A highly sensitive mass sensor using high-mode resonant cantilever with step change in thickness

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Abstract

Resonant cantilever mass sensors (RCS) can quantitatively detect the unknown analyte by measuring the frequency shift induced by attached the mass of analyte. With the characteristics of high sensitivity, long term stability and easy interfacing with digital signal processing, such sensors have been successfully applied in various application fields such as proteomics, genomics, gas sensing, food contamination, cancer detecting, chemical or fluidic detection. Recently, a great deal of attention has been paid on developing high sensitivity sensors that can be used outside of laboratory environments. According to the operating principle of the resonant sensor, the mass sensitivity is mainly depending on the ratio of the resonant frequency at mode number *n* to the corresponding effective mass of the resonant cantilever. Simultaneously increasing the resonance frequency and reducing the effective mass can effectively improve the mass detection sensitivity. As reported in many references, reducing geometry dimensions to increase the natural frequency by MEMS or NEMS technology is an effective way to improve sensitivity and spatial resolution of resonance mass sensors. Due to the limitation of the equipment for measuring the nano-sized cantilever deflection, the geometrical size reduction method encounters a bottleneck in improving the sensitivity. Different from the method of reducing the cantilever dimension, a new sensitivity improving method was proposed to inspire high-order mode by optimizing the stiffness and mass distribution of the cantilever. Through theoretically analyzing the relationship among structural parameters, vibration modes and the sensitivity, a novel piezoelectric resonant mass sensor working in the fourth order vibration mode is designed and fabricated. The experimental sensitivity of the proposed sensor is 188.41×10^4 Hz/g nearly 19.5 times greater than that of the custom rectangular cantilever sensor of 9.85×10^4 Hz/g. The simulated sensitivity is nearly consistent with that of the experiment with the deviation of 1.28%. Meanwhile, the quality factor is 82.65, which is about 3.5 times as great as that of the rectangular uniform cantilever sensor with the same size, hence, validates the feasibility and effectiveness of the newly proposed sensitivity improving method.

Key words: mass sensor, cantilever, high mode resonance, sensitivity, quality factor

1 Introduction

Resonant cantilever mass sensors (RCS) can quantitatively detect the unknown analyte by measuring the induced frequency shift by itself. With the characteristics of high sensitivity, long term stability and easy interfacing with digital signal processing, such sensors have been successfully applied in various application fields such as proteomics [Liu et al.(2007); Ekinci and Roukes(2005); Frederic et al.(2005)], genomics, gas sensing, food contamination, cancer detecting, chemical or fluidic detection [Murali et al.(2007); Margarita et al.(2009);Degertekin et al.(2001)].

Recently, a great deal of attention has been paid on developing high sensitivity sensors that can be used outside of laboratory environments. According to the operating principle of the resonant sensor, the mass sensitivity is mainly depending on the ratio of the resonant frequency at mode number n to the corresponding effective mass of the resonant cantilever, which can be indicated by

$$S_n = \frac{\Delta f_n}{\Delta m^e} \cong \frac{f_n}{2M_n^e} \qquad (1)$$

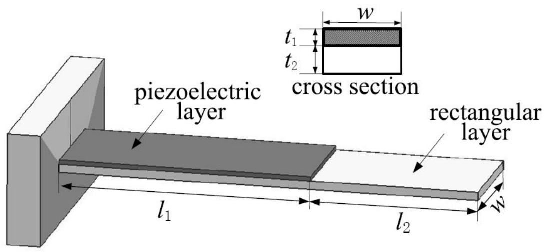
In Eq. (1), Δf_n is the frequency shift for per unit mass change of Δm^e, f_n is the *n*th flexural-mode resonance frequency for a cantilever of effective mass M_n^e. From equation (1), it can be seen that simultaneously increasing the resonance frequency and reducing the effective mass can effectively improve the mass detection sensitivity. As reported in many references, reducing geometry dimensions to increase the natural frequency by MEMS or NEMS technology is an effective way to improve sensitivity and spatial resolution of resonance mass sensors[Shen et al.(2006; 2007); Kumar et al.(2010)]. Due to the limitation of the equipment for measuring the nano-sized cantilever deflection, the geometrical size reduction method encounters a bottleneck in improving the sensitivity. In other words, for nano-sized cantilevers, the high sensitivity is achieved at the expense of complex supplementary equipments for accurate detecting and low anti-jamming in vacuum. Alternatively, another method of using high-order resonant modes has been introduced to improve sensitivity and quality factor[Lochon et al.(2005); Maraldo and Mutharasan (2010)]. Unfortunately, as the most often used method to achieve high order mode vibration, the geometric size reduction method will directly reduce the cantilever surface area and make it difficult to manipulate the particles in the detecting system, Actually, the cantilever profile can also affects the vibration modes of the uniform cantilevers by means of changing the cantilever profile as well as by changing the effective stiffness and the mass distribution. Intuitively speaking, the cantilever profiles can be optimized to lightweight in the manner of reducing the stiffness by a smaller factor, thus resulting in higher resonance frequencies than that of the uniform cantilevers.

In this paper, different from the methods of reducing the cantilever dimension, a new sensitivity improving method was proposed to inspire high-order mode by optimizing the resonant cantilever with step change in thickness of the mass sensor. By theoretically analyzing the relationship among structural parameters, vibration modes and the sensitivity, a novel piezoelectric resonant mass sensor inspired with fourth order mode vibration is designed and fabricated. The experimental and simulated sensitivities of the proposed sensor are 188.41×10^4 Hz/g and 192.50×10^4 Hz/g respectively, which are nearly 19.50 times greater than that of the custom rectangular cantilever sensor of 9.85×10^4 Hz/g. Additionally, the simulation results of the sensitivity is nearly consistent with that of the experiments with the deviation within 1.28%. Meanwhile, the quality factor is 82.65, which is about 3.50 times as great as that of the rectangular uniform cantilever sensor with the same size, hence, validates the feasibility and effectiveness of the newly proposed sensitivity improving method. Furthermore, the influence of the length and thickness of the step change section of the resonant cantilever on the sensitivity is analyzed.

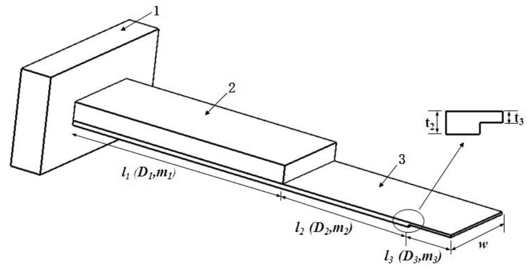
2 Structure of the high-order mode resonant mass sensor

Commonly, rectangular uniform cantilever is the most widely used spring element in designing resonant mass sensor, as shown in Fig.1(a), and the corresponding elastic constant is determined by the structural geometry and material properties. Actually, the sensitivity is deeply related to the cantilever resonance frequency and the deflection at the free end. Therefore, it is an effective way to reduce the thickness at the free end properly to change both the stiffness and mass distribution, as shown in Fig.1(b), in which, the non-piezoelectric extension is set to be the detection area, and the characters of l_1, l_2 and l_3 represent the lengths of the piezoelectric layer, and the cantilever and the step part , w is the width of the cantilever, t_1, t_2 and t_3 are the thicknesses of the piezoelectric layer and the cantilever and the step part.

In Fig.1(b), the cantilever can be divided into three distinguished parts with different cross sections and non-uniform lengths of l_1, l_2 and l_3. The bending vibration equation for constant cross-section beam based on Euler–Bernoulli beam theory is established as follows.



(a) Rectangular section cantilever sensor



(b) high-order mode resonant cantilever sensor with step change in thickness of the cantilever

Fig.1 Comparison between two kinds of cantilever sensors

$$D_i \frac{d^4 Y(x)}{dx^4} - \omega^2 m_i Y(x) = 0 \tag{2}$$

In Eq. (2), D_i and m_i represent the bending modulus and mass per unit length of each part, respectively. ω is the angular frequency, $Y(x)$ is the displacement amplitude.In Eq. (3), D_1 and m_1 are the bending modulus and mass per unit length in section 1, D_2 and m_2 in section 2, and D_3 and m_3 in section 3.Then, the natural frequencies of the cantilever are obtained as

$$f = \frac{k_{1n}^2}{2\pi}\sqrt{\frac{D_1}{m_1}} = \frac{k_{2n'}^2}{2\pi}\sqrt{\frac{D_2}{m_2}} = \frac{k_{3n''}^2}{2\pi}\sqrt{\frac{D_3}{m_3}} \tag{3}$$

where, k_{1n}, $k_{2n'}$ and $k_{3n''}$ are the wave numbers of the n th, n' th and n'' th order mode in section 1, section 2, and section 3.

When single piezoelectric layer is utilized, the effective stiffness and mass of the three lengths (D_1 , m_1 , D_2 , m_2 , D_3 , m_4) can be expressed as Eq. (4) to Eq. (9).

$$D_1 = \frac{E_1^2 t_1^4 + E_2^2 t_2^4 + 2E_1 E_2 t_1 t_2 \left(2t_1^2 + 2t_2^2 + 3t_1 t_2\right)}{12\left(E_1 t_1 + E_2 t_2\right)} \tag{4}$$

$$m_1 = \rho_1 t_1 w + \rho_2 t_2 w \tag{5}$$

$$D_2 = \frac{E_2 t_2^3}{12} \tag{6}$$

$$m_2 = \rho_2 t_2 w \tag{7}$$

$$D_3 = \frac{E_3 t_3^3}{12} \tag{8}$$

$$m_3 = \rho_3 t_3 w \tag{9}$$

where, ρ_1 , ρ_2 and ρ_3 are the mass densities of each section, w is the width.

If the added mass is a point mass loaded at the cantilever tip, the flexural-mode resonance frequency thus can be approximated in terms of Rayleigh Quotient.

$$f_n' = \frac{1}{2\pi}\sqrt{\frac{K_n^e}{M_n^e + \Delta m \cdot Y_{2n}^2\left(x'\right)}} \tag{10}$$

In Eq. (10), x' is the position of the added point mass, Y_{2n} is the nth mode shape, K_n^ε and M_n^ε are effective flexible stiffness and effective mass.

For $\Delta m << M_n^\varepsilon$, the resonance frequency shift Δf_n , due to the added point mass at the cantilever tip, is thus Eq. (11)

$$\Delta f_n = f_n - f_n^{'} = \frac{1}{2} f_n \frac{\Delta m \cdot Y_{2n}^2\left(x'\right)}{\sqrt{M_n^e\left(M_n^e + \Delta m \cdot Y_{2n}^2(x')\right)}} \cong \frac{1}{2} f_n \frac{\Delta m \cdot Y_{2n}^2\left(x'\right)}{M_n^e} \tag{11}$$

If the added mass Δm is distributed on the surface of the non piezoelectric extension part, the resonance frequency shift can be expressed as Eq. (12).

$$\Delta f_n = f_n - f_n^{'} \cong \frac{1}{2} f_n \frac{\Delta m^e}{M_n^e} \tag{12}$$

The detailed expression of the mass sensitivity of the proposed cantilever sensor can be obtained, which can be expressed as Eq. (13).

$$S = \frac{\Delta f}{\Delta m} = \frac{1}{4\pi w}\sqrt{\frac{D_1}{m_1}} \cdot \frac{k_1^2 Y_{1n}^2(x)}{m_1\int_0^{l_1} Y_{1n}^2(x)dx + m_2\int_{l_1}^{l_1+l_2} Y_{2n}^2(x)dx} \tag{13}$$

3 Simulation and Experiments

Through numerical simulation, the detection sensitivity for different vibration modes can be obtained in Fig.2. It can be seen that the sensitivity of the fourth order vibration mode is much higher than those of the lower order vibration modes. Then, the influences of the step parameters including step length and thickness on the sensitivity were analyzed, as shown in Fig.3 and Fig.4. The output sensitivity is varying with the step length nonlinearly while the thickness and the total length keep constant. When the length l_3 reaches 6.00×10^{-3} m, the sensitivity can achieve the highest value, as shown in Fig.3. Meanwhile, keeping the step length and the total cantilever thickness constant, the sensitivity also varies nonlinearly with the thickness ratio of the step part to the cantilever. In the thickness ratio range from 3.33×10^{-1} to 5.00, the sensitivity deceases with the increasing of the thickness ratio.

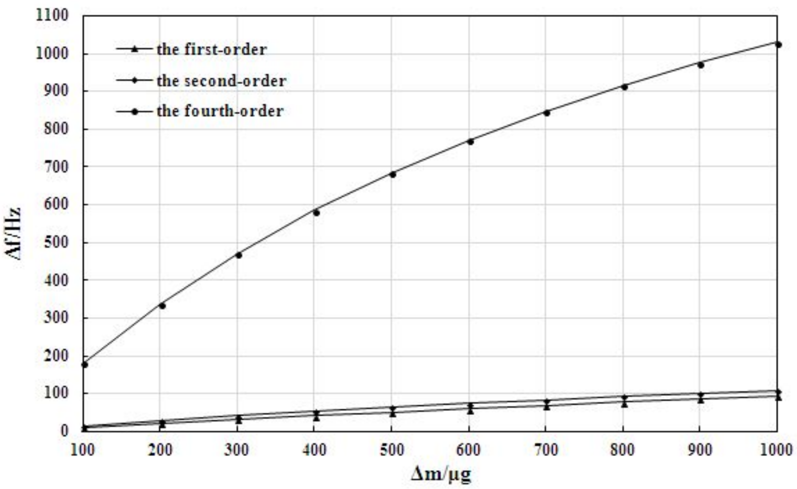


Fig.2 Sensitivity curve of each order with step change in thickness of the cantilever sensor

More importantly, to evaluate the sensitivity improvement, we fabricated a novel grooved cantilever type mass sensor by using electric discharge wire-cutting technology, as shown in Fig.5. The mass sensor mainly consists of a fixture and a cantilever which was made by bonding a PZT layer with 4.00×10^{-3} m width, 2.50×10^{-4} m thickness, 1.00×10^{-2} m length to a two-layer steel sheet with $t_2 = 6.00\times10^{-5}$ m, $l_2 = 1.62\times10^{-2}$ m , $t_3 = 3.00\times10^{-5}$ m, $l_3 = 1.80\times10^{-3}$ m using conductive epoxy. The experiment results in Fig.6 show that the sensitivity is about 188.41×10^4 Hz/g with a quality factor of 82.65, which is much higher than that of the rectangular uniform cantilever sensor. Also, the simulation result is nearly consistent with that of the experiments with a maximum deviation of 1.28%. Finally, the proposed design model for the high order modes resonant sensors is adequately validated.

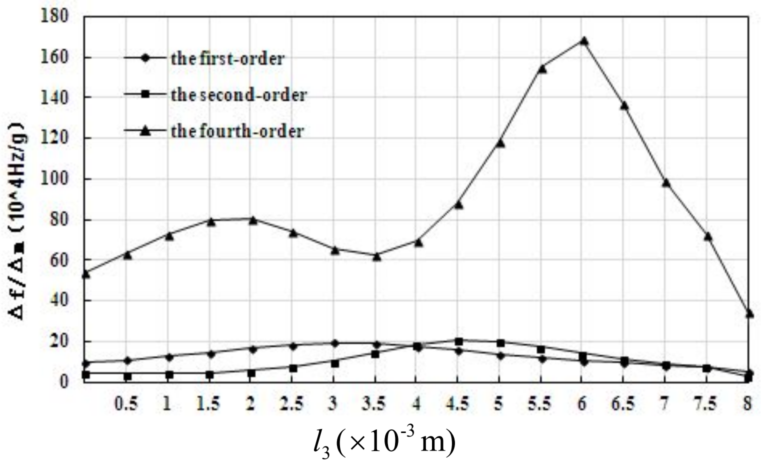


Fig.3 The influence of the length l_3 on the sensitivity of the sensor

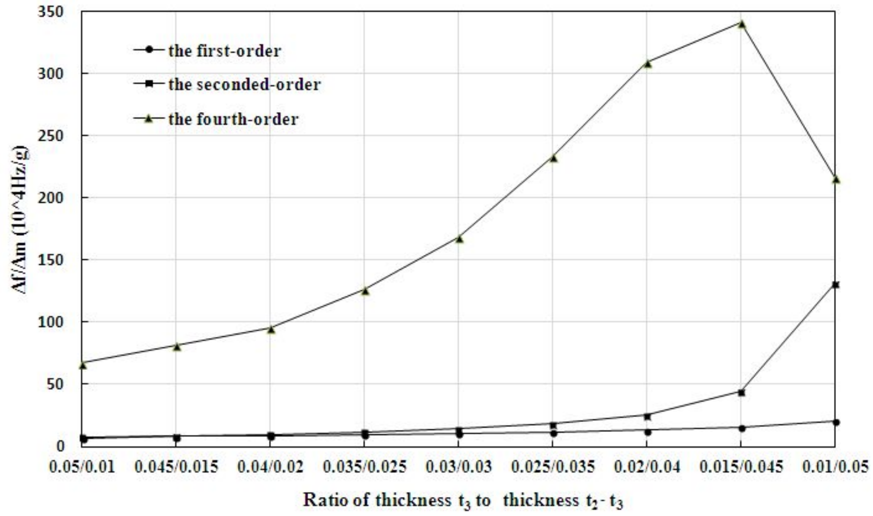


Fig.4 The influence of the ratio of thickness $t_3/(t_2-t_3)$ on the sensitivity

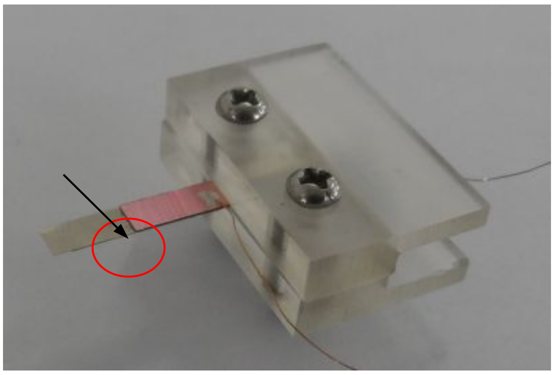


Fig.5 Non-uniform thickness sensor with large thickness ratio

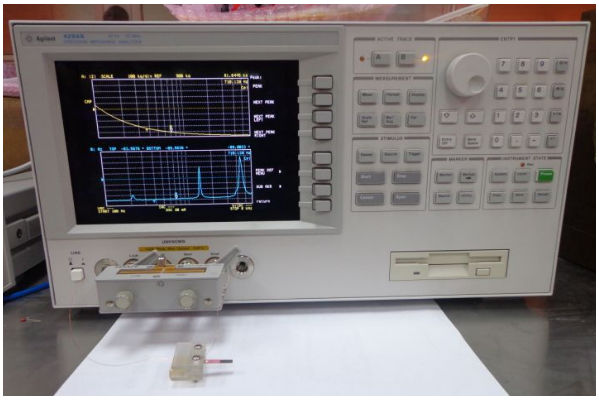


Fig.6 Test system for sensor sensitivity measurement

4 Conclusion

As the key spring element in the resonant mass sensor, a new cantilever profile has been proposed to inspire high-order mode by optimizing the mass and stiffness distribution at the free end. The analytical model was established for theoretically analyzing the relationship among structural parameters, vibration modes and the sensitivity. Through the linear search optimization, a novel piezoelectric resonant mass sensor inspired with fourth order mode vibration is designed and fabricated. The experimental and simulated sensitivities of the proposed sensor are 188.41×10^4 Hz/g and 192.50×10^4 Hz/g respectively, which are nearly 19.50 times greater than that of the custom rectangular cantilever sensor of 9.85×10^4 Hz/g. Additionally, the simulation results of the sensitivity is nearly consistent with that of the experiments with the deviation within 1.28%. Meanwhile, the quality factor is 82.65, which is about 3.50 times as great as that of the rectangular uniform cantilever sensor with the same size, hence, validates the feasibility and effectiveness of the newly proposed sensitivity improving method.

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