Multi-scale Multiplicative Perturbation Average Method for FPU Equation

F. Wu, Q. Gao, and *W.X. Zhong

State Key Laboratory of Structural Analysis of Industrial Equipment,
Department of Engineering Mechanics, Dalian University of Technology, Dalian 116023, China

*Corresponding author: wxzhong@dlut.edu.cn

This paper is devoted to a numerical study of the classical nonlinear FPU system. The considered model consists of a chain of $2m$ mass points, connected with alternating soft nonlinear and stiff linear springs, and fixed at the end points. A method which combines the multiplicative perturbation method and average method is proposed. The FPU system is firstly approximated by using two independent approximate models. The first is a linear model with high frequency which can be solved analytically. The second is a nonlinear model with low frequency which can be solved with numerical method with a step size that are not small compared to the period of the linear model. Then the multiplicative perturbation method and the average method are used to the two models to produce a perturbation nonlinear system which can be solved with a big time step. The numerical tests show that the proposed method can overcome the difficulties of numerical resonance and stiff problem, and can display the details of the high-frequency vibration in the multiple-scale way.

Keywords: FPU, multiplicative perturbation, average, multi-scale, numerical resonance