SPH simulation of sound propagation and interference

Y.O. Zhang[1,2], *T. Zhang[1,2], H. Ouyang[3] and T.Y. Li[1,2]

[1]School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, Wuhan 430074, China.
[2]Hubei Key Laboratory of Naval Architecture and Ocean Engineering Hydrodynamics, Huazhong University of Science and Technology, Wuhan 430074, China
[3]School of Engineering, University of Liverpool, The Quadrangle, Liverpool L69 3GH, UK.

*Corresponding author: zhangt7666@mail.hust.edu.cn

Abstract

Complicated domain topologies and moving boundaries in acoustic simulation are difficult to be described with mesh based methods. On the contrary, the meshfree SPH (Smoothed Particle Hydrodynamics) method does not have much trouble in dealing with these problems. Therefore, the present paper aims to simulate sound propagation and interference in time domain with the SPH method by solving linearized acoustic wave equations. Firstly, linearized acoustic wave equations are represented in the form of particle approximation. After that, a standard SPH numerical method for simulating sound waves in time domain is built by adding the leapfrog integration and the nearest neighbor particle searching method. Finally, both one dimensional sound propagation and interference models are simulated with the SPH method and results are validated and compared with theoretical data. Numerical results show that the SPH method can simulate acoustic waves accurately.

Keywords: SPH, sound propagation, sound interference, acoustic wave, time domain

Introduction

Mesh-based methods are widely used in modeling acoustic waves and these methods include some classic numerical methods such as the Finite Element Method (FEM) [Ihlenburg (1998)] and the Boundary Element Method (BEM) [Kythe (1995)]. However, these method is not perfect in solving problems with moving or deformable boundaries or interfaces due to its mesh-based properties.

Meshfree methods can handle these problems with a set of arbitrarily distributed nodes instead of mesh and many methods have been used in solving acoustic problems. As a meshfree, Lagrangian method, the SPH method not only has almost all advantages that meshfree methods have, but it is also suitable for solving problems with large ranges of density and object separation as shown in recent reviews by Springel [Springel (2010)], Liu and Liu [Liu and Liu (2010)] and Monaghan [Monaghan (2012)] due to its Lagrangian property. Introducing this method to acoustic simulation would also bring its advantages to some specific fields like combustion noise, bubble acoustic, sound propagation in multiphase flow et al. Therefore, this paper focuses on the application of SPH in the simulation of acoustic waves.

The SPH method was first pioneered independently by Lucy [Lucy (1977)] and Gingold and Monaghan [Gingold and Monaghan (1977)] in 1977 to solve astrophysical problems. It computes with a set of particles which possess individual material properties. Owing to its properties that have mentioned, the SPH method has been used in the fields of astrophysics, structure deformation, fluid dynamics etc. [Springel (2010); Liu and Liu (2010); Monaghan (2012)]. However, no literature is found about solving linearized acoustic equations with SPH, except for few researches [Wolfe and

Semwal (2007); Hahn and Negrut (2009)] discussed solving fluid dynamic equations to obtain sound waves.

Solving the fluid dynamic equations can model acoustic problems, but it is not the only way. Since there are large differences in the length scale between the values and variations of velocity and density, solving the linearized acoustic equations requires lower computational resource compared with solving the fluid dynamic equations and it has been widely used in modeling engineering problems [Bruneau (2010)]. However, no literature was found to use the SPH method to solve the linearized acoustic equations.

The paper is organized as follows. In section 2, the linearized acoustic equations are solved with the standard SPH theory and then the acoustic modeling method is built by adding the time integration and neighbor particles searching method. In section 3 and 4, one-dimensional sound propagation and interference model are simulated with the standard SPH method and the results are validated and compared with the theoretical solution.

SPH Formulations of Sound Waves

The linearized continuity and momentum equations governing sound waves can be written as

$$\frac{\partial(\delta\rho)}{\partial t} = -\rho\,\nabla\cdot\boldsymbol{u} \tag{1}$$

$$\frac{\partial\boldsymbol{u}}{\partial t} = -\frac{1}{\rho}\nabla p \tag{2}$$

The linearized state equation for ideal air is

$$p = c_0^2\delta\rho \tag{3}$$

where $\delta\rho$ is the change of density, ρ is the density, u is the velocity vector, t is the time, p is the sound pressure, c0 is the sound speed. The particle approximation equation of the continuity of acoustic waves is written as

$$\frac{\partial(\delta\rho_i)}{\partial t} = (\rho_0+\delta\rho_i)\sum_{j=1}^{N}\frac{m_j}{(\rho_0+\delta\rho_j)}\boldsymbol{u}_{ij}\nabla_i W_{ij} \tag{4}$$

The momentum equation in SPH method is obtained as

$$\frac{\partial\boldsymbol{u}_i}{\partial t} = \sum_{j=1}^{N} m_j\left[\frac{p_i}{(\rho_0+\delta\rho_i)^2}+\frac{p_j}{(\rho_0+\delta\rho_j)^2}\right]\nabla_i W_{ij} \tag{5}$$

Particle approximation of the equation of state is

$$p_i = c_0^2\delta\rho_i \tag{6}$$

The second order leap-frog integration [Kelager (2006)] is used in the paper. All-pair search approach [Liu and Liu (2003)], as a direct and simple algorithm, is used to realize the neighbor particles searching in acoustic waves simulation.

Sound Propagation

Sound Propagation Model

A one-dimensional sound propagation in a pipe with uniform cross section is used. The sound propagation model is shown in Figure 1.

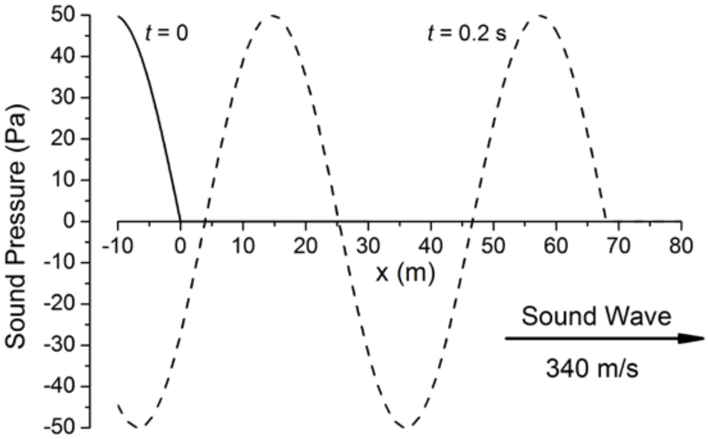


Figure 1 One-dimensional sound propagation model

The sound pressure of the acoustic wave transmitted in Figure 1 is

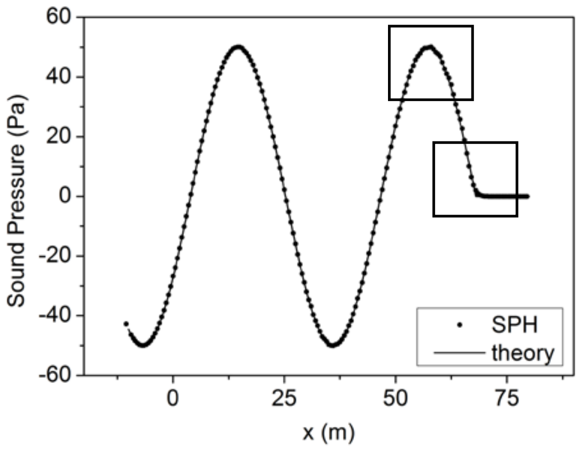
$$p(t, x < 0) = p_A \sin(wt - kx) \qquad ()$$

where t is the time (propagation starts when t = 0), x is the geometric position, pA is the amplitude of the acoustic wave (in this section, pA = 50 Pa), w is the circular frequency of wave (in this section, w = 50 rad/s), k = w/c0, the sound speed c0 is 340 m/s and the density of the propagation medium is 1.0 kg/m3.

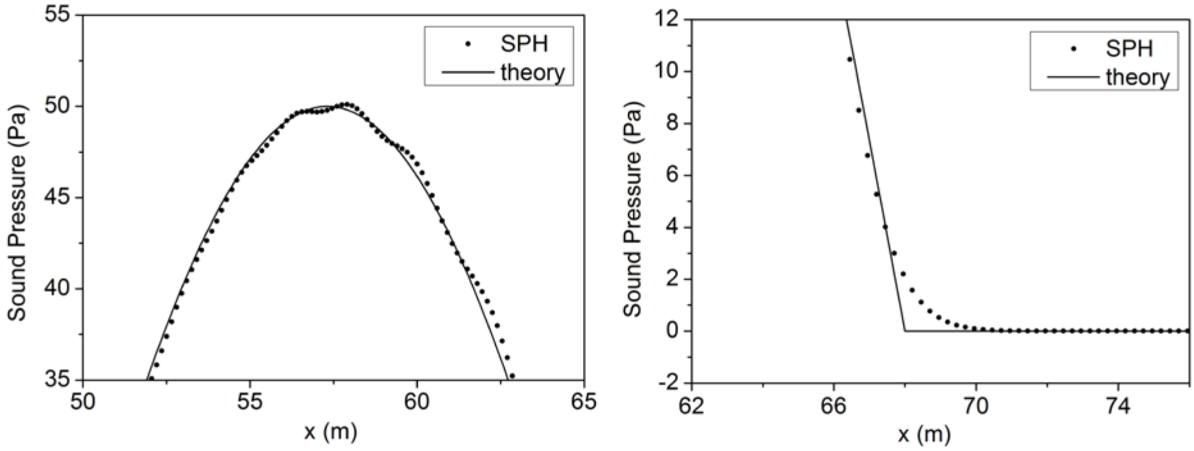
The sound propagates from x < 0 to x > 0 and the computational domain is from -10 m to 80 m. The simulation results at the time t = 0.2 s are used to compare with the theoretical resolution.

SPH Simulation

The simulation results of sound pressure at the time t = 0.2 s are shown in Figure 2 (a) while the theoretical solution is also plot in the figure. It can be seen from the line graph that there are two peaks appear in the propagation and one of them is shown in Figure 2 (b). At the same time, a detail view of the start of the sound is also given in Figure 2 (c).



(a) sound pressure at t = 0.2 s



(b) detail view 1 (c) detail view 2

Figure 2 Sound pressure comparison between the SPH and the theoretical results at t = 0.2 s

As can be seen from the figure, the SPH simulation results have almost the same trend compared with the theoretical solution. Values of the sound pressure can also be obtained accurately by using the SPH method. However, Figure 2 (b) and (c) show the effects of unphysical oscillations and it mainly appears at the place with large changes of sound pressure.

Sound Interference

Sound Interference Model

An interference model of two different sound waves is used as shown in Figure 3.

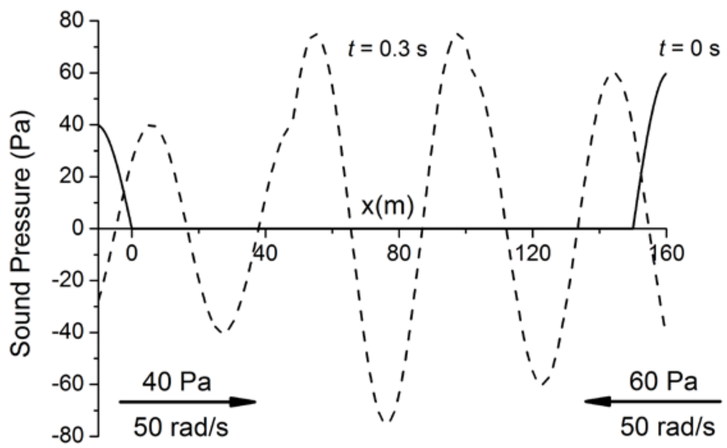
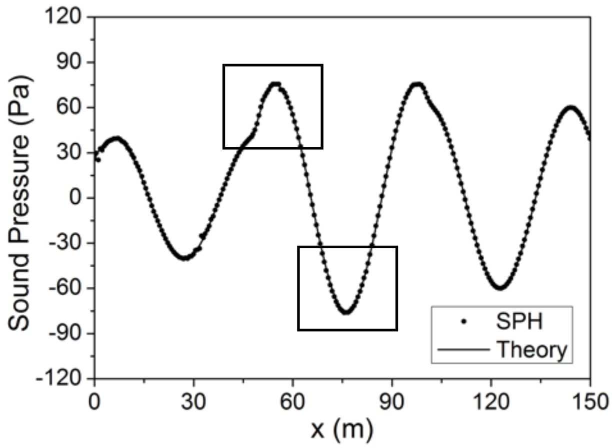


Figure 3 One-dimensional model of sound interference between two different sound waves

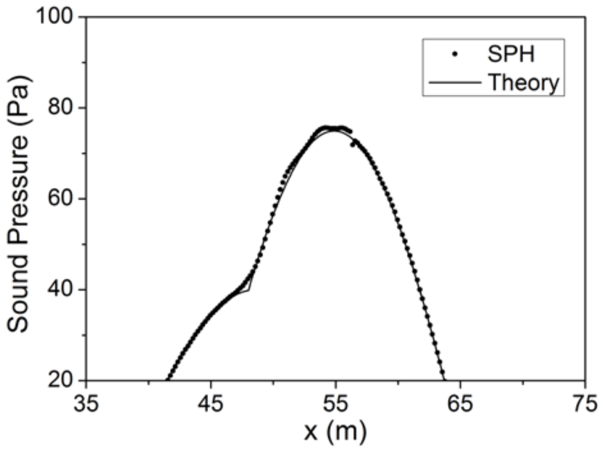
A sound wave with 40 Pa sound pressure and 50 rad/s circular frequency transmits from the left side while another sound wave with 60 Pa and 50 rad/s comes from the right side. After 0.3 s, the sound pressure along the x axis is shown in dash line in Figure 3.

SPH Simulation

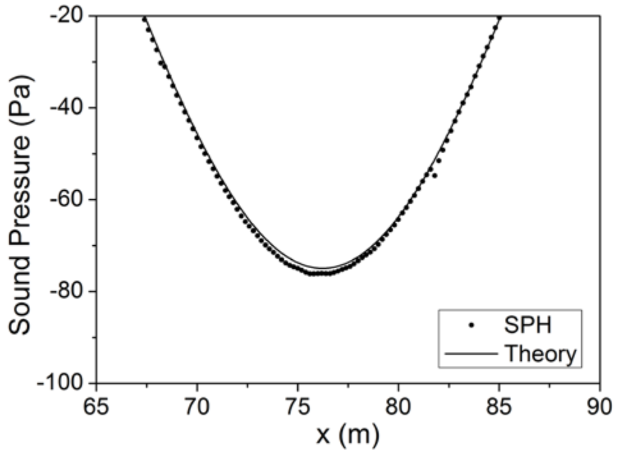
The simulation results and theoretical solution of sound pressure at the time t = 0.3 s are shown in Figure 4 (a). Two detail views of a peak and a valley are given in Figure 4 (b) and (c).



(a) sound pressure at t = 0.3 s



(b) detail view 1



(c) detail view 2

Figure 4

Considering the computational time is 0.3 s, the interference happens at $48 < x < 102$ m. It can be seen from the line figure that the SPH simulation results agree well with the theoretical solution. As shown in the detail views, the standard SPH method can compute sound pressure accurate comparing with the theoretical results.

Conclusions

Linearized acoustic equations are solved with the standard SPH theory and the simulation method is built by adding the time integration and neighbor particles searching method. One dimensional sound propagation and interference models are simulated with the SPH method and computational results are compared with theoretical data. Sound pressure results show that the standard SPH method can achieve accurate solution, although unphysical oscillations cannot be ignored.

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