coarse truss structures. Kiousis et al. (2010) using the model of Papadopoulos & Xenidis (1999) also studied concrete columns in compression. A random particulate model for fracture of aggregate and fiber composites was used by Bazant et al. (1990) assuming the particles to be elastic and having only axial interaction as in a truss. In Salem (2004), a fine micro truss model was proposed for the analysis of reinforced concrete using isotropic truss members by a generalization of the strut and tie model. Nagarajan et al. (2010) studied the mesoscopic numerical analysis of reinforced concrete using a modification of the micro truss model of Salem (2004). Hence from these works it becomes apparent that the size of the truss bar element to be used could directly be related to the scale of interest and detailed analysis required in any simulation which may have an adverse effect on the result.

In the investigation of new materials and structure, the ultimate goal of the engineer is to be able to obtain qualitative and reasonably quantitative analytic results. In order to achieve this, simple but effective models need be adopted to accommodate the micro deformation mechanisms of the structure. However, with the truss model increasing the fineness of the truss bars usually increases the computational cost. Therefore the effect of the size of the truss bar element and its relation to the scale of interest become a vital consideration in the simulation and analysis process.

The interpretation of size and scale effects can be approached in different ways. Several theoretical models have been developed to interpret size effects such as the strain gradient theories of Aifantis, 1999a and 1999b, Gao et al. 1997 and Fleck and Hutchinson 1997 with the formulation of the latter in Fleck and Hutchinson 2001. Other works include models with dislocation confined in thin films (Freund 1987, Thompson 1993 and Nix 1998); Theories on discrete dislocation dynamics include the works of Zbib and Aifantis 2003 and Needleman and Van der Giessen 2003. Atkins, 1999 and Bazant 1999 presented their study on fracture mechanics theories (especially for concrete) and statistical models was initially proposed by Irwin 1964 and later by Liu and Zenner 1995 and Seifried, 2004.

The theory of gradient elasticity is a simple approach to include microstructure deformation in the analysis of a material/structure, since it becomes particularly useful for small volumes, where the internal length introduced by the gradient coefficients is comparable to the characteristic dimension of the system. Mindlin (1964) showed that by isolating a typical unit cell element from the grid of say a crystal lattice (local representative volume), the modeling of a continuum with micro deformation can be developed. Ben-Amoz (1976) by assuming a particulate composite material as consisting of the matrix and inclusion (unit cell) also showed it is possible to classify composite media by the degree of inhomogeneity with the ratio of the length of the local representation and the length of the unit cell. Hence a relation is established between the deformation within the local representative volume and the unit cell by using higher-order strain/stress gradients to represent the micro deformation within the macro structure/material. In line with this same concept, but in a different manner, the theory of gradient elasticity proposed by Aifantis (1984) included the higher order strain gradient directly into the constitutive relations and introduced an internal length parameter, which relates to the micro unit of the material. A recent review of this theory is given in Askes and Aifantis [2011].

In recent years this theory has gained more increasing interest amongst researchers and the engineering community due to its ability to provide additional information, which the classical elasticity theory is incapable of providing. The failure of the classical elasticity theory to include higher order strain gradient contributions can lead to underestimates of stresses and inadequacy in capturing any scale and size dependent behavior in small-scale structures: since classical elasticity theory possesses no characteristic length (i.e. material parameter with internal length scale), which consider the interaction between macro and micro length scales in the constitutive response and the corresponding interpretation of associated scale and size effect.

Many authors have studied the gradient theories using various computational methods. Amongst others include: In the framework of gradient plasticity Pamin and de Borst (1998) used the finite element method to simulate the crack spacing problem with a reinforced concrete model. Chang, et al. (2002) applied higher-order strain/higher order stress gradient models derived from a discrete microstructure to fracture and related their constitutive equations to that of Aifantis (1984). By using the gradient elasticity theory of Aifantis (1984), Dessouky et al. (2003) presented a finite element model for the microstructure analysis of asphaltic materials. In a similar manner, Akarapu and Zbib (2006) also considered the analysis of plane cracks in elastic materials using the finite element methods.

Motivated by this and using the constitutive stress-strain relation of the gradient elasticity model of Aifantis [1984], the gradient truss model was first studied in Akintayo (2011) and later presented in Akintayo et al. (2012). Subsequently, a more detailed study was presented in Akintayo 2014, in which different boundary conditions were imposed at the support of the bar element and the corresponding force-displacement relations were derived for a robust application in the proposed gradient truss model. It is shown that the gradient elastic bar element is able to support strain gradient along its length such that simulation of the micro-scale deformation is included and a means of relating the macrostructure bar length to the microstructure internal length is established.

In this paper based on these findings, in a simple manner by using the gradient bar element stiffness expression and by considering the ratio between the size of the truss bar and the internal length of the material being simulated, the effect of changes in this ratio on the simulation is investigated. As an illustration, the gradient enhanced bar element is used to simulate a simple truss structure. Subsequently, by considering different bar length to internal length ratios, the response of the truss structure to scale and size dependence behavior can be examined.

Brief Review of the Classical Elasticity Bar Element and its Local Stiffness Matrix

Consider the generic truss element shown in Figure 1(a). The force and displacement components are linked by the element stiffness relations

$$\bar{\mathbf{f}} = \bar{\mathbf{K}}\bar{\mathbf{u}} \tag{1}$$

which written out in full is

$$\begin{bmatrix} \bar{f}_{xi} \\ \bar{f}_{yi} \\ \bar{f}_{xj} \\ \bar{f}_{yj} \end{bmatrix} = \begin{bmatrix} \bar{K}_{xixi} & \bar{K}_{xiyi} & \bar{K}_{xixj} & \bar{K}_{xiyj} \\ \bar{K}_{yixi} & \bar{K}_{yiyi} & \bar{K}_{yixj} & \bar{K}_{yiyj} \\ \bar{K}_{xjxi} & \bar{K}_{xjyi} & \bar{K}_{xjxj} & \bar{K}_{xjyj} \\ \bar{K}_{yjxi} & \bar{K}_{yjyi} & \bar{K}_{yjxj} & \bar{K}_{yjyj} \end{bmatrix} \begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{yi} \\ \bar{u}_{xj} \\ \bar{u}_{yj} \end{bmatrix} \tag{2}$$

There are several ways to construct the stiffness matrix $\bar{\mathbf{K}}$ in terms of the bar length l_o, modulus of elasticity E and bar cross-sectional area A. The most straightforward technique is the unit displacement method.
By viewing the truss element in Figure 1(a) as a spring in Figure 1(b), we can set the element stiffness $k_s = \bar{K}_{ijij}$, with

$$k_s = \frac{AE}{l_o} \tag{3}$$

Consequently the force-displacement equation is

$$F = \frac{AE\left(u_j - u_i\right)}{l_o} = k_s d = \frac{AE}{l_o} d \tag{4}$$

where F is the internal axial force and d the relative axial displacement, which physically denote the bar elongation.

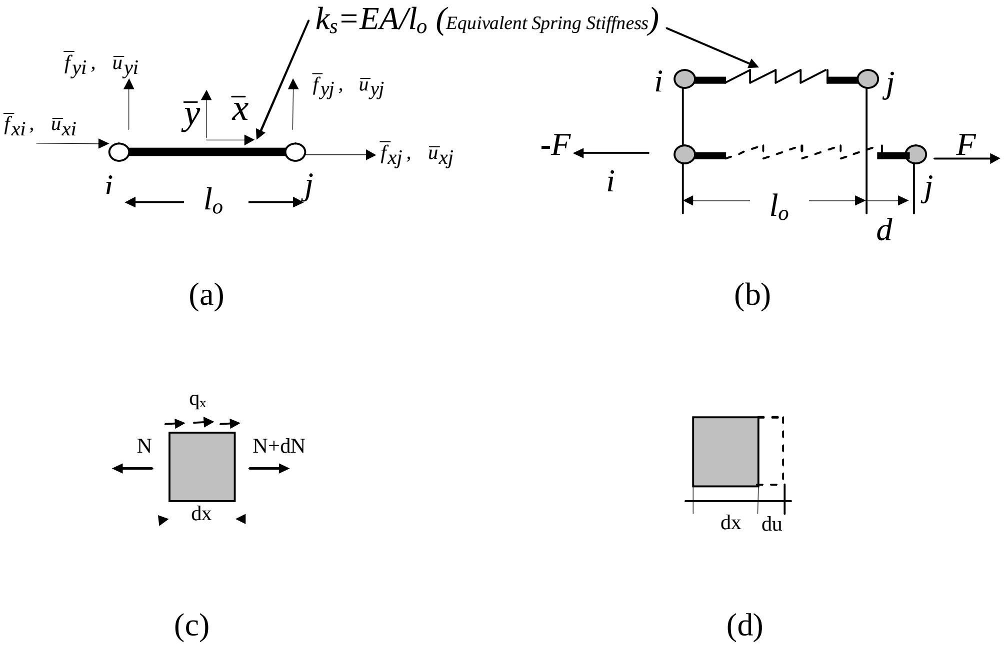


Fig.1.(a). Generic 2-node truss bar element referred to its local coordinate system $\{\bar{x},\bar{y}\}$ with nodal forces f_{ij} and displacements u_{ij}. (b) Interpretation of a Generic truss element as equivalent spring (c) Equilibrium for infinitesimally small truss element. (d) Kinematics for infinitesimally small truss element.

By assuming the displacement is of equal magnitude and direction at each node as well as within the element (i.e. constant along the bar), the strain takes the form

$$\varepsilon = \frac{du}{dx} = \frac{\Delta L}{l_o} = \frac{\left(u_j - u_i\right)}{l_o} \tag{5}$$

and on the basis of the one dimensional Hooke's law the stress-strain relation is

$$\sigma = E\varepsilon \tag{6}$$

where σ is the stress, ε is the strain and E is the Elastic or Young Modulus.

The elastic 2-node bar element of Fig. 1a is prismatic, weightless, and isotropic; the Poisson's effect is not considered and the axial load is applied at the centroid. Then equilibrium in the x-direction for the infinitesimally small length of the truss bar element, shown in Fig 1c. gives

$$q_x = -\frac{dN}{dx} \tag{7}$$

The normal constant stress σ of a one-dimensional truss is the force F applied on the truss per unit cross-sectional area;

$$\sigma = \frac{F}{A} = E\varepsilon \tag{8}$$

By considering the infinitesimal element Fig. 1c and denoting the infinitesimal elongation, by *du*, the relationship between the strain and the displacement is obtained as

$$\varepsilon = \frac{du}{dx} \tag{9}$$

and the governing differential equation for truss members reads:

$$q_x = -EA\frac{d^2u}{dx^2} \tag{10}$$

which in terms of the force applied per unit cross-sectional area can be rewritten as

$$N = EA\frac{du}{dx} \tag{11}$$

The element stiffness is given by Eq. 3 and since equilibrium suggests that $f_i = -f_j$, hence the force-displacement relation for the bar is given by

$$\underbrace{\begin{Bmatrix} f_i \\ f_j \end{Bmatrix}}_{\boldsymbol{f_e}} = \underbrace{\frac{AE}{l_o}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{\boldsymbol{K_e}} \underbrace{\begin{Bmatrix} u_i \\ u_j \end{Bmatrix}}_{\boldsymbol{u_e}} \tag{12}$$

where $\mathbf{f_e}$, $\mathbf{K_e}$ and $\mathbf{u_e}$ are the force, stiffness and displacement matrices respectively.
However, in plane classical elasticity structural analysis, the simple local stiffness matrix of an isolated bar, with respect to reference axes (x,y), is usually split into the elastic or material stiffness $\boldsymbol{k_e}$ and the geometric stiffness $\boldsymbol{k_g}$ and is written as

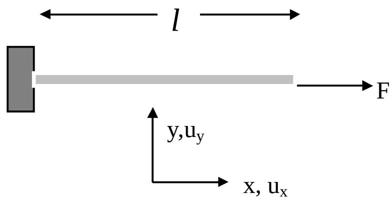
$$\begin{aligned} \boldsymbol{k}_\ell &= \boldsymbol{k}_e + \boldsymbol{k}_g \\ &= \frac{EA}{l_o}\begin{pmatrix} c_x^2 & c_x c_y \\ c_x c_y & c_y^2 \end{pmatrix} + \frac{N}{l}\begin{pmatrix} c_y^2 & -c_x c_y \\ -c_x c_y & c_x^2 \end{pmatrix} \end{aligned} \tag{13}$$

Where (l_o , l) are the undeformed and deformed length of the bar, and (c_x , c_y) are direction cosines of the bar.

Gradient Elastic Bar

Consider an elastic bar length l_o shown in Fig. 2, with modulus of elasticity E, and a cross-sectional area of A, fixed at one end and subject to an axial tensile force F at the right end $x = l_o$. The one-dimensional gradient elasticity stress-strain constitutive equation presented in Aifantis 1984 and used in Altan & Aifantis 1997, Akintayo 2011, Akintayo et al. 2012 is given by

$$\sigma_x = E\left(\varepsilon_x - c\frac{d^2\varepsilon_x}{dx^2}\right) \tag{14}$$



`Fig.2. A bar under axial tensile load F

where σ_x is the stress and $\varepsilon_x = du/dx$ is the strain, while c is the gradient elastic coefficient ($c \equiv \ell^2$ i.e. a microstructure-dependent internal length).
From the stress-strain relation of Eq. 14, in relation to the force applied per unit cross-sectional area of Eq. 8, the following is obtained:

$$EA\left(\frac{du}{dx} - c\frac{d^3u}{dx^3}\right) = F \tag{15}$$

and from the equilibrium condition of Eq. 11 the corresponding governing differential equation for the gradient elastic case takes the form

$$EA\left(\frac{d^2u}{dx^2} - c\frac{d^4u}{dx^4}\right) + q_x = 0 \tag{16}$$

The solution of Eq. 16 is easily obtained as

$$u = C_1 + C_2 x + C_3\left(\cosh(\frac{x}{\sqrt{c}}) + \sinh(\frac{x}{\sqrt{c}})\right) + C_4\left(\cosh(\frac{x}{\sqrt{c}}) - \sinh(\frac{x}{\sqrt{c}})\right) \tag{17}$$

where C_1, C_2, C_3 and C_4 are constants to be determined from the boundary conditions.

Boundary Conditions and Determination of Constants

In order to solve for the constants C_1, C_2, C_3 and C_4, classical and extra non-classical boundary conditions need be determined. The following classical boundary condition is used

$$u\big|_{x=l_o} = 0\,; \qquad EA\left(\frac{du}{dx}\right)_{x=l_o} = F \tag{18}$$

A general discussion of the extra boundary conditions for gradient elastic bar was provided in Akintayo (2011, 2012a). Hence this is not recapitulated, or expanded upon here, as it is out of the scope of the present paper. We proceed instead, with the consideration of the following simple case where zero strain is imposed at the free end and zero strain gradient at the fixed end given below

$$u\big|_{x=0}=0; \qquad EA\left(\frac{du}{dx}-c\frac{d^3u}{dx^3}\right)_{x=l_o}=F; \qquad \frac{du}{dx}\bigg|_{x=l_o}=0; \qquad \frac{d^2u}{dx^2}\bigg|_{x=0}=0 \tag{19}$$

Using these boundary conditions, the constants C_1-C_4 are determined as follows:

$$C_1=0; \qquad C_2=\frac{F}{EA}; \qquad C_3=-C_4=-\frac{Fe^{\frac{l_o}{\sqrt{c}}}\sqrt{c}}{\left(1+e^{\frac{2l_o}{\sqrt{c}}}\right)EA} \tag{20}$$

Displacement, Strain and Axial Force

With the obtained constants the displacement and axial force within the assumed gradient elasticity framework are obtained as

$$u=\frac{Fx}{EA}-\frac{F\sqrt{c}}{EA}\left(\sinh\left(\frac{x}{\sqrt{c}}\right)\operatorname{sech}\left(\frac{l_o}{\sqrt{c}}\right)\right); \quad \varepsilon^g=\frac{F}{EA}-\frac{F}{EA}\left(\cosh\left(\frac{x}{\sqrt{c}}\right)\operatorname{sech}\left(\frac{l_o}{\sqrt{c}}\right)\right); \quad F=\frac{EAu}{x-\sqrt{c}\left(\sinh\left(\frac{x}{\sqrt{c}}\right)\operatorname{sech}\left(\frac{l_o}{\sqrt{c}}\right)\right)} \tag{21}$$

It is easily seen that the axial displacement, strain and force expressions contain the classical term and the gradient elastic contribution (the micro-scale deformation) which includes the internal length √c. Hence, at the free end of a bar x = l_o, the following relations are obtained for the displacement and axial force:

$$u=\frac{Fl_o}{EA}-\frac{F\sqrt{c}}{EA}\left(\tanh\left(\frac{l_o}{\sqrt{c}}\right)\right); \qquad \varepsilon=0; \qquad F=\frac{EAu}{l_o-\sqrt{c}\tanh\left(\frac{l_o}{\sqrt{c}}\right)} \tag{22}$$

In Eq. (22) the classical displacement and strain values can be retrieved in the absence of the gradient elastic contribution.

Gradient Elastic Bar Element Stiffness

In this section, the gradient elastic bar element stiffness is derived. With this the simulation of a bar with a gradient enhanced bar element can be analysed. From Eq. 4 and Eq. 22, the corresponding gradient elastic bar stiffness is obtained as

$$k_i^g=\frac{EA}{l_o-\sqrt{c}\tanh\left(\frac{l_o}{\sqrt{c}}\right)} \tag{23}$$

It is readily seen that unlike the classical case for the gradient elasticity case the bar stiffness does not only depend on the conventional geometric and material properties of the bar, but also on the introduced micro-scale parameter √c of the underlying microstructure. From Eq. 23 in the absence of the gradient elastic contribution the classical bar element stiffness is retrieved. By considering the following arbitrary properties of the bar element: Area A = 1cm^2, Elastic Modulus E=100MPa; bar length l_o=10cm, the stiffness values for the classical case and the gradient elastic cases are given in Table 1 for different bar length and internal length ratios l_o /√c. The different bar stiffness are represented as k^c (classical elastic case) and k^g_1 - k^g_5 (gradient elastic cases) and are given in Table 1 for the changes in l_o/√c from 20 to 4.

Table 1. The following properties are used arbitrarily to obtain the bar stiffness k^c (classical elastic case) and k^g_1 - k^g_5 (gradient elastic cases): A = 1cm, E=100MPa and l_o=10cm

Cases	Bar Stiffness Values for $l_o/\sqrt{c}=20$	Bar Stiffness Values for $l_o/\sqrt{c}=10$	Bar Stiffness Values for $l_o/\sqrt{c}=6.7$	Bar Stiffness Values for $l_o/\sqrt{c}=5$	Bar Stiffness Values for $l_o/\sqrt{c}=4$
k^c	10	10	10	10	10
k^g_{1a}	10.526	11.111	11.765	12.4997	17.6259

This numerical example reveals that as the bar length to internal length ratio $l_o/\sqrt{c}$ changes from 20 to 4 i.e. the bar length becomes comparable to the internal length the simulation gives higher stiffness values. Moreover since the simulation values of the displacement at the nodes of a truss model depend on the stiffness of the bar, hence from Table 1, for any particular specimen size, bar elements sizes comparable to its material micro-scale characteristic length should give lower displacement values. Consequently, it is implied that the simulation result of a specimen of particular size for any scale of interest will be different for different bar element sizes used. Smaller bars will be stiffer than those with larger ones, hence finer truss models would give lower displacement values than coarse models. Thus it is required first that the specimen size and a particular reference scale of interest be identified in order to choose the appropriate bar length that will adequately simulate the material or structure.

Numerical Example

In this numerical example, the gradient truss model is applied to the truss structure of Fig 3 simply supported. The vertical and horizontal bar length l_o = 10cm and the diagonal bars length $l_o\sqrt{2}$ = $10\sqrt{2}$cm, and all bars have the same Young's modulus E =100 Mpa.

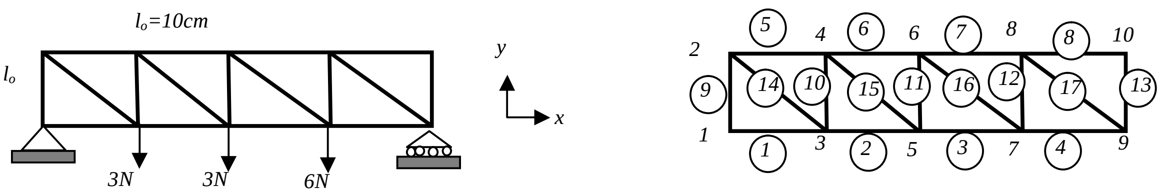


Fig. 3. (a). Loaded Seventeen element truss structure (b) Numbering of nodes and elements.

Nodal coordinate
{{0,0},{0,10},{10,0},{10,10},{20,0},{20,10},{30,0},{30,10},{40,0},{40,10}}

Applied node forces

{{0,0},{0,0},{0,-3},{0,0},{0,-3},{0,0},{0,-6},{0,0},{0,0},{0,0}}

Boundary Conditions
u = {0,0, 0,0, 0,-0.10, 0,-0.10, 0,-0.15, 0,-0.15, 0,-0.10, 0,-0.10, 0,0, 0,0}

For the gradient truss model the gradient elastic coefficient values considered are $\sqrt{c}$ = 0.5cm ($l_o/\sqrt{c}$ = 20); 1.0cm ($l_o/\sqrt{c}$ = 10); 1.5cm ($l_o/\sqrt{c}$ = 6.7); 2.0cm (($l_o/\sqrt{c}$ = 5); 2.5cm ($l_o/\sqrt{c}$ = 4), $\sqrt{c}$ = 3.0cm ($l_o/\sqrt{c}$ = 3.3); 3.5cm ($l_o/\sqrt{c}$ = 2.85); 4.0 cm ($l_o/\sqrt{c}$ = 2.5); 5.0cm (($l_o/\sqrt{c}$ =2); 6.0cm ($l_o/\sqrt{c}$ = 1.7).The cross-sectional area is taken as the same for all the bars: A = $1cm^2$. We proceed to obtain the element stiffness matrix in global coordinates.

By using the Direct Stiffness Method, a simple Mathematica program is used for this analysis. It comprises of three major processing stages: (1) the pre-processing, (2) processing, and (3) post processing.

The pre-processing stage is implemented by the driver program which puts the data structure in place by defining the model and directly setting the data structures.
The processing stage involves three major stages and the in-built Mathematica function *LinearSolve* is used: Firstly the master stiffness matrix is assembled with a subroutine element stiffness module; secondly, the master stiffness matrix and the node force vector are modified for the displacement boundary conditions; thirdly, the solution of the modified displacement equations is then obtained. Once these three processing stages are executed and the displacements made available the post processing stage follows.

At the post processing stage, through a **Ku** matrix multiplication, the forces are recovered to include the reactions. The internal (axial) forces in the truss elements are computed, and then the deflected shapes can be plotted.

Below the deflected shapes indicating the displacement are given for the classical and gradient elastic cases.

Classical Elastic Case

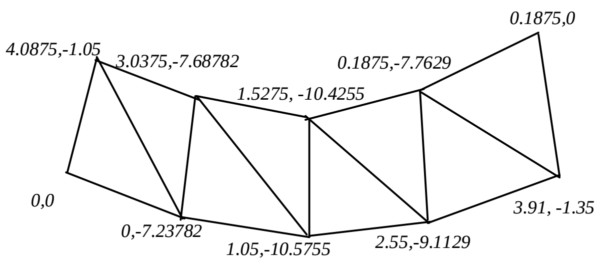


Fig. 4. Classical Elastic deformed configuration of truss structure.

Gradient Elastic Cases

For the truss structure of Fig. 3 the global stiffness matrices for Cases 1 – 10 and the deformation configurations are shown in Fig. 5.

Here also it can be observed that as the $l_o/\sqrt{c}$ ratio changes from 20 to 1.7 (i.e. the bar length becomes comparable to the internal length), the structure increases in stiffness significantly and the displacement values are much less than the classical ones.

Consequently, the truss model simulation result of a specimen of a particular size can be related to the particular scale of interest by identifying a ratio between a characteristic length scale of the specimen and the truss bar length to be used. This result is indicative that finer truss models of a particular specimen size would give lower displacement values than coarse models of the same specimen.

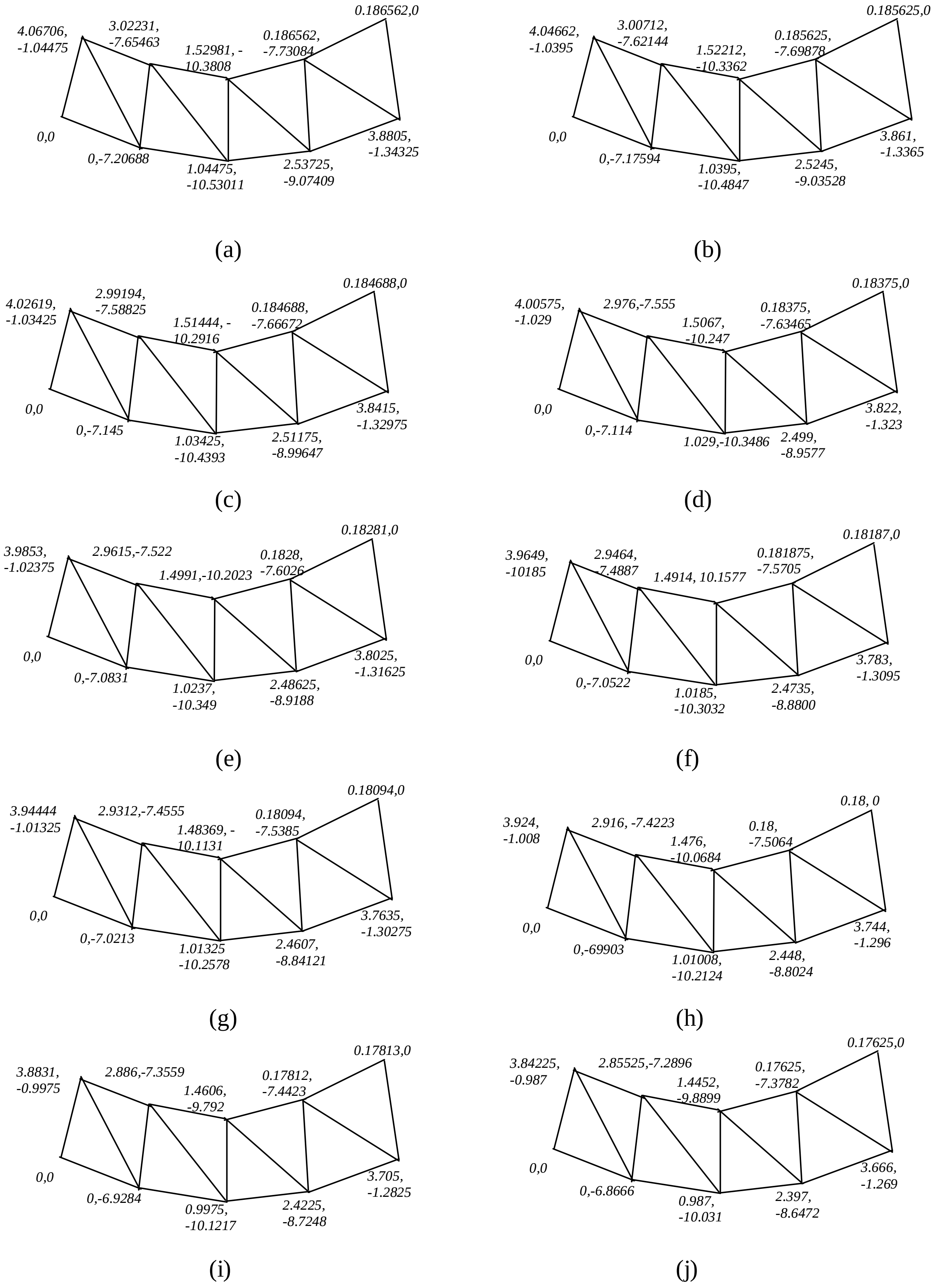


Fig. 5. Gradient Elastic deformed configuration of structure. (a). (l_o / √c = 20); (b). (l_o / √c = 10); (c). (l_o / √c = 6.7); (d). ((l_o / √c = 5; (e). (l_o / √c = 4); (f). (l_o / √c = 3.3); (g). l_o / √c = 2.85); (h). (l_o / √c = 2.5); (i). ((l_o / √c =2); (j). (l_o / √c = 1.7).

Conclusion

In this exploratory study size and scale dependent behaviour of truss models was examined using the gradient elasticity theory. With the introduction of a characteristic internal length parameter of a specimen at a defined scale, a ratio can be established with the truss bar element length.

It is shown that the simulation result of a truss structure of fixed size differ according to the ratio of the truss bar element length to the internal length of the material. The simulation results with the bar length comparable to the chosen internal length are shown to be stiffer. Hence smaller bars will be stiffer than larger ones and finer truss models comparable to the characteristic length scale will give lower displacement values at the nodes than coarser models.

Consequently, in order to adequately simulate a material or structure, a particular characteristic length scale of interest need be identified in relation to the specimen to be simulated and thus the appropriate corresponding truss bar element size can be identified and used.

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