A reliability optimization allocation method considering differentiation of

functions

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Abstract

The reliability optimization has achieved great concern in recent years. Nowadays, many researchers obtain allocation results which can maximize the system reliability subject to the system budget. In these researches, the effect of system's functions is always neglected or only considering the single main function of system. In addition, there are also not obvious evidences in results to distinguish the importance level of different units. However, complex systems tend to perform multiple functions. What's more, the use frequency of each function and the combinations of units to realize different function is not the same. In addition, the use demand of different function is decided by different task environment, the demand differentiation of functions has led to the use frequency of various functions should have different levels about reliability. Therefore, the reliability optimization allocation that only considering cost constraint conditions is not accuracy and will results in disaccord between the obtained results with actual situation.

Focusing on the problem mentioned above, a reliability optimization allocation method that considering cost constraint and importance factor is proposed. In this paper we consider systems consisting of units characterized by different reliability and importance factors. Such systems are multi-function because they must perform different tasks depending on the combination of units. Different functions may be work simultaneously. Firstly, the concept of importance factor is defined to describe the importance of a unit and the required importance factor level of system functions in the task is also given. To deal with the differentiation of system functions, the corresponding bound about importance factor are executed when looking for the optimal solution. Similarly, the cost constraint is also forced. Finally, in order to reduce the randomness of intelligent algorithm, a number of optimization are conducted and a rule is proposed to select the most optimal solution from all the optimal solutions which are obtained in every iterative loop.

Example of the comprehensive transmission equipment is presented. To begin with, we establish the reliability function of system as the objective optimization function. Then, the restraint of budget and different demands of importance factor of system functions are posed. Furthermore, using a genetic algorithm as the optimization tool, the optimization result can be obtained. Finally, the most optimal solution is selected. The results show that the method we proposed is more correct and more approximate the reality.

The reliability optimization allocation method presented in this paper can not only consider the constraint of cost, but also can consider the diversities of functions, and thus the optimization results will more approximate actual situation. At the same time, this paper can also provide guidance for the similar reliability optimization problem.

Keywords: reliability optimization, differentiation, importance factor, system function, genetic algorithm

Introduction

Reliability is an important index of a system, and system reliability allocation is prerequisite for reliability design and analysis of system. The reliability design of system must be done under several constraints such as cost, volume, weight etc. In engineering, some researchers often make reliability allocation which can maximize the reliability of system under the condition of budget constraint. The early researchers also study the method of reliability allocation for all kinds of system structure, such as series, parallel, series-parallel etc., and the related reliability allocations subject to the budget of system are done under the condition that units of a system are independent of each other: [Chen (1992); Bueno (2005); Levitin and Lisnianski (1999); Ramirez-marquez and Coit (2004)]. In these research work for reliability allocation, people often only consider one single main function of the system and ignore other functions. That may lead to a great error for system reliability allocation. G. Levitin etc. conduct the research about the reliability allocation of multistate system, and they make a conclusion that all units in the system must be involved in to realize any state of the system while the reliability of system is allocated to all the units: [Levitin and Lisnianski (2003); Levitin (2003); Levitin et al. (2003); Levitin and Lisnianski (2001)]. However, with the development of science and technology, the structure of the system is more and more complex and the system that has multiple main functions is also more and more popular. In these complex systems, the usage frequency of each function and the combination of units to realize different functions are not the same. In addition, different use demand of each system function is determined by different environment of task, and the differentiation of use demand of each system function results in that the reliability level of various functions should be different. But the existing research results cannot solve the reliability optimization allocation problem of systems with differentiation of functions. Meanwhile, as the complexity of reliability model of system and the complex relation between reliability index and constraints, it is difficult to obtain the solution for the reliability optimization allocation problem. A large number of researches can prove that applying artificial intelligence methods (such as neural networks, ant colony algorithm, genetic algorithm, hybrid genetic algorithm, simulated annealing algorithm, etc.) into reliability optimization allocation can achieve good effect: [Altiparmak et al. (2003); Nabil and Mustapha (2005); Chen and You (2005)]. Way Kuo made a good overview about the related content of reliability allocation before 2006: [Kuo and Rin (2007)]. Nevertheless, the artificial intelligence methods mentioned above also have their disadvantages, such as: premature phenomena may occur in the course of using, swinging near optimal solution when approaching optimal solution, slow convergence and easy to fall into local extremum, or optimization result is not ideal, and so on. Therefore, it is meaningful to improve the basic algorithm so that it is applicable for specific problems and can obtain the optimal solution efficiently.

In order to solve the above problem, a reliability optimization allocation method for systems with differentiation of functions is proposed in this paper. Firstly, a constraint considering the differentiation of functions is presented, namely a new index named importance factor is used to measure the usage frequency of each function and the use demand of each function determined by different environment of task. Then, the objective function of optimal allocation of system reliability is built according to the new cost model proposed in this paper, and further put forward the description of reliability optimization allocation of systems with differentiation of functions. In addition, the operation of code, evolution and selection in the genetic algorithm is also transformed to improve the accuracy and efficiency of solving the optimization problem. Finally, the proposed method is applied into the reliability optimization allocation of an integrated transmission device

whose goal is to minimize the cost of system. The result shows that the obtained solution is more in line with the actual situation.

A reliability optimization allocation method considering differentiation of functions

Assumption that a system that consists of m units can execute n functions.

Establishment of objective function

The reliability allocation is usually done according to the requirement of task. Not only the final reliability of system must meet a certain standard R_s , but also the allocation results must minimize the cost of system. In fact, reliability and cost are often nonlinear relationship. A cost model is presented under the assumption that their relationship submits to exponential distribution, it is shown as Eq. (1).

$$c_{i}\left(P_{i}, R_{i}, R_{i,\min}\right) = P_{i}e^{\left(\frac{R_{i}}{R_{i,\min}}-1\right)}$$

$$\tag{1}$$

Where, $c_i(\cdot)$ represents the cost model of i th unit, P_i is the basic price of i th unit, R_i is the reliability of i th unit, $R_{i,\min}$ is the lower limit value of reliability of i th unit. The total cost of system is the objective function of reliability optimization allocation, and its mathematical form is shown as Eq. (2).

$$C_{S}(R) = \sum_{i=1}^{m} c_{i} \left(P_{i}, R_{i}, R_{i, \min} \right)$$

$$\tag{2}$$

Where, $C_s(\cdot)$ is the total cost function of system.

Constraint considering the differentiation of functions

To deal with the differentiation of functions in the process of reliability allocation, the more reasonable approach is to convert it into a constraint condition and to select an optimal solution in the optimization process. As to it, a new index named importance factor is proposed. Obviously, the units in a system not only have reliability index, but also have the parameter of importance factor. The importance factor is used to measure the importance of a unit or its usage frequency. The importance factor is related to the reliability of the unit. Generally, the higher usage frequency of a unit, the greater importance of the unit and the higher reliability allocated to the unit. Therefore, according to the positive correlation between importance factor and reliability, a model is proposed based on the experience in engineering application, shown as in Eq.(3).

$$g_i(R_i) = e^{(R_i - 1)} \quad 0 < g < 1$$
 (3)

Where, $g_i(\cdot)$ is the importance factor of i th unit.

When importance factor is applied to deal with the differentiation of functions, if the importance factor of j th function cannot reach the scheduled request, as shown as in Eq.(4).

$$F_{f_i}\left(G_{f_i}, W_{f_i}\right) < 0 \tag{4}$$

The Eq. (4) indicates the allocated result is disqualified, and new iteration has to be done to make it meet the requirement, as shown as in Eq.(5).

$$F_{f_i}\left(G_{f_i}, W_{f_i}\right) \ge 0 \tag{5}$$

Where, F_{f_j} is a function to determine whether j th function can be realized or not, G_{f_j} is the importance factor of j th function, W_{f_j} is the predetermined level of importance factor of j th function.

Because a function of system is implemented by combination of units and importance factor reflects the important degree of each unit, the calculation method of importance factor of each function is shown as in Eq. (6).

$$G_{f_j} = \prod_{i=1}^{l} g_i(R_i) \quad k \in [1, l] \quad l \le m$$

$$(6)$$

Where, l is the number of units involved in j th function.

Compare G_{f_j} with W_{f_j} , the state of j th function can be determined. When a system has several functions, only all functions can be realized successfully does a system succeed. In other words, the importance factor of each function should satisfy the requirement of importance factor of itself, shown as in Eq. (7).

$$F_{f_i} \ge 0 \quad j \in [1, n] \tag{7}$$

Problem formulation

Combining the objective function and constraint conditions mentioned above, the reliability optimization allocation problem with differentiation of functions can be formulated as in Eq. (8).

$$\min C_{S}(R) = \sum_{i=1}^{m} C_{i}(R_{i}) = \sum_{i=1}^{m} P_{i}e^{\left(\frac{R_{i}}{R_{i,\min}}-1\right)}$$

$$s.t.$$

$$R_{i,\min} \leq R_{i} \leq R_{i,\max}$$

$$\prod_{i=1}^{m} R_{i} \geq R_{s}$$

$$i = 1, 2, \dots, m$$

$$F_{f_{j}}(G_{f_{j}}, W_{f_{j}}) \geq 0 \quad j \in [1, n]$$

$$(8)$$

Where, R_s is the reliability goal of system which needs to be achieved.

The improvement of genetic algorithm

The concrete operation process of basic genetic algorithms is shown as in Fig. 1. The operation of code, evolution and selection in the genetic algorithm are also transformed to improve the accuracy and efficiency for solving the optimization problem.

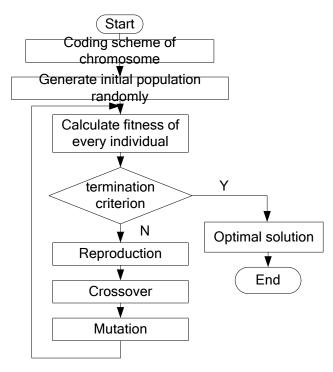


Figure 1. Operation process of basic genetic algorithm

1. Coding scheme

The real number coding is used for its several advantages compared with binary coding, such as: unnecessary to be transformed from the chromosome to performance value which can improve the efficiency of the genetic algorithm, reduces the requirements on memory and the use of different genetic operator is very free and so on.

2. Initial population

The characteristics of the initial population have important influences on the calculated results and efficiency of calculation. To achieve the global optimal solution, initial population should be dispersed in the solution space.

In order to ensure the uniform distribution in the solution space and get good initial population, homogeneous array can be constructed artificially. Firstly, the solution space is divided into A subspaces. Then, B chromosomes are produced in each subspace based on the predetermined or random method. Finally, C chromosomes whose fitness is the largest are selected from all these chromosomes.

3. Design of fitness function

Fitness indicates that the ability of an individual to adapt to the environment, and how to select fitness function depends on different situations. The objective function is taken as the fitness function in this paper and the best individual adapt to the environment has the largest value of fitness, which equal to the minimum of objective function. In addition, in order to guarantee that individuals produced randomly are within the above established constraints and remove the unsuitable individuals in the process of searching, penalty strategy is adopted. For the unsuitable individual, the fitness of it is set to be zero. The formulation of penalty function is shown as in Eq. (9).

$$\begin{cases} fit(\alpha) = 0 & F_{f_j} < 0 \quad or \quad \prod_{i=1}^{m} R_i < R_s \\ fit(\alpha) = 0.5 + 2 \cdot \frac{S - \alpha}{S - 1} & F_{f_j} \ge 0 \quad and \quad \prod_{i=1}^{m} R_i \ge R_s \end{cases}$$
 (9)

Where, $fit(\cdot)$ is the function of fitness, S is the size number of population, α is the ranking in the descending order of reliability.

4. Reproduction

The value of fitness is used to judge whether the chromosome is good or not. The larger the individual's fitness is, the larger the chance of being selected is. Fitness assignment based on rank and the strategy of roulette are adopted to select the excellent individuals in this paper.

5. Crossover operation

Crossover operation is the main genetic operator in genetic algorithm, which is acting on the individuals selected for reproduction. In this paper, the discrete restructuring and randomly selecting parent individuals with the same probability are selected as the crossover operation to determine the value of offspring individuals.

6. Mutation

Selection and crossover basically complete most work of the search function of genetic algorithm, and the mutation can enhance the ability of genetic algorithm to find the optimal solution. The mutation operation in this paper is that an individual is selected randomly among population and the value of a character in the gene string is changed randomly with a certain probability. Variation can increase the diversity of population, and adaptive mutation is operated to make the individual whose fitness is larger to do searching in the small scope while the individual whose fitness is smaller to do searching in the wide scope.

7. Termination criterion

Because genetic algorithm optimization can be unlimited evolution, and it is often difficult to have a big improvement after a certain degree of evolution, so a specific number of iteration need to be predetermined as the standard to decide whether the optimization process will go on or not. The certain generation of evolution is selected as the optimization criterion in this paper.

8. Optimization of multi-population

Because selection operation may make the population evolution to be controlled by superior individual, and crossover operation might destroy the good genes, so the basic genetic algorithm has the problem of premature convergence. Therefore, the optimization of multi-population is carried out to improve the performance of genetic algorithm.

Genetic algorithm of multi-population allows the child population to evolve along different directions, and the outstanding individuals in the solution space of different area can be obtained. It also can prevent the occurrence of premature convergence from searching in expanding scope. On

the other hand, with the migration and spread of excellent individual among different populations, the convergence rate and precision of solution can be improved.

Different optimal solutions can be obtained through optimization of multi-population. According to the actual situation, we can formulate some suitable guidelines to select the final optimal solution from these different optimal solutions.

Reliability optimization allocation process for systems with differentiation of functions under the goal of minimizing the cost of systems

For systems with differentiation of functions, the reliability optimization allocation to minimize the cost of systems can be done according to Fig. 2.

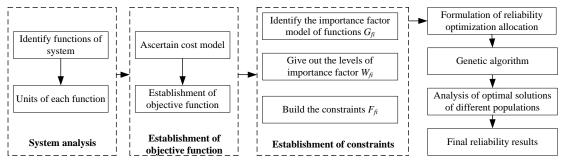


Figure 2. Reliability optimization allocation process for systems with differentiation of functions under the goal of minimizing the cost of systems

Example

The analysis of an integrated transmission device

There are 19 subsystems in an integrated transmission device, and they are 1-overall gearing, 2-body parts, 3-middle bracket, 4-torque converter assembly, 5-transmission assembly, 6-hydraulic torque converter, 7-planet before shift gear, 8-auxiliary drive, 9-hydraulic gear reducer, 10-hydraulic retarder control valves, 11-left side cover, 12-right side cover, 13-fan drive assembly, 14-liquid viscous clutch assembly, 15-oil pump group, 16-couplet of pump motor, 17-oil supply system, 18-hydraulic control system and 19-manipulation of the electronic control system. The integrated transmission device needs to implement five functions: straight driving, swerve, braking, fan cooling and compressor drive, and the reliability requirement of system is 0.7. The relationship among system functions and subsystems is shown as in Fig. 3.

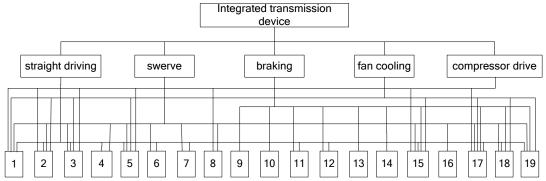


Figure 3. Relationship among system functions and subsystems

Establishment of the objective function

Compared with the data of engineering, the lower limit of reliability of each subsystem is all equal to 0.8, the upper limit of reliability of each subsystem is all equal to 0.9999, and the basic price of each subsystem is shown as in Table 1.

Table 1. The basic price of each subsystem

No	1	2	3	1	5	6	7	8	9
P	320	170	60	200	230	260	80	80	240
10	11	12	13	14	15	16	17	18	19
110	220	220	120	240	540	420	90	270	210

According to Eq. (2, 3), the objective function of integrated transmission device can be established as the sum of cost of each subsystem.

$$C_{S}(R) = \sum_{i=1}^{19} P_{i} e^{\left(\frac{R_{i}}{R_{i,\min}} - 1\right)}$$

Establishment of constraint condition considering differentiation of functions

Only the five functions of integrated transmission device all meet the requirements in terms of importance factor, the system can be denoted as success. The values of W_f about five functions are shown in Table 2.

Table 2. The values of W_f about five functions

No.	straight driving	swerve	braking	fan cooling	compressor drive
W_f	0.52	0.39	0.32	0.2	0.35

Naturally, the constraint conditions can be obtained as follows.

$$\begin{cases} R_{i,\min} \le R_i \le R_{i,\max} \\ \prod_{i=1}^m R_i \ge 0.7 \end{cases} i = 1, 2, \dots, 19$$
$$\begin{cases} F_{f_j} \left(G_{f_j}, W_{f_j} \right) \ge 0 & j \in [1, 5] \end{cases}$$

Problem formulation of reliability optimization allocation of integrated transmission device

Reliability is allocated for each subsystem under the constraint conditions of reliability index of system with differentiation of functions, so as to make the cost of system to be minimum.

$$\min C_{S}(R) = \sum_{i=1}^{19} C_{i}(R_{i}) = \sum_{i=1}^{19} P_{i}e^{\left(\frac{R_{i}}{R_{i,\min}}-1\right)}$$

$$\begin{cases} s.t. \\ R_{i,\min} \le R_i \le R_{i,\max} \\ \prod_{i=1}^{19} R_i \ge 0.7 \end{cases} \quad i = 1, 2, \dots, 19$$

$$\begin{cases} F_{f_j} \left(G_{f_j}, W_{f_j} \right) \ge 0 & j \in [1, 5] \end{cases}$$

Genetic algorithm and selection of results of optimization allocation

The suitable parameters of genetic algorithm are determined and shown as in Table 3.

Table 3. Parameter of genetic algorithm										
Size numbe	r Probability of	Probability of	Maximum							
of population	n crossover	mutation	generation							
120	0.9	0.01	500							

The reliability optimization allocation of integrated transmission device is done according to the reliability optimization allocation process in section 2.5. The optimization allocation process with a single population is shown as in Fig. 4.

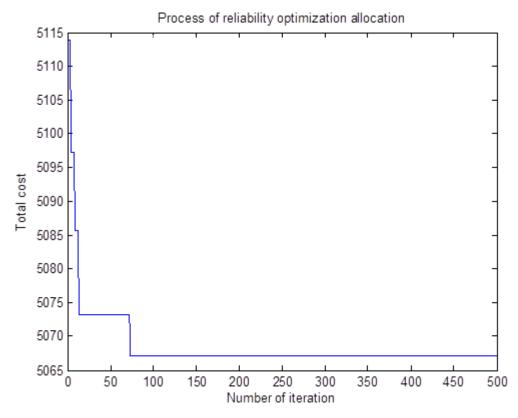


Figure 4. Process of reliability optimization allocation with a single population

At last, the additional 20 populations are selected to do the reliability optimization allocation of the integrated transmission device, and the results are shown in Table 4, β_i indicates the i th population. The results are shown in Table 4. The obtained reliability of each subsystem is ranked every time. Each line represents a set of optimal solutions, and the number of subsystem whose

reliability is the lowest is set at far left, and the number of subsystem whose reliability is the highest is set at far right. In other words, the degree of reliability value increase gradually from left to right.

Table 4 The rank of reliability of subsystem

	Degree of reliability (decrease gradually)																		
β	_	_	_	_	_		_								_	_		-	_
,-	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D ₁₄	D ₁₅	D_{16}	D ₁₇	D_{18}	D_{19}
1	14	15	6	16	1	12	5	18	19	11	9	4	2	13	17	10	8	3	7
2	16	15	1	9	19	18	5	13	6	10	4	12	7	17	3	14	8	2	11
3	1	16	15	18	14	12	2	6	9	5	19	4	7	8	17	13	3	10	11
4	15	16	1	11	18	14	12	9	4	19	6	8	10	5	7	2	17	13	3
5	16	15	14	1	9	6	18	12	5	4	19	2	11	17	10	13	7	8	3
6	15	1	16	18	6	11	12	9	14	10	19	4	5	13	2	8	3	17	7
7	15	18	16	6	9	11	1	5	19	2	14	12	4	13	10	8	7	17	3
8	15	1	11	6	18	16	12	5	19	4	9	14	10	3	7	13	17	2	8
9	16	9	15	1	6	14	18	12	11	4	5	2	19	17	8	10	7	13	3
10	6	15	16	1	5	14	9	11	18	19	3	2	4	13	8	10	7	17	12
11	16	15	1	6	4	9	18	14	12	11	13	2	5	10	19	17	7	3	8
12	16	15	12	11	1	6	9	14	4	18	5	7	2	10	13	17	19	8	3
13	15	9	16	1	5	11	14	6	12	4	19	18	17	10	13	2	3	7	8
14	1	15	18	5	16	9	2	19	6	14	12	11	7	13	10	17	8	3	4
15	16	12	15	18	1	11	19	6	14	9	5	4	2	7	8	10	17	13	3
16	15	1	14	16	6	9	18	11	5	4	12	10	13	19	7	2	8	3	17
17	15	6	14	16	1	18	5	9	19	11	17	12	13	4	10	7	3	8	2
18	15	1	16	14	6	9	18	5	11	12	19	4	13	8	2	10	3	7	17
19	16	6	15	18	1	14	12	9	11	5	19	4	13	17	2	10	3	7	8
20	5	15	16	1	12	19	18	6	11	4	9	7	2	3	14	8	17	10	13

In addition, the total costs with different populations are shown in Table 5.

Table 5. Total costs with different populations

NO.	1	2	3	4	5	6	7	8	9	10
Total cost	5073.9	5070.4	5075.5	5067.2	5078.4	5077.5	5077.2	5072.5	5076.3	5074.8
NO.	11	12	13	14	15	16	17	18	19	20
Total cost	5062.9	5076.7	5078.4	5078.6	5066.4	5077.9	5078.3	5069.1	5077.7	5076.0

The improved genetic algorithm is applied in the reliability optimization allocation of the integrated transmission device. We can see from Fig. 4 that the total cost of system become stable after iterating 75 times and the optimal solution is also obtained. What's more, Table 5 shows that the total costs of different population are very close. This result illustrates the correctness and accuracy of the algorithm proposed in this paper. Observation can be seen from Table 4, the reliabilities assigned to the 15th and 16th subsystems are smaller, and reliabilities assigned to the 3rd, 7th, 17th subsystems are bigger. This shows that the method proposed in this paper can well deal with the issue of the reliability optimization allocation of complex system with differentiation of functions. Based on the correct reliability allocation method, a better and more correct optimal solution can be obtained through several times of reliability optimization allocation, and the randomness is also can be reduced. At last, the rule for selecting the final solution for the reliability optimization allocation can be formulated with reference of the allocation results of different population.

In this paper, the allocation results of the population that its cost of system is less and smaller reliabilities are assigned to the 15th, 16th subsystems and the larger reliabilities are assigned to the 3rd, 7th, 17th subsystems, will be selected to be the final solution. From Table 4 and Table 5, we can see that the allocation value of 19th population can be determined as the final solution for the integrated transmission device. The corresponding cost of system is 5062.9 and the reliability of each subsystem is shown in Table 6.

Table 6. Final reliability of each subsystem

No.	1	2	3	4	5	6	7	8	9	10
Reliability	0.9841	0.9972	0.9995	0.9621	0.9911	0.9658	0.9740	0.9610	0.9832	0.9776
No.	11	12	13	14	15	16	17	18	19	
Reliability	0.9890	0.9630	0.9692	0.9885	0.9745	0.9745	0.9889	0.9807	0.9795	

Conclusion

In this paper a reliability optimization allocation method for systems with differentiation of functions is proposed. Firstly, a cost model is presented, in which the meaning and value of its parameters are easy to ascertain. What's more, the index of importance factor is proposed to deal with the differentiation of functions. Finally, the system model of reliability optimization allocation is built and the improved genetic algorithm is used to solve the problem of system reliability optimization allocation under the target of minimum total cost.

In order to verify the validity and accuracy of the proposed method in this paper, the reliability optimization allocation of an integrated transmission device is done. The results show that the proposed method can solve the problem of differentiation of functions, and the allocated results also indicate the differentiation of reliability requirements. This method has better guidance than traditional reliability allocation method and it is easy to be applied in engineering.

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